Do Global Virtual Axionic Gravitons Exist?

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ABSTRACT

Recently, the author has proposed the global one-dimensionality conjecture which, throughout making use of the method of the primary canonical quantization of General Relativity according to the Dirac-Arnowitt-Deser-Misner Hamiltonian formulation, formally avoids the functional differential nature which is the basic obstacle of quantum geometrodynamics, a model of quantum gravity based on the Wheeler-DeWitt equation and resutling from quantization of the Einstein field equations. The emergent quantum gravity model is straightforwardly integrable in terms of the Riemann-Lebesgue integral, has an unambiguous physical interpretation within quantum theory, and introduces a consistent concept of global graviton. In this article, the axionic nature of the global gravitons, that is the hypothetical particles responsible for gravitation in the global one-dimensional quantum gravity model, is shortly described and, making use of fundamental methods of quantum field theory, the virtual states of such gravitons are generated.

Key Words: global one-dimensionality conjecture; quantum gravity; quantum geometrodynamics; graviton; axion

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1. Introduction

Recently the Wheeler-DeWitt equation was considered in the framework of the global one-dimensionality conjecture (Glinka, 2012; 2010). Recall that this conjecture formally reduces the functional differential nature of the Wheeler-DeWitt quantum geometrodynamics throughout the primary canonical quantization of General Relativity in the Dirac-Arnowitt-Deser-Misner Hamiltonian formulation (Glinka, 2012; 2010).

Let us sketch the state of affairs. The reduction follows from a simple and well-known Jacobi's formula connecting the variations of a metric and its determinant, and results in replacement of the functional evolution parameter which is the metric of space by a variable which is the metric determinant. The ramifications of this change are highly non-trivial. Namely, the resulting model of quantum gravity significantly differs from the Wheeler-DeWitt equation, because has a manifestly well-defined mathematical consistency defined in terms of classical integrals and, therefore, is straightforwardly integrable. Moreover, the model has both a clear and unambiguous physical interpretation within the formalism given by quantum field theory and, consequently, establishes a new concept of graviton. This “global graviton” arises from a certain specific global optimization of the Wheeler-DeWitt quantum geometrodynamics and, for this reason, its physical existence is much more fundamental than the quanta of gravitation defined by the standard quantum geometrodynamics based on the Wheeler-DeWitt equation, as well as more likely than the gravitons understood in terms of the Yang-Mills theories, which constitute the leading reasoning line of modern particle

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physics (Cf. relevant references discussed in Glinka 2012; 2010).

In this short article, the axionic nature of the global graviton is consistently described throughout the global one-dimensional version of the quantum geometrodynamics and, making use of certain fundamental methods of quantum field theory, the existence of the non-trivial states of such global gravitons, which we call the virtual states of global gravitons, are deduced.

2. Global Quantum Gravity

Let us focus our attention on the global one-dimensional quantum gravity, recently investigated in (Glinka 2012; 2010), in the version appropriate for any Lorentzian higher-dimensional space-time which is strictly analogous to the case of 3+1 dimensional space-time. Making use of certain standard classical in conventions, one can show that in such a situation the quantum gravity is described by the one-dimensional Klein-Gordon equation:

$$\left(\frac{\delta^2}{\delta h^2}+\omega^2\right)\Psi=0, \qquad (1)$$

where ω^2 is the following parameter

$$\omega^2=-\frac{{}^{(D)}R-2\Lambda-2\kappa\rho}{2\kappa^2D^2(D-2)}\frac{1}{h}, \qquad (2)$$

where ${}^{(D)}R$ is the Ricci scalar curvature of the D-dimensional space embedded in the $D+1$ space-time, Λ is the cosmological constant, $\rho=T_{\mu\nu}n^\mu n^\nu$ is the energy density of Matter fields created by the double projection of the stress-energy tensor of Matter fields $T_{\mu\nu}$ onto the unit normal vector n^μ,

$$\kappa=\frac{8\pi G}{c^4}$$

is the Einstein constant, and $h=\det h_{ij}$ is the determinant of a metric of space h_{ij} defined by the D+1 decomposition of a space-time metric

$$g_{\mu\nu}=\begin{bmatrix}-N^2+N_iN^i & N_j\\ N_i & h_{ij}\end{bmatrix}. \qquad (3)$$

The model (2) makes sense if and only if for the dimension of space D is different from 0 and 2.

Let us notice that, in general, axions can be understood as "axial" particles, that is the particle-like solutions to an arbitrary one-dimensional quantum evolution equation. The single dimension is then considered as a global affine parameter of the quantum evolution which plays the role of either time variable or single space dimension. Interestingly, such property gives automatically a supersymmetric nature to axions. The quantum theory (1) is manifestly a one-dimensional quantum mechanics, where the evolution parameter is the determinant h of a metric of space. According to the investigations (Glinka 2010; 2012), this parameter is the global dimension of the model, and, consequently, the gravitons understood as the particle-like solutions to the evolution equation (1), have axionic nature. The one-dimensional nature gives naturally inherited supersymmetry in such a model of quantum gravity, what means that the global gravitons are new kind of supersymmetric axionic particles.

In the context of quantum field theory, which is the mathematical basis of particle physics, the model of quantum gravity (1) is the Klein-Gordon equation. Its solutions have the physical sense of particles if and only if the gravitational frequency is constant, that is $\omega=const$. In such a situation, a geometry of the D -dimensional space is determined through the following Ricci scalar curvature

$${}^{(D)}R=2\Lambda+2\kappa\rho-2\kappa^2D^2(D-2)\omega^2h, \qquad (4)$$

in which the corrections due to the frequency effects can be treated as small in the light of the fact that $\kappa^2\sim10^{-86}N^{-2}$. In such a situation, the particle solutions to the quantum gravity (1) can be constructed by making use of the Fourier transforms and the secondary quantization in the Fock space.

3. Axionic Graviton

The particle-like solution of the quantum theory (1) can be constructed straightforwardly in terms of the annihilation and creation operators $\mathbb{G}$ and $\mathbb{G}^\dagger$ which constitute the appropriate Fock space

$$\left[\mathbb{G}(h),\mathbb{G}^\dagger(h)\right]=1, \tag{5}$$

$$\left[\mathbb{G}^\dagger(h),\mathbb{G}^\dagger(h)\right]=0, \tag{6}$$

$$\left[\mathbb{G}(h),\mathbb{G}(h)\right]=0, \tag{7}$$

or, in other words, by the secondary quantization of the wave function Ψ

$$\Psi(h)=\frac{\mathbb{G}(h)+\mathbb{G}^\dagger(h)}{\sqrt{2\omega}}. \tag{8}$$

This is the global one-dimensional quantum gravity in the case of constant ω, which is the quantum field theory, that is the quantum theory of the global graviton.

It is easy to see that in the case of a constant value of ω the quantum theory (1) has the unique general solution in the form of a pattern quantum superposition

$$\Psi(h)=e^{i\omega(h-h_0)}A+e^{-i\omega(h-h_0)}B, \tag{9}$$

where A and B are the constants of integration, and h_0 is a certain initial value of the global dimension. Writing

$$A=\frac{\mathbb{G}_0}{\sqrt{2\omega}}, \tag{10}$$

$$B=\frac{\mathbb{G}_0^\dagger}{\sqrt{2\omega}}, \tag{11}$$

where $\mathbb{G}_0$ and $\mathbb{G}_0^\dagger$ are the initial values of the creation and annihilation operators, one obtains the dynamical Fock basis which spans the Hilbert space in the model of quantum gravity

$$\mathbb{G}(h)=e^{i\omega(h-h_0)}\mathbb{G}_0, \tag{12}$$

$$\mathbb{G}^\dagger(h)=e^{-i\omega(h-h_0)}\mathbb{G}_0^\dagger, \tag{13}$$

and, therefore, the Heisenberg-type equations of motion hold

$$\frac{\delta\mathbb{G}}{\delta h}=i\omega\mathbb{G}, \tag{14}$$

$$\frac{\delta\mathbb{G}^\dagger}{\delta h}=-i\omega\mathbb{G}^\dagger, \tag{15}$$

and the initial data satisfy the relations

$$\left[\mathbb{G}_0,\mathbb{G}_0^\dagger\right]=1, \tag{16}$$

$$\left[\mathbb{G}_0^\dagger,\mathbb{G}_0^\dagger\right]=0, \tag{17}$$

$$\left[\mathbb{G}_0,\mathbb{G}_0\right]=0. \tag{18}$$

Consequently, the graviton which is the axion has the form

$$\Psi(h)=\frac{1}{\sqrt{2\omega}}\left(e^{i\omega(h-h_0)}\mathbb{G}_0+e^{-i\omega(h-h_0)}\mathbb{G}_0^\dagger\right), \tag{19}$$

and, for this reason, the relations hold

$$\Psi^\dagger(h)=\Psi(h), \tag{20}$$

$$\left[\Psi(h),\Psi^\dagger(h')\right]=i\frac{\sin\omega(h-h')}{\omega}, \tag{21}$$

$$\left[\Psi(h),\Psi^\dagger(h)\right]=0. \tag{22}$$

One sees also that the conjugate momentum field

$$\Pi_\Psi(h)=\frac{\delta\Psi(h)}{\delta h}=i\sqrt{\frac{\omega}{2}}\left(e^{i\omega(h-h_0)}\mathbb{G}_0-e^{-i\omega(h-h_0)}\mathbb{G}_0^\dagger\right) \tag{23}$$

$$=i\sqrt{\frac{\omega}{2}}\left(\mathbb{G}(h)-\mathbb{G}^\dagger(h)\right), \tag{24}$$

has the following properties

$$(\Pi_\Psi(h))^\dagger=\Pi_\Psi(h), \tag{25}$$

$$\left[\Pi_\Psi(h),\Psi(h')\right]=i\cos\omega(h-h'), \tag{26}$$

$$\left[\Pi_\Psi(h),\Pi_\Psi^\dagger(h')\right]=i\omega\sin\omega(h-h'). \tag{27}$$

4. Virtual States

The equation (21) can be used for generation of the quadratic equations

$$(e^{i\omega(h-h_0)})^2 - 2\omega\left[\Psi(h),\Psi^\dagger(h_0)\right]e^{i\omega(h-h_0)} - 1 = 0, \quad (28)$$

$$(e^{-i\omega(h-h_0)})^2 + 2\omega\left[\Psi(h),\Psi^\dagger(h_0)\right]e^{-i\omega(h-h_0)} - 1 = 0, \quad (29)$$

which can be solved immediately

$$e^{i\omega(h-h_0)} = \omega\left[\Psi(h),\Psi^\dagger(h_0)\right] \pm \sqrt{1+\omega^2\left(\left[\Psi(h),\Psi^\dagger(h_0)\right]\right)^2}, \quad (30)$$

$$e^{-i\omega(h-h_0)} = -\omega\left[\Psi(h),\Psi^\dagger(h_0)\right] \pm \sqrt{1+\omega^2\left(\left[\Psi(h),\Psi^\dagger(h_0)\right]\right)^2}, \quad (31)$$

and, consequently, the graviton solution (23) possesses four nonequivalent quantum states which by making use of the definitions

$$\Psi^0 = \frac{1}{\sqrt{2\omega}}\left(\mathbb{G}_0 + \mathbb{G}_0^\dagger\right) = \Psi(h_0), \quad (32)$$

$$\Pi_\Psi^0 = i\sqrt{\frac{\omega}{2}}\left(\mathbb{G}_0 - \mathbb{G}_0^\dagger\right) = \Pi_\Psi(h_0), \quad (33)$$

can be written in the following form

$$\Psi_{1,2}^a(h) = -i\Pi_\Psi^0\left[\Psi(h),\Psi^\dagger(h_0)\right] \pm \Psi_0\sqrt{1+\omega^2\left(\left[\Psi(h),\Psi^\dagger(h_0)\right]\right)^2}, \quad (34)$$

$$\Psi_{3,4}^b(h) = -i\Pi_\Psi^0[\left[\Psi(h),\Psi^\dagger(h_0)\right] \pm \frac{1}{\omega}\sqrt{1+\omega^2\left(\left[\Psi(h),\Psi^\dagger(h_0)\right]\right)^2}]. \quad (35)$$

Now, by taking into account the formula (21), one can present these states in the following form

$$\Psi_{1,2}^a(h) = \pm\Psi^0\cos\omega(h-h_0) + \frac{\Pi_\Psi^0}{\omega}\sin\omega(h-h_0), \quad (36)$$

$$\Psi_{3,4}^b(h) = \mp i\frac{\Pi_\Psi^0}{\omega}e^{\pm i\omega(h-h_0)}. \quad (37)$$

In this manner, this is clear that $\Psi_{1,2}^a(h)$ and $\Psi_{3,4}^b(h)$ are independently existing quantum states of the same graviton. Moreover, one can easily see that

$$\left[\Psi_1^a(h),(\Psi_1^a(h'))^\dagger\right] = i\frac{\sin\omega(h-h')}{\omega}, \quad (38)$$

$$\left[\Psi_2^a(h),(\Psi_2^a(h'))^\dagger\right] = -i\frac{\sin\omega(h-h')}{\omega}, \quad (39)$$

$$\left[\Psi_3^b(h),(\Psi_3^b(h'))^\dagger\right] = 0, \quad (40)$$

$$\left[\Psi_4^b(h),(\Psi_4^b(h'))^\dagger\right] = 0, \quad (41)$$

and, consequently, the only state $\Psi_1^a(h)$ agrees with the commutation relation (21), while the remaining states $\Psi_2^a(h)$, $\Psi_3^b(h)$, $\Psi_4^b(h)$ do not satisfy this relation. This suggests that the quantum states $\Psi_2^a(h)$, $\Psi_3^b(h)$, $\Psi_4^b(h)$ of the graviton are virtual, while the state $\Psi_1^a(h)$ is physical.

5. Summary

The global one-dimensional quantum gravity (Glinka 2012; 2010), in itself is a consistent quantum theory of gravitons defined as the axionic particles carrying gravitation in a global way. On the background of modern models of quantum gravity, such like the attempts offered by gauge field theories, loop quantum gravity, string theory, and other much more advanced mathematical physics for presently technologically available experimental verification (Cf. discussion in Glinka 2012; 2010), the existence of global gravitons is a unique potentially verifiable physical situation.

In this paper, constructive application of the fundamental methods of quantum field theory, such celebrated in particle physics, straightforwardly demonstrated that the unique hypothetical features of the global gravitons create an inherent and coherent scenario for existence of their virtual quantum states. The virtual gravitons could be responsible for the specific particle-like gravitational interactions rather than measurable inertial effects analogous to the astronomical phenomena which give the support to validity of the Mach principle on the macroscopic scales.

However, this question needs further clarification due to both the phenomenology and experiments of the joined best effort of observational astronomy, high energy physics, particle astrophysics, and space science. Nevertheless, looking from the present-day theoretical point of view, the model reasoning presented in this paper allows to make use of the hypothetically existing virtual axionic particle-like global gravitons in order to search, detection, and description of the microscopic gravitational waves, possibly at high or even ultra-high energetic regions.

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