Disentanglement Dynamics of a Data Driven Quantum Neural Network

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ABSTRACT

This study examines the disentanglement evolution of a quantum neural network (QNN) model locally in contact with data environments. As a valuable resource, duration of entanglement in quantum systems is extremely significant. Therefore, the effect of various initial states to the generation or decay of entanglement has been investigated under pure and maximally mixed environmental states as two limit cases. Numerical results show that initial state preparation has a profound impact on the fate of entanglement even in the course of maximally noisy environments. The obtained results reveal that the decay of entanglement of the quantum neural network (with flip-flop type interaction) is affected by the initial preparation of the Bell state even in the presence of pure state environmental monitoring. Depending on the results it's also suggested to begin with coherent product states since it provides robust entanglement generation with longer disentanglement time during the open quantum system evolution.

Key Words: Quantum Entanglement, Quantum Neural Network, Central Spin Model

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Introduction

Networks composed of quantum matter are known as quantum networks, which are significant for quantum communication and distribution of quantum resources (Cirac *et al.*, 1997). Particularly, the quantum resources make the scheme appealing over classical communication tasks. Entanglement (Horodecki *et al.*, 2009) is one of the well-known and well-studied quantum resources with important applications such as secure communication and transfer of an unknown quantum data (Bennett *et al.*, 1993; Scarani *et al.*, 2009; Galindo A, Martín-Delgado MA, 2002). Despite these exciting applications, entanglement is fragile and preservation and distribution of entanglement for sufficiently long times is a challenge.

In this study, entanglement dynamics of a simple QNN is investigated. Quantum neural nets (Schuld, Petruccione and Sinayskiy, 2014) are special types of quantum networks which are the quantum versions of the artificial neural networks. As artificial neural networks are prominent components of artificial intelligence, quantum versions of these networks desire special interest due to the possibility of quantum artificial intelligence or possible advantages of QNNs (Banchi, Pancotti and Bose, 2016). This study focuses on the entanglement dynamics based on a small-scale quantum neural network architecture. Though entanglement dynamics are well studied and understood for open and closed quantum system dynamics in this small scale, the dynamical behaviour of quantum entanglement in a QNN architecture under particular assumptions needs to be reassessed due to the reasons explained above. Moreover, by the rapid progress of the current state-of-art, on demand generation of entanglement has been achieved faster than the entanglement decay (Humphreys *et al.*, 2018).

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Due to these advances, such as deterministic entanglement generation, the estimation of the dynamical behaviour of entanglement becomes essential for the particular architectures of the quantum neural networks.

More particularly, the current study deals with the numerical simulation of the dynamical evolution of an open quantum neural network with and without initial entanglement. The basic assumption of the manuscript is that the QNNs are open quantum systems and they exist in an information environment. Therefore the objective of the study is to investigate the quantum data driven entanglement dynamics of the QNNs. Open system dynamics modelled via repeated interactions (Ziman *et al*., 2002) between the local nodes of the network and the units representing the reservoir. It's been shown that this type of repeated interactions are the dynamical maps equivalent to the Markovian master equation approach due to the divisibility of quantum channels (Wolf and Cirac, 2008). We find that the entanglement dynamics are strongly related to initial state of the quantum network. Both asymptotic decay and the decay of entanglement in finite time was observed due to the variation of environmental states. The results of the numerical simulations could provide useful information in the course of the design and preparation of initial states aiming preservation or generation of entanglement on demand.

Model and system dynamics

QNN model we investigate is equivalent to the central spin model (also known as spin-star network) which has various applications in the decay of quantum coherence (Zurek, 2006) and quantum communication (Chen *et al*., 2006). Most recently, we have studied the dynamical evolution of the central spin quantum coherence of this model under Markov dynamics depending on different types of spin couplings (Türkpençe *et al*., 2018). It's also been shown that non-equilibrium nature of the model could have advantages for thermodynamic work harvesting in the quantum scale (Türkpençe *et al*., 2017). In this study, central spin model was investigated as a quantum version of neural networks inspired by biological models. Fig.1 summaries the basic ideas of the proposed model dynamics provided that the system is locally in contact with an environment contains single qubit quantum information. The nodes of the network are in contact with the central node which is the readout node via conventional flip-flop spin interaction. The time independent Hamiltonian representing the system dynamics reads

$$H = \frac{\omega}{2}\left(\sum_i^N \sigma_z^i + \sum_i^{N-1} \sigma_z^{u_i}\right) + \frac{J}{2}\left(\sum_i^{N-1} \sigma_+^1 \sigma_-^i + \sum_i^{N-1} \sigma_+^{u_i} \sigma_-^i + H.c.\right) \quad (1)$$

where σ_z, σ_+, σ_- are Pauli-z, -raising and –lowering operators respectively. Here, σ^1 is the Pauli operator representing the readout node, σ^i is the Pauli operator representing an individual node in contact with the information environment and σ^{u_i} is the Pauli operator of the individual unit representing the information environment. The Bohr frequency ω of each spin and single qubit information unit has been taken equal for simplicity. On the other hand, the couplings between the nodes of the spin-star system with the environment and the inter-nodes couplings J have also been assumed equal.

As stressed above, open system dynamics has been treated by adopting a repeated interaction process also known as a collisional model (Türkpençe, Akıncı, Şeker, 2017). Figure 1(b) presents the particular use of the collisional model for simulating the network's open dynamics. Here, the initially and identically prepared qubit states sequentially interacts with the nodes of the neural network. The dynamical evolution of the neural network would be obtained by tracing out the reservoir (environment) degrees of freedom as

$$\rho_s(t+\tau) = \mathrm{Tr}_{u_i}[U(\tau)\rho_s(t)\otimes\rho_{u_i}U(\tau)^\dagger] \quad (2)$$

where $U(\tau) = e^{-iH\tau}$ is the unitary operator representing the system plus environment dynamics for time independent Hamiltonians, τ is the duration of each unit-node interaction and Tr_{u_i} stands for partial trace over environmental degrees of freedom. Thus, the open quantum evolution has been simulated iteratively by discrete steps of quantum channels represented by Equation 2 and the calculations have been performed by exact diagonalization. Here, ρ_s represents the state of the system of interest (quantum neural network) and ρ_{u_i} represents the quantum state of each single qubit environment unit. In this work state vector and density matrix formalisms has been used interchangeably in order to represent the quantum states of the relevant systems.

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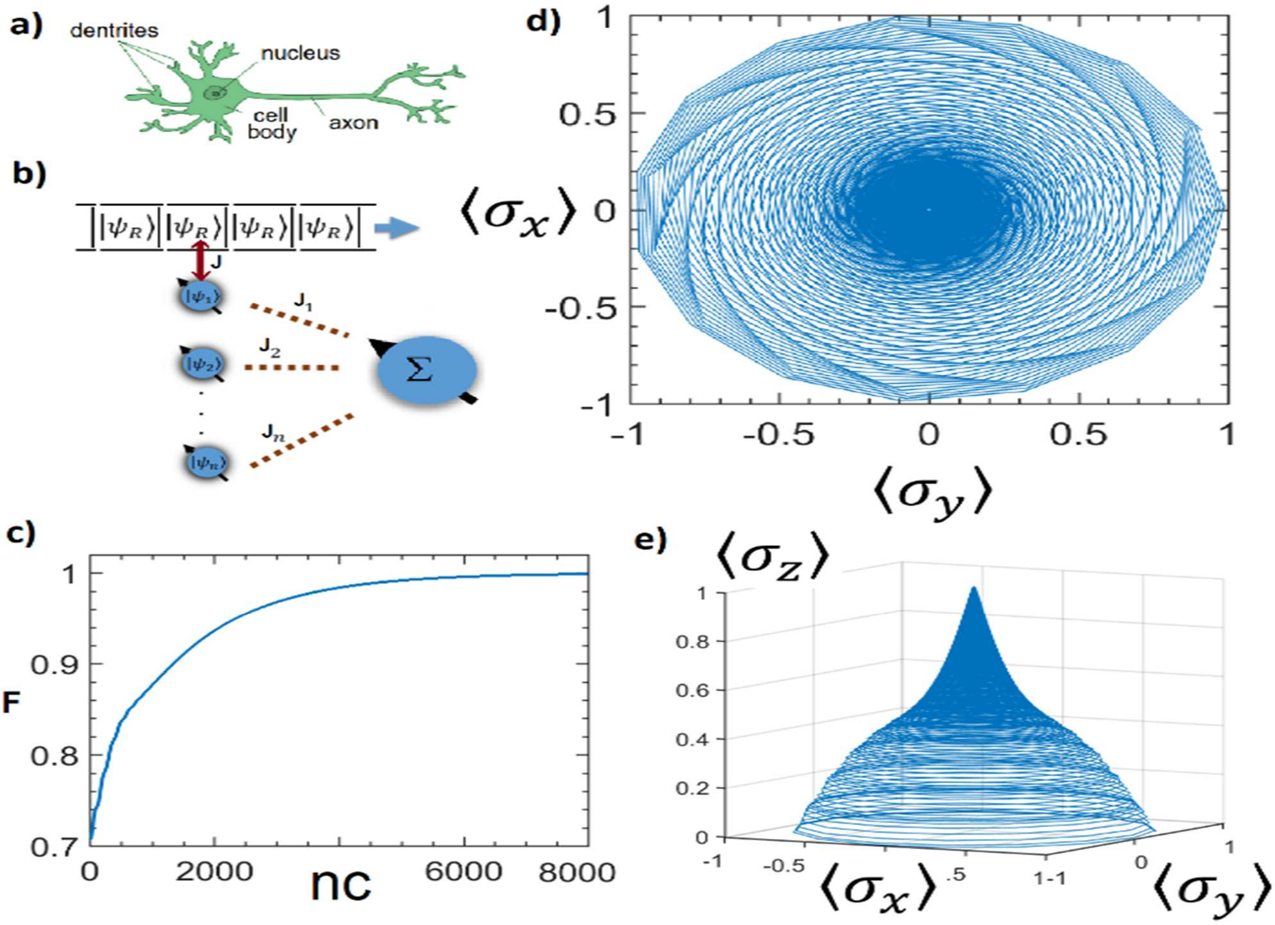

Figure 1. Visualization of time resolved data driven quantum neuron. (a) A biological neuron. (b) A quantum neuron in a central spin architecture. Repeated interactions between the nodes and identically prepared units model the open system dynamics. (c)-(d)-(e) Representations of the equilibration of the quantum neuron (for a single input node) by a data environment. (c) Fidelity between the state of the readout node of the quantum neuron and the spin up state of the environment. (d) Transverse plane of the Bloch ball during equilibration. (e) 3D visualisation of the Bloch sphere during the equilibration process. The coupling between the environment unit and the input node is equal to the coupling between the input and the readout node J=0.1. The duration of the each unit interaction τ between the units and the input node is τ = 5× 10^{-2}/J.

The objective of the paper is to analyse the fate of entanglement for various initial states. To this end, we investigate two cases. The first one is the case that there is no initial entanglement between the readout node and one of the input nodes in contact with the environment. Second, system starts from one of the maximally entangled Bell states. We define maximally entangled states in the generic form as $|\Phi^{\pm}\rangle = [|00\rangle \pm |11\rangle]/\sqrt{2}$ or $|\Psi^{\pm}\rangle = [|01\rangle \pm |10\rangle]/\sqrt{2}$ where $|0\rangle$ and $|1\rangle$ are orthogonal basis states. In this section, we apply some preliminary calculations in order to benchmark the system dynamics and the parameters for later calculations. As the Benchmark calculation, the system was assumed to have only one node therefore can be represented by a linear chain with two nodes. These nodes were initialized in cat states $|+\rangle = [|0\rangle \pm |1\rangle]/\sqrt{2}$ and the state of units representing the information reservoir initialized in $|0\rangle$ or spin up state. Therefore in this case, the initial system density matrix is $\rho_s(0) = |++\rangle\langle++|$ and the density matrix representing the units is $\rho_u = |0\rangle\langle 0|$.

After the system has been subjected to information reservoir, it's expected that environment will send information to the system, that is, the system will be equilibrated by the environment consist quantum information. This evolution could be monitored by using the conventional measure fidelity in a time dependent manner as

$$\mathcal{F}(t) = \mathrm{Tr}\sqrt{\sqrt{\rho_u}\rho_1(t)\sqrt{\rho_u}}\,. \tag{3}$$

Here, $\rho_1(t)$ is the quantum state of the readout node at some instant and ρ_u is the quantum state of the reservoir units as target state. As explicit in Figure 1 (c) time evolution of fidelity of the readout node with the reservoir state converges to unit fidelity. Here, initial state of the readout node has been driven from $|+\rangle$ towards $|0\rangle$ which is the state of the reservoir. This is the most straightforward way to present the equilibration dynamics of the system with the subjected reservoir. On the other hand, Bloch sphere representation of the readout qubit evolution was illustrated for 2 and 3-dimensions in Figures 1 (d) and (e) respectively. Particularly in Figure 1 (e) evolution from transverse plane (coherent state) towards $|0\rangle\langle 0|$ state is explicit. These have been plotted by using expectation values as $\langle\sigma_\nu(t)\rangle = \mathrm{Tr}[\rho_1(t)\sigma_\nu]$ with $\nu : x, y, z$. These preliminary results show that the system evolution faithfully represents the open quantum system dynamics with this collisional model. The reason to pick the collisional model for open system dynamics is that the flexibility to choose the quantum state of the reservoir.

Results

In this section, time resolved entanglement dynamics of the discussed neural network has been investigated in the course of an information environment. Information environment has been represented by $|0\rangle\langle 0|$ or $|1\rangle\langle 1|$ that are single qubit pure states. Also $\rho_{wn} = \frac{1}{2}\mathbb{1}$ was taken into account to represent the environmental state. Here, $\mathbb{1}$ is 2×2 identity matrix and ρ_{wn} is a maximally mixed state and stands for environmental white noise (Luo, 2018). In this study concurrence was adopted as the entanglement measure and calculated as $C = \max[0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4]$ where λ_i are the square roots of non-Hermitian matrix $\rho\tilde{\rho}$ in the descending order. Here, ρ is the density matrix to be calculated and $\tilde{\rho}$ is its spin flipped form such as $\tilde{\rho} = (\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y)$ where ρ^* is the complex conjugated form of ρ.

Figure 2 represents the evolution of concurrence as a function of time in the presence or in the absence of the initial entanglement. The QNN again has only one node and the initial states (including environment plus system) were, respectively examined with $|\Theta\rangle = |\mathbf{0}\rangle \otimes |0\rangle \otimes |1\rangle$, $|\mathbf{0}\rangle \otimes |\Phi^+\rangle$, $|\mathbf{0}\rangle \otimes |\Psi^+\rangle$. Here, the state depicted by bold stands for the environmental state and the remaining states represent the interacting nodes. In the first case there is no initial entanglement as all the nodes are initially separable. As in Figure 2 (a) nodes become entangled during the time evolution. In this case, concurrence has highly oscillatory behaviour during its decay. On the other hand, in the second and third cases of Figure 2 (a), one can observe that the decay of initially entangled states depict an exponential behaviour. Moreover, due to our results, one observes different decay constants for different types of Bell states during this exponential decay. Exponential entanglement decay versus entanglement decay in finite time in the presence of spontaneous emission was reported elsewhere (Yu T and Eberly JH, 2004).

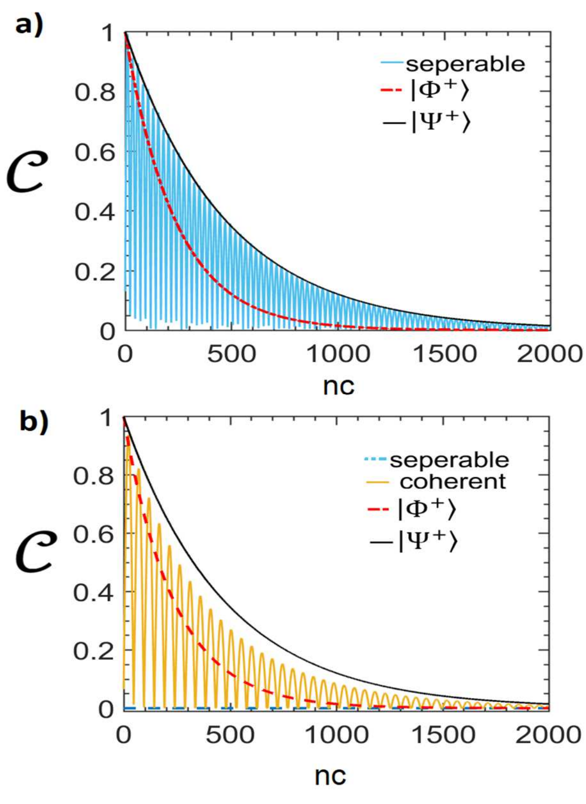

Figure 2. Evolution of concurrence for different initial states of the quantum neuron depending on the number of collisions (nc). (a) The quantum state of the quantum neuron initialized as respectively, $|10\rangle$ separable states and maximally entangled $|\Phi^+\rangle$, and $|\Psi^+\rangle$ Bell states. (b) The quantum state of the quantum neuron initialized as respectively, separable $|00\rangle$, coherent $|++\rangle$ and $|\Psi^+\rangle$ states. The state of the units representing the environment was set as $|0\rangle$ for both cases. The coupling between the environment unit and the input node is equal to the coupling between the input and the readout node J=0.1. The duration of the each unit interaction τ between the units and the input node is $\tau = 5\times 10^{-2}/\mathrm{J}$.

We report disentanglement mechanisms in the presence of information reservoir in which the states are in pure computational basis states. Additionally we report that different types of Bell states have different relaxation times for the same environment and parameters i.e. the Bell state $|\Psi^+\rangle$, which is performing better in entanglement lifetime. Moreover, its exponential decay curve acts as if the envelope of oscillatory separable case. When the initial states of the system has been replaced by $|00\rangle$ as in Figure 2 (b) for separable case, no entanglement generation observed during the evolution process. In addition, also the evolution of entanglement has been examined for $|++\rangle$ as the initial state of the system. Again an oscillatory decay behaviour is apparent in this coherent product state. But this time decay is faster than that of the separable initial $|00\rangle$ state case.

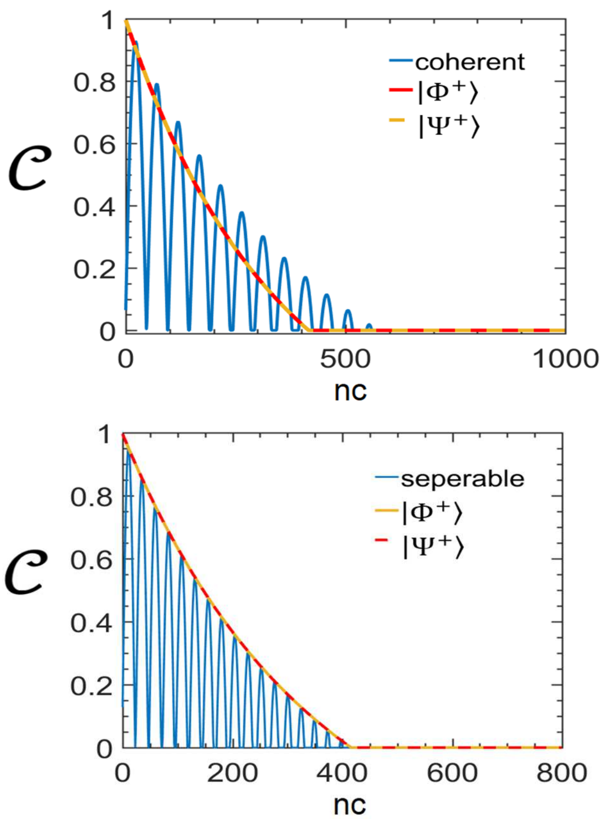

Figure 3. Evolution of concurrence for different initial states of the quantum neuron depending on the number of collisions (nc). (a) The quantum state of the quantum neuron initialized as respectively, coherent $|++\rangle$, $|\Phi^+\rangle$ and $|\Psi^+\rangle$ Bell states. (b) The quantum state of the quantum neuron initialized as respectively, $|01\rangle$ separable, $|\Phi^+\rangle$ and $|\Psi^+\rangle$ Bell states. The state of the units representing the environment was fixed as a maximally mixed state. The coupling between the environment unit and the input node is equal to the coupling between the input and the readout node J=0.1. The duration of the each unit interaction τ between the units and the input node is $\tau = 5\times 10^{-2}$/J.

The maximally entangled Bell states $|\Phi^+\rangle$ and $|\Psi^+\rangle$ have also been plotted again in Figure 2 (b) in order to compare with the previous initial states. These results reflect the strong dependence of entanglement generation to initial states of the system with the nearest neighbour flip-flop interaction.

So far, only pure states were addressed as environmental states. Though this is reasonable for quantum data environments, always there exist noises that deteriorate the purity of quantum states. In order to analyse the entanglement dynamics for more realistic situations, the following calculations have been performed under maximally mixed single qubit units representing the environmental states. Figure 3 is an example of these calculations in which the results dramatically change compared that of the pure state data environment case. Generation of entanglement for the separable initial node states $|01\rangle$ and $|++\rangle$ were compared in Figures 3 (a) and (b). First in this case, it's obvious that finite time entanglement decay comes out which is also known as entanglement sudden death (Al-Qasimi, 2008). Although, finite disentanglement time referred to as temperature-dependent thermal reservoirs we choose the maximally mixed qubit reservoir equivalent to temperature $T = \infty$. One reason we also choose the maximally mixed state as the environment is that the lower limit disentanglement time could be calculated with this type of environment which is referred to as a white noise (Luo, 2018). Secondly, the entanglement behaviour of various initial states in this limit becomes important in terms of experiment design parameters for deterministic entanglement generation. Both, in Figures 3 (a) and (b) there is sudden decrease in the disentanglement time during the evolution in the presence of white noise. Beyond this expected result, what is interesting is that the disentanglement time difference between the initially entangled $|\Phi^+\rangle$, $|\Psi^+\rangle$ states no longer exists. As clear in Figures 3 (a) and (b) their time dependent evolution overlap. The overlap line is the envelope of the oscillatory concurrence function with initially $|01\rangle$ separable state. On the other hand, separable coherent $|++\rangle$ initial state induced entanglement generation evolution performs better than the initial $|01\rangle$ state. Note that no entanglement generation occurs starting from $|00\rangle$ state. Therefore, we report coherent $|+\rangle$ (cat) states as high-performance initial states under noise dominant environmental conditions for entanglement based quantum neuro computing. Disentanglement evolution in the presence of pure and maximally mixed environments are two limit cases of the problem. Our neural network with two and more than two modes in contact with various types of environments yields results between these two limit cases.

Conclusions

This brief study examines the dynamical evolution of entanglement in a simple data driven quantum neural network with a flip-flop type interaction. Quantum data environment was modelled as two limit cases, one with pure states and the other one with maximally mixed states in which no information exist. The open quantum dynamics treated by a collisional model which is available for implementing different environmental states. The study also examines the generation of entanglement due to initially separable states. According to the results obtained by the numerical calculations, dynamical evolution of disentanglement varies dramatically with respect to the initial states. The results show that initially entangled $|\Psi\rangle$ state has longer lifetime than $|\Phi\rangle$ state in a minimum noisy data environment. On the other hand, it's reported that this finding is not valid with highly noisy environments. In contrast to general expectations, it's found that in highly noisy environments decay of entanglement generated due to coherent $|+..+\rangle$ product states performs better than initially prepared entangled states.

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References

Al-Qasimi A, James DF. Sudden death of entanglement at finite temperature. Physical Review A 2008; 77(1): 012117.

Banchi L, Pancotti N, Bose S. Quantum gate learning in qubit networks: Toffoli gate without time-dependent control. NPJ Quantum Information 2016; 2: 16019.

Bennett CH, Brassard G, Crépeau C, Jozsa R, Peres A, Wootters WK. Teleporting an unknown quantum state via dual classical and Einstein-Podolsky-Rosen channels. Physical review Letters 1993; 70(13): 1895-99.

Chen Q, Cheng J, Wang KL, Du J. Optimal quantum cloning via spin networks. Physical Review A 2006; 74(3): 034303.

Cirac JI, Zoller P, Kimble HJ, Mabuchi H. Quantum state transfer and entanglement distribution among distant nodes in a quantum network. Physical Review Letters 1997; 78(16): 3221.

Galindo A, Martin-Delgado MA. Information and computation: Classical and quantum aspects. Reviews of Modern Physics 2002; 74(2): 347.

Horodecki R, Horodecki P, Horodecki M, Horodecki K. Quantum entanglement. Reviews of Modern Physics 2009; 81(2): 865.

Humphreys PC, Kalb N, Morits JP, Schouten RN, Vermeulen RF, Twitchen DJ, Markham M, Hanson R. Deterministic delivery of remote entanglement on a quantum network. Nature 2018; 558(7709): 268.

Luo MX. Typical Werner states satisfying all linear Bell inequalities with dichotomic measurements. Physical Review A 2018; 97(4): 042301.

Scarani V, Bechmann-Pasquinucci H, Cerf NJ, Dušek M, Lütkenhaus N, Peev M. The security of practical quantum key distribution. Reviews of Modern Physics 2009; 81(3): 1301.

Schuld M, Sinayskiy I, Petruccione F. The quest for a quantum neural network. Quantum Information Processing 2014; 13(11): 2567-86.

Türkpençe D, Akıncı TÇ, Şeker S. Decoherence in a quantum neural network. NeuroQuantology 2018; 16(6): 1-5.

Türkpençe D, Akıncı TÇ, Şeker S. Quantum neural networks driven by information reservoir. arXiv preprint arXiv:1709.03276. 2017.

Türkpençe D, Altintas F, Paternostro M, Müstecaplioğlu ÖE. A photonic Carnot engine powered by a spin-star network. EPL (Europhysics Letters) 2017; 117(5): 50002.

Wolf MM, Cirac JI. Dividing quantum channels. Communications in Mathematical Physics 2008; 279(1): 147-68.

Yu T, Eberly JH. Finite-time disentanglement via spontaneous emission. Physical Review Letters 2004; 93(14): 140404.

Ziman M, Štelmachovič P, Bužek V, Hillery M, Scarani V, Gisin N. Diluting quantum information: An analysis of information transfer in system-reservoir interactions. Physical Review A 2002; 65(4): 042105.

Zurek WH. Decoherence, einselection, and the quantum origins of the classical. Reviews of Modern Physics 2003; 75(3): 715.