On Some Explanations and Analysis of the Basic Foundations of Quantum Cognition: Comments on a paper by Pothos, Busemeyer and Trueblood

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ABSTRACT

The aim of such paper is to explain in detail the foundations of quantum cognition and to evidence the manner in which our pioneering experiments on quantum interference in humans should be correctly interpreted in the framework of a correct quantum mechanical elaboration. Often, studies on quantum cognition are elaborated from some authors evidencing that presently we have empirical evidence that the application of quantum rules gives better results respect to classical ones. Consequently, such studies are delineated as empirical tentative. It is not true. Quantum cognition is a scientific realization structured about robust quantum foundations and this is the reason because it gives such important results. Consequently, we discuss the evidence to correctly explain the foundations of this emerging discipline. In order to reach this objective we analyze the fundamental role of the abstract entity that is called quantum mechanical wave function. Soon after we describe the manner in which quantum operators must be entered in cognition studies by using the projectors that represent logic statements and thus relate directly our cognitive functions. We also discuss in detail the logical origins of quantum mechanics outlining that it does not represent a physical theory looking at the matter at microscopic level but a God Giano two faces looking simultaneously to material and to mental reality. We also discuss results establishing the nature of our consciousness and the unquestionable result that it obeys to the basic rules of quantum mechanics. We also give a further and pressing indication: in quantum cognition studies the quantum wave function must be intended as reconstructed experimentally a posteriori in our cognitive studies and cannot be confused with the standard examining elaboration that is used in usual textbooks of quantum mechanics. Also the question of the order effect is analyzed in detail using the important notion of reaction time of psychology. In substance, we give a total reformulation of the current quantum cognition studies evidencing that previous elaborations finalized to present the current status of this discipline as empirical tentative to characterize a quantum cognitive modelling result to be only approximate, incorrect and confusing prospects that damage the information about the robust structure of quantum cognition and therefore need to be rejected.

Key Words: quantum cognition, quantum mechanics, cognition, quantum cognitive models

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1. Introduction

In the last time many authors published their elaborations about the field of quantum cognition on several journals. Often they recall some theoretical and experimental studies that we obtained on quantum cognition, one quoted paper is often (Conte, 2008b) but we complain that other our studies contribute in a valuable manner to this new discipline as (Conte *et al.,* 2003; Conte *et al.,* 2006; Conte *et al.,* 2007; Conte, 2008b; Conte, 2009; Conte *et al.,* 2010; Conte *et al.,* 2011; Conte, 2012; Conte *et al.,* 2015c) and they are never quoted. We retain that a criteria is adopted in science: everybody is free to publish anything desired but one must provide that

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preceding publications directly relevant to the topic are quoted. At the moment this does not occur. In the tentative to find a possible explanation, it is also possible to speculate the very unlikely event that an author does not mention the results of another author because considers them to be of unsatisfactory level and this is a condition extremely unlikely since all publications are now properly analyzed today by the peer review system. Also in this case it is appropriate that the author cites the results and his criticism to which the quoted author may respond. This is an ethical criterion in science that specifically prevents damage to the profile of authors who contributed in a fundamental manner and currently continue to contribute to the structuring of an emerging discipline and averts the risk that some authors present their elaborations in a form that is too personally self-resonant. Given these remarks of a general nature, it is necessary to go examining in depth the basic consistency and the rigour with which some processing on the quantum cognition are presented.

Although quantum mechanics just started with its founding fathers and in particular von Neumann (von Neumann, 1935) to illustrate a possible role of mental entities and of consciousness in the foundations of this theory, in our opinion our studies contributed to determine the advent of the field currently named quantum cognition because as pioneering authors we formulated a theoretical elaboration and for the first time this elaboration was supported from experimental data and verifications, inducing in this manner an increasing interest and credibility owing to the presence for the first time of experimental results based on the exploration of a peculiar feature of quantum mechanics that is the quantum interference effect. To be clear: everyone is free to believe science is done in the manner that one considers more appropriate, but the limit case would be at least that some authors could avoid to quote our results at all if subsequently they are not used to evidence the attempts that we subsequently are performing to delineate a definite, robust and well founded structure of such arising discipline. Some authors instead quote our initial results; do not quote all our subsequent results that move in the direction to delineate a robust structure of the discipline and aim instead to present the current studies on quantum cognition as initial attempts to delineate a novel empirical approach to cognitive modelling. In brief, we do not retain that in quantum cognition we are at the stage of an initial approach that is being used since in some cases it seems that gives better results respect to classical cognitive modelling. We retain instead, as we shall evidence in the present paper that this discipline, if following our elaborations, starts having solid basic foundations that we have contributed to demonstrate. The basic key here is the word “demonstration”. The arising situation is that our theoretical and experimental work, following our initial results and currently produced until today, frequently is not mentioned from these authors and we remain as authors initially contributing but subsequently totally ignored. Basically, it appears that specialists on quantum cognition would be going performing only tentative approaches based on the fact that some fundamentals of quantum mechanics seem to fit better than classical cognitive models. In short, we should be still missing of robust fundamentals while empirical attempts should result promising. We read elaborations that result so distant from what we believe to be the correct understanding of the discipline.

The disadvantage to give such kind of presentation of quantum cognition is so serious since as example it opens consequently the window to a lot of criticism or reservations. This is as example the case of D. Mender (Mender, 2013).

The aim of the present short paper is to give detailed explanation of the foundations of quantum cognition. In the course of our explanation we will recall statements and elaborations of those authors that, in our opinion, following an alternative direction of exposition, open the possibility of severe criticisms.

2. The basic Foundations of Quantum Cognition

We will start considering a statement that appeared in (Trueblood *et al.,* 2012). The only finality to recall such statement is to evidence the importance to avoid analogies that could not be supported in a rigorous framework. We find the following statement:

> *Before measurement, cognition behaves more like a wave than a particle allowing for individuals to feel a sense of ambiguity about different belief states simultaneously. According to quantum probability theory, beliefs remain in a superimposed state until a*

final decision must be reached, which resolves the uncertainty and produces a collapse of the wave to a specific position like a particle.

We cannot escape to observe that this statement is confusing and difficult to be interpreted both from a physicist and from a psychologist.

Cognition behaves more like a wave? Where the authors found such result! Are they attempting to use some analogy? They could induce confusion in the reader. This risks appearing a bold attempt to capture a kind of analogy that should relate cognition and wave/particle dualism that of course represents a specific and long debated and well delineated problem of quantum mechanical studies. In quantum cognition explication, there is a need for clarity, accuracy and rigour and not the requirement of pale analogies as one may conjecture using a standard quantum mechanical textbook. The fundamental thesis is that when a person is called upon to take a decision, as example in consequence of a given task, the subject has in itself a state of intrinsic ambiguity, intrinsic indetermination and an inner conflict and, as we will explain later, he has still a lot of psychological contributions and mechanisms acting about the decision to be taken and in relation to the arising alternatives. All these features are context dependent and this mental state at the level of our consciousness follows the principle of superposition of states (quantum wave function and its collapse) that of course is the representation of the intrinsic and irreducible indetermination that is the basic foundation of quantum theory. We have produced a lot of evidences and demonstrations that our consciousness obey rules of quantum mechanics as well as during perception and cognition (Conte, 2008; Conte, 2010; Conte, 2011b; Conte, 2014; Conte, 2015) and thus we do not speak of a simple assumption but a datum that is rigorously supported from a series of theoretical and experimental elaborations. Let us state about such superposition principle of states that in itself has not difficulties to be explained in detail at cognitive level in a rigorous neurological framework. Let us attend to some expositions given by Jansen (2015).

We repeat here his words: Extra-mental reality can only be perceived by the brain with the help of sense organs, which transmit all information from extra-mental reality with the help of physical factors, such as electromagnetic waves for the eyes or other physical factors for all other sense organs. Physical factors are transformed by the sense organs into neural activity called sensory transduction. Light enters the eye as electromagnetic waves and stimulates sensory neurons in the retina, which transforms the physical stimulation into depolarization of neurons and transmit their information to specialized brain regions. Mechanical waves enter the ear and stimulate specialized receptors, which transmit their activation to specialized regions in other brain regions. All sensory organs function in a similar way, since they are stimulated by physical factors and transmit their activity to their corresponding special brain regions. With information perceived by all sensory organs, the brain constructs a mental representation of extra-mental reality. However, if the perception organs are inactivated such as by closing the eyes or the ears, there is no longer any direct physical contact between the extra-mental reality and its mental representation in the brain and then no observation of extra-mental physical events is still possible. Now the brain is cut from its outside environment and functions only with its memory, such as perceptions from the past encoded in the memory and retrieved again. The distinction between sensory perceptions through direct physical contact with the extra-mental reality and memorized perceptions of the past is very important, since pure bottom-up perception from sensory organs to the brain remain unchanged. The intensity of powerful light flashes or unsupportable noise cannot be voluntarily changed, whereas perceptions of the same events after their memorization become modifiable, for instance they can be forgotten or recalled a day later and do no longer induce the same pain feelings. Since memorized past perceptions can be cognitively modified, they allow rearrangements in the representation of extra-mental reality by imagining new situations, which no longer correspond to prior perceived extra-mental reality. These functions are essential for imagining the future by reorganizing past memory perceptions in a different way and projecting them mentally into the future. Since the future is generally uncertain, it can only be imagined with potentiality. Thus several possibilities have to be imagined simultaneously, although with different probabilities, which correspond in humans to mental superposition for the prediction of an unknown future.

The key features are since the future is generally uncertain; it can only be imagined with potentiality. Thus several possibilities have to be imagined simultaneously, although with different probabilities, which correspond in humans to mental superposition for the prediction of an unknown future. This is precisely the quantum superposition principle of quantum mechanics applied to quantum cognition. Also ignoring the consistent number of times in which we have explained such basic concept, in Jansen we find again an illuminating and excellent exposition of the status of the matter. According to our results we have to add only some observation and some light modifications but the standard requirement to acknowledge the essential role of the superposition quantum principle at the level of mental states cannot be questioned. Consequently, when we use the superposition quantum principle in quantum cognition we are not using an ad hoc procedure that we use since quantum mechanical model in cognition in an empirical mode gives better results respect to classical cognitive models but because this is a consequence of the fact that quantum foundations enter in brain dynamics at cognitive level as basic and fundamental structure. Let us indicate what we have to modify in Jansen statements. The first is an observation useful as confirmation: the brain has constantly a basic function; it is that one to arrange and/or to conjecture about the future. Jansen says: *"With information perceived by all sensory organs, the brain constructs a mental representation of extra-mental reality."* We specify in detail that brain realizes a superposition of mental states as evidenced in particular in (Conte, 2014b; Conte 2015) and thus a final mental representation. Finally, when this author uses the term "imagining" we would substitute it with "arranging" outlining in detail that the basic brain dynamics is constantly based at mental level on arranging a future perspective. Finally, when this author says that the future is generally uncertain we would substitute this term with that one of indeterminate.

There is more. In the paper signed in (Trueblood *et al.,* 2012; Pothos *et al.,* 2013), we find still that the authors use frequently the term *uncertainty.* It is important to outline that from its starting, quantum mechanics had two peculiar features, one is the intrinsic, irreducible indetermination and the other is the quantum interference. The term uncertainty is light years away from quantum mechanics and from the intrinsic and irreducible indetermination that it delineates. To use this term it means then push quantum mechanics in the grounds of classical physics. The term uncertainty has nothing to do with quantum mechanics and the most puzzling is that in the course of the text the authors in (Pothos *et al.,* 2013), after using the term uncertainty, at some point also recall the Kochen Specker theorem that really moves in quantum mechanical direction and not in that one of uncertainty.

In a paper these authors still continue (Pothos *et al.,* 2013): *"In quantum theory, there is a fundamental distinction between compatible and incompatible questions for a cognitive system".* Note that the terms compatible and incompatible have a specific, technical meaning in QP theory, which should not be confused with their lay use in language. If two questions, A and B, about a system are compatible, it is always possible to define the conjunction between A and B. In classical systems, it is assumed by default that all questions are compatible. Therefore, for example, the conjunctive question "are A and B true" always has a yes or no answer and the order between questions A and B in the conjunction does not matter. By contrast, in QP theory, if two questions A and B are incompatible, it is impossible to define a single question regarding their conjunction. This is because an answer to question A implies a superposition state regarding question B (e.g., if A is true at a time point, then B can be neither true nor false at the same time point). Instead, QP defines conjunction between incompatible questions in a sequential way, such as "A and then B." Crucially, the outcome of question A can affect the consideration of question B, so that interference and order effects can arise. This is a novel way to think of probability, and one that is key to some of the most puzzling predictions of quantum physics. For example, knowledge of the position of a particle imposes uncertainty on its momentum.

However, incompatibility may make more sense when considering cognitive systems and, in fact, it was first introduced in psychology. The physicist Niels Bohr borrowed the notion of incompatibility from the work of William James. For example, answering one attitude question can interfere with answers to subsequent questions (if they are incompatible), so that their relative order becomes important. Human judgment and preference often display order and context effects, and we shall argue that in such cases quantum theory provides a natural explanation of cognitive process".

These are only indicative statements. Note that they pertain to quantum mechanics on a general plane relating the basic foundations of quantum theory. In the case of quantum cognition studies one cannot think to do a simple translation of some quantum mechanical foundations into quantum cognition, thinking in this manner to introduce and to delineate the general framework of a model of the human cognitive dynamics as it enters in quantum cognition. If all should be so simple we had solved from many time basic unsolved problems in neuroscience and in psychology. First of all, in some considerations the authors are inspired from expositions that we previously performed in a properly and rigorous manner and discussed elsewhere (Conte, 2010; Conte, 2011; Conte, 2011b; Conte 2011c; Conte *et al.,* 2012) and not quoted. In addition, the level of exposition that is reported may still generate only confusion and misunderstanding.

They say "If two questions, A and B, about a system are compatible, it is always possible to define the conjunction between A and B. In classical systems, it is assumed by default that all questions are compatible. Therefore, for example, the conjunctive question "are A and B true" always has a yes or no answer and the order between questions A and B in the conjunction does not matter. By contrast, in QP theory, if two questions A and B are incompatible, it is impossible to define a single question regarding their conjunction. This is because an answer to question A implies a superposition state regarding question B (e.g., if A is true at a time point, then B can be neither true nor false at the same time point). Instead, QP defines conjunction between incompatible questions in a sequential way, such as "A and then B." Crucially, the outcome of question A can affect the consideration of question B, so that interference and order effects can arise."

The comment here is in a general consideration. Incompatibility in quantum mechanics relates only simultaneous considered observables. The simultaneous posing tasks to a subject do not exist in neuroscience and in psychology at least at cognitive level. We could realize it technically but there are very limited cases in which two posed questions A and B may be examined by our brain simultaneously. Always the basic tendency is that, receiving the inputs, one first answers to A and soon after he/she answers to B. In the greatest part of the cases we have always A and soon after B or B and soon after A.

Remaining in the purpose to give arguments with definite rigour we need to remember here that standard quantum mechanics runs about the notions of well valuable observables to which definite Hermitean operators are connected. They are *verified* to be compatible or incompatible. Where are here, in the case of the quantum cognition studies, the observables? But basically, where are here the operators? May be the posed task! May be some inner observable at the level of our mental states! Again, in the attempt to elucidate the matter, we observe a racking of basic foundations of quantum mechanics into quantum cognition as an adventure. This is a standard procedure that appears also in several other papers. We have a constant mixing of advanced assumptions mixed to other moments in which instead it is evidenced that only empirical attempts are executed. Often the recurrent thesis is that they are only pointing out that in studies on cognitive models some things seem better interpreted if we adopt the quantum probability calculation rather than the classic. They escape to the real need to represent the scientific studies rigorously and systematically, pale and highly questionable analogies or adventurous attempts are shown Certainly, attempts in science are always acknowledged and legitimated but the first requirement is the scientific rigour and this is the case that is strongly required in quantum cognition. If we do not ignore some basic studies, such requirement of rigour exists. One has to observe that presently brain sciences are going through a very important conflict. From one hand we have neuroscience. This discipline is giving every day new basic knowledge also thanking neuroimaging contributions. Unexpected fundamental results neuroscience is giving constantly. However, in spite of such relevant results, it does not arrive to give the most expected result that is to explain the actual nature of mental entities. On the other hand we have the empirical results of the psychology. Also in this case we have constant new contributions but often they result conflicting with those of neuroscience. In this way the situation is that in the middle between neuroscience and psychology there is the darkness that has to be overcome by the presence of a bridge that should be represented by a new model. Quantum mechanics is not a possible candidate but the basic foundation to represent this model not for speculations, not for epistemological considerations, not for tentative and attempts but because the science of quantum mechanics demonstrates that this is the case. Of course quantum mechanics has so many possibilities to overcome the difficulties that are often presented when we consider the possibility to include quantum effects in a so large and macroscopic system as brain.

We have now to consider still the question of compatible and incompatible observables. To maintain the required scientific level, we have to start from the only well fixed and established fact. It responds to the basic pillar demand of quantum mechanics to include in the analysis of each quantum system in nature the so called wavefunction. This is the first and important statement of quantum cognition. Wavefunction is an abstract entity that we will discuss in detail. A datum is certain, all quantum mechanics runs about this basic admitted entity, called the wave function. The basic foundation is here: we cannot ignore its existence as abstract entity (reality is not only matter!) and it represents a "factor of knowledge" and thus relates directly our mental state during the cognition. This is demonstrated not admitted (Conte *et al.,* 2009, Conte, 2010; Conte *et al.,* 2011; Conte, 2012; Conte, 2014; Conte, 2014b; Conte, 2015; Conte 2015 b). There are several results that move in the same direction: the wave function represents an abstract entity whose direct counterpart at neurological and psychological level is representative, roughly speaking, of the mental state of the subject. This is the basic point whose relevance we have attempted to illustrate in several papers (Conte *et al.,* 2009; Conte, 2010; Conte *et al.,* 2011; Conte, 2012; Conte, 2014; Conte, 2014b; Conte, 2015; Conte, 2015b). Let us recall these papers illustrating as we have to reason in quantum cognition studies.

States of mental entities are represented by wavefunctions. The states of our consciousness may be represented by a wave function and thus respond to all the requirements that have been fixed in quantum mechanics by this abstract entity.

Be care! We are not saying that we have finally obtained the greatest result of science that is to represent a mental state or our consciousness by a mathematical function. We are simply considering that following some results (Conte *et al.,* 2009; Conte, 2010; Conte *et al.,* 2011; Conte, 2012; Conte, 2014; Conte, 2014b; Conte, 2015; Conte, 2015b), by using wave function, we have a rough but rigorous representative counterpart of what we intend for state of mental entity and state for consciousness and that quantum cognition, if attempted to be realized on robust foundations, starts from this result.

Let us explain about consciousness also if we have exposed this argument elsewhere (Conte, 2014).

Some considerations about the mechanism of perception are necessary. The human eye is very sensitive but we examine here if we may see a single photon. We retain that under some definite conditions, the answer is positive. The sensors in the retina respond to a single photon. Of course we have that neural filters only enable a signal to reach the brain to trigger a conscious response when at least about five to nine photons arrive within less than 100 ms. If we could consciously see single photons we would experience too much visual noise in very low light, so this filter is a necessary adaptation, not a weakness. The retina has two types of receptors, cones and rods. The cones are responsible for colour vision and are less sensitive to low light than the rods. In bright light the cones are active and the iris is stopped down. This is called photopic vision. When we enter a dark room, the eyes first adapt by opening up the iris to allow more light in. Over a period of about 30 minutes, there are other chemical adaptations that make the rods become sensitive to light at about a 10,000th of the level needed for the cones to work. After this time we see much better in the dark, but we have very little colour vision. This is known as scotopic vision. The active substance in the rods is rhodopsin. A single photon can be absorbed by a single molecule that changes shape and chemically triggers a signal that is transmitted to the optic nerve. It is possible to test our visual sensitivity by using a very low level light source in a dark room. The experiment was first done successfully by Hecht, Schlaer and Pirenne (1942). They concluded that the rods can respond to a single photon during scotopic vision. In their experiment they allowed human subjects to have 30 minutes to get used to the dark. They positioned a controlled light source 20 degrees to the left of the point on which the subject's eyes were fixed, so that the light would fall on the region of the retina with the highest concentration of rods. The light source was a disk that subtended an angle of 10 minutes of arc and emitted a faint flash of 1 millisecond to avoid too much spatial or temporal spreading of the light. The wavelength used was about 510 nm (green light). The subjects were asked to respond "yes" or "no" to say whether or not they thought they had seen a flash. The light was gradually reduced in intensity until the subjects could only guess the answer. These authors found that about 90 photons had to enter the eye for a 60% success rate in responding. Since only about 10% of photons arriving at the eye actually reach the retina, this means that about 9 photons were actually required at the receptors. Since the photons would have been spread over about 350 rods, the experimenters were able to conclude statistically that the rods must be responding to single photons, even if the subjects were not able to see such photons when they arrived too infrequently. In 1979, Baylor, Lamb and Yau were able to use toads' rods placed into electrodes to show directly that they respond to single photons. Of course just thirty three years ago they observed that the solution is already at hand (for details see Conte, 2014).

As previously said, we have given demonstration recently that binocular vision and/or binaural hearing raise interesting explanations at the level of consciousness. As Woo outlined in 1981 (Woo, 1981) and as of course it is well known, since the stimuli that the two eyes receive, are compared in the brain to yield depth and motion, since the vision of split brain subjects shows clear left –right differences, we may conclude that we are subconsciously aware, some of time, of which eye sees which. This is the central point. In fact, it follows immediately that we may perform an experiment in a dark room. We may arrange the signal so that it reaches one and only one of our eyes. In the case of such experiment, we are normally unaware which eye has actually seen the signal. In this condition we are forced to acknowledge that there is a state of the consciousness which corresponds to what we have previously indicated as quantum superposition principle, in fact it responds to a coherent superposition of seeing the signal by the right eye and seeing the same signal by the left eye. As previously outlined the rods must be responding to single photons. Since the light quantum coming into the left eye would excite the left retina, we have to answer which excitation can in turn be checked. It would seem that even in principle there can be no interference between the wave function ψ_r corresponding to seeing a flicker by excitation of the right retina. A similar reasoning may be developed by us for ψ_l. We have not so much alternatives in the arising conclusion. The whole visual system is so tightly correlated that there exists a state of awareness ψ_0 which is a coherent superposition of ψ_r and ψ_l with the same energy for the total system. This is the quantum superposition that we mentioned previously and this experiment evidences in a robust manner that our consciousness responds consequently to the basic quantum rules. The wavefunction, as previously explained, develops its basic role.

As the second point, still according to Woo (1981), when we turn to human observers and conscious awareness the flickers, we may compare the number N_0 of signals seen in a given time interval when the same quantum may pass through either eye with the sum N_l+N_r , where N_l is the number of signals seen when the light path to the right eye is blocked and similarly we may reason for N_r. The total number of photons directed to open eyes in the latter case is arranged in a manner to be equal to that directed to both eyes in the binocular run and for the same time. The present techniques may help us in reducing unexpected negative effects reducing the number of photons involved until there are only enough quanta to excite one retina at a time. The arising result of the experiment is

$$N_0 = N_l+N_r$$

as we expect in the classical case ? No, we expect

$$N_0 \neq N_l+N_r$$

since our previous results include the quantum role.

We may now pass to consider another feature of quantum cognition. What is the reason to insist so much to insert quantum mechanics in cognitive studies? The profound reason is that quantum mechanics contains in the inner of its formalism logical statements and logic statements are the basic foundation of cognition and thus of mental entities.

Only quantum mechanics has this peculiar feature. Consider the physical case of a system. We have the system and some its connected observables that are physical quantities that we intend to measure. In quantum theory ,to each observable we connect an Hermitean operator and thus on this basis we may estimate the values that such observable may probabilistically assume in consequence of an intrinsic and irreducible indetermination that reality has according to its dynamics. Before of the measurement all the alternative states, relative to all the possible results, coexist, after the measurement we find a result with a certain probability. This is quantum mechanics. Now, we cannot transfer this matter automatically in quantum cognition. In the case of quantum cognition studies, we have only tasks that we give to a subject. We do not know the nature of the involved mental observable and, in particular, we are unable to connect to it an operator. Owing to some peculiar features of quantum mechanics, however, we have the manner to solve this problem and to represent by an operator, the variable that we consider in the posed task. This is to say that this operator is represented by other operators that represent logic statements and thus relate directly cognitive dynamics. This is the basic foundation. Thus, not forced analogies given as examples and poorly holding, but a direct and pertinent treatment of the basic structure of the problem. As example, consider that we pose a task to a subject and that the characterizing variable is a “dichotomous variable” that we call A and that may assume, just to say, or the value (+1) or the value (-1). The corresponding operator connected to the variable relating the task may be represented by combining such values (+1) or (-1), (possible results of the considered task) with the logic statement that represents our cognitive performance that, during the decision, the value (+1) results to be true or false and the same thing happens for the value (-1). In other terms we may write the operator as $A = a_1P_1 + a_2P_2$ where a_1 values (+1) (result (+1) of the task) and a_2 values (-1) (result (-1) of the task). P_1 and P_2 are the logical statements, the projectors are called technically, and realize ab initio that cognition is involved in the characterization of the operator A that is connected to the task A (Conte, 2013; Conte, 2014; Conte, 2015b).

The very interesting feature is that P_1 and P_2 have a given mathematical structure that we do not report here to avoid mathematical difficulties to the reader but are directly related to the wavefunction, and thus they are representative of the state of the mental entity during the administration of the task A. Technically we use the representation $|\psi\rangle\langle\psi|$ being ψ the starting admitted psi function relating our mental state. Let us examine: P_1 and P_2 may assume only two values, respectively or 1 or 0. Let us consider P_1. What does it mean that it assumes the value 1. It means that posed the task A to the subject and being the corresponding state of the mental entity of the subject in the superposition of alternatives to decide or $a_1 = +1$ or $a_2 = -1$, it is true that he/she answers +1 . 0 for P_1 means that it is false that he/she answers $a_1 = +1$. In the same manner, P_2 has possible values or 1 or 0. What does it mean that it assumes the value 1. It means that posed the task A to the subject and being the corresponding state of the mental entity of the subject in the superposition of alternatives to decide or $a_1 = +1$ or $a_2 = -1$, it is true that he/she answers $a_2 = -1$. 0 for P_2 means that it is false that he/she answers $a_2 = -1$. In this manner the wave function psi, the state $|\psi\rangle$, the state of the mental entities, and cognition, logic statements $|\psi\rangle\langle\psi|$, are fused ab initio and we may have compatible (or not) mental conditions. This is the state of the matter. These are basic foundations. We have a physics in which some peculiar features of cognition are present ab initio. They are only in quantum mechanics. Consequently, when we started with our experiments and elaborations, we did not introduce models, we did not perform attempts to climb on mirrors or even using confounding attempts to adapt with trivial examples quantum mechanics within the general framework of cognitive models. Quantum mechanics itself is a theory that is made in this way. Using the concept of the wavefunction and projectors, this theory delineates *ab initio* a model of reality in which material of the one part and cognition, on the other part, coexist. It is a theory “two faces Giano” who on the one hand looks at the matter and on the other hand looks at cognition. The example that we have given, evidences that this is the situation without any possibility to contradict it. Basically, as already outlined in other publications, it sets out a roadmap of reality that is completely new and radical. The arising structure indicates that there are levels of our reality where we no more can separate the material from cognition. It is a level of reality in which there is no longer the reality of an object that exists in it independent of the cognition that we have about it. Quantum mechanics does not reason as in the old our standard manner, matter is there in space and time and mental entities are suspended in some other abstract space that we do not identify. Both such features coexist a priori and we have levels of our reality in which no more we may admit something material independently from the cognition that we have about it. We have outlined such results several times and it is sufficient to read all the references to look at. Let us go still deepening the argument. Quantum theory focalizes the question of quantum interference. We have quantum interference in some celebrated experiments of quantum physics so that we will not perform further discussion on this. It has been continuously mentioned and discussed so that we will not discuss it. It is important of course to outline the following feature. In order to demonstrate experimentally that quantum mechanics has a role during perception and cognition in humans we performed some experiments with the finality to evidence that we have quantum interference also in humans. We used ambiguous figures to demonstrate existing quantum interference, we used also the well-known Stroop effect, we used some cognitive anomalies as conjunction fallacy and recently we demonstrated it also in integration of emotion and cognition in children. We demonstrated (Conte, 2011; Conte, 2011b) , following one of the greatest logic of this time (Orlov, 1996), that if we use the logic statements, the projectors $|\psi\rangle\langle\psi|$, we obtain the same kind of quantum interference as well as when particles are considered in the celebrated two slits experiment of physics on quantum interference. Here we did not use particles, we used logic statements and we obtained the same quantum interference. This is a result clearly elucidating about the logical origins of quantum mechanics. We have basic foundations here. Quantum theory holds by itself on cognition and thus we expect that it enters properly in cognitive science.

We have not finished. Problems still remain. As said, wave function is the basic starting key. Projectors, having cognitive and logic basic features, derive from wave function. The question is how we determine such wave function when we are involved in studies on quantum cognition.

On a general plane this question may be posed in two completely different ways and ignoring such two basic features we arrive after to expositions that, as previously said, result misunderstanding and confusing and they open the side of every possible criticism. The first way: how is one to determine experimentally a situation (as example, of a physical system) describable by a given wave function? Scholars having competence in quantum mechanics know that this problem may be solved by performing a complete measurement of a set of commuting observables (here the question of compatible and incompatible observables is directly posed!) upon a SINGLE physical system. Scholars in quantum mechanics know well that this cannot be the case of studies on quantum cognition.

The second way is posed instead in the following manner: how is one to acquire *a posteriori* a knowledge about the fact that a system was described by a wavefunction? This is the case of quantum cognition studies. Note that confounding the basic difference posed between such two completely different questions, means to reset completely the full understanding around the ongoing studies on quantum cognition .The wave function, as representative of a mental state, cannot be measured in the usual sense of this word. As a mental state, wave function is an abstract entity and it cannot be observed directly. Nevertheless, it may be determined *a posteriori* provided that one has an ensemble of similar systems, each described by the same wave function, psi (ψ). This was the basic statement that induced us to starts in 1986 (Conte, 1983) and after in (Conte, 2003) with theoretical and experimental studies that determined the advent of quantum cognition. We report here the approach.

Let us start measuring a task A on a large number of subjects and get a statistical distribution of the set of results. According to our basic statement we have to connect a wave function to such mental cognitive dynamics.

Suppose the task has only two outcomes or $|+\rangle$ or $|-\rangle$.

These are quantum states and thus represented by a final state (roughly speaking, the wave function) that will be the simultaneous superposition of the previous mentioned ones. It is then written in the following manner

$$|\psi\rangle = c_1|+\rangle + c_2|-\rangle$$

This is to say: we start giving the subject the task A. He/she is in a “ready” mental condition. We give the task A and he/she assumes both the existing alternatives having the ambiguity and the inner conflict (and a lot of other psychological contributions) that the task induces contextually. After decision or the mental state $|+\rangle$ or $|-\rangle$ will arise.

The mental condition is of an intrinsic indetermination before a decision is assumed and the previously written superposition is the mirror mathematical representation of such situation of psychological ambiguity and conflict that in the subject is established. Depending from the context in which the task A will be given and from the inner psychological conditions of the subject at the moment in which the task A is given, with a probability p(+) he/she will answer (+) and with probability p(-) he will answer (-). The advantage is that we may estimate such probabilities experimentally. In fact, if we are conducting the experiment we will obtain, as said, the statistical distribution of the answers and thus directly the probabilities. If we need c_1 and c_2 to write explicitly the previous superposition we will provide extracting the root square of the probabilities obtained experimentally. Really, c_1 and c_2 are complex numbers, so that we will estimate the modulus experimentally but not the phase of each of the two complex numbers.

In order to determine $|\psi\rangle$ completely, we have to know not only the modulus or the absolute values of such coefficients but also the phases. To this end we repeat a new experiment. By the same previous procedure we give now to the ensemble of subjects a new task B having again two alternatives (+) or (-) as possible answer. The task is new, we expect the cognitive dynamics will be different, will have different contextual probabilities in answer and thus call this time the states as $|+d\rangle$ and $|-d\rangle$. Again we will have a superposition of mental states and the wave function will be

$$|\psi\rangle = d_1|+d\rangle + d_2|-d\rangle.$$

Again we will obtain the statistical distribution of the answers, again the probabilities p(+d) and p(-d) and again the modulus but not the phase of the two new complex numbers d_1 and d_2 will be determined. Note that all the results are obtained experimentally. Consider that our objective is to reconstruct “a posteriori” the wave function from experimental data. This is important for two reason: the first is that we reconstruct by such wave function this rough mathematical representation of the mental state but, by this way, we reconfirm that we are on the correct way. Wavefunction is in fact an abstract entity that is contained only and only in quantum mechanics. Consequently, if our experiment enables us to reconstruct it, we also reconfirm that in the experiment the quantum mechanics has a role. If instead it results that we cannot determine the wave function, we conclude that quantum mechanics has not a role.

In order to go on in the calculation of such wave function we have to recombine the previous written superpositions by some mathematical procedure that we do not report here to avoid to give mathematical difficulties to the reader.

The result is that finally we have in our hand a mathematical system and as all the mathematical systems that we started to study in our first years of our school, it may be soluble, indeterminate or impossible. Only such three cases may arise from our experimental study.

What is the conclusion if the system results impossible? Technically speaking, using the quantum mechanical language we say that in the course of the experiment giving first the task A and after the task B, we realized a mixture. Said in current language, the conclusion is that if the system is impossible, this means that quantum mechanics had not a role in our experiment.

If on the other hand the equations of our system result dependent to such an extent that the phase could not be determined uniquely, this means that the sets of the task A and B result not independent of each other in the mental representation of the subjects. This means that we will not arrive to establish the wave function in a correct and unique form.

Finally, it remains the third case. It is that we arrive to a unique solution of our mathematical system and thus to a valid reconstruction “a posteriori” of the wave function. In this case we reach our objective; we evidence existing wave function in the mental representation of the subject. In this case we arrive also to estimate the quantum interference.

This is the real explanation of the experiments that we performed in the advent of quantum cognition. This is the basis of theoretical and experimental studies on quantum cognition and analysis of quantum interference at the cognitive level as we intended in our pioneering studies and presently intend it. In this manner, we have outlined as possibly quantum wave function may be reconstructed. We used the two basic and peculiar features of quantum mechanics that are the intrinsic irreducible indeterminism and the quantum interference.

This above discussion evidences that it is possible to determine post factum such wave function. But in order to be able to do so, one needs an ensemble of similarly prepared systems as we would say in quantum language. This was perhaps the main reason for confusion in the past, also among physicists, and led some authors astray to a conclusion that quantum mechanics applies exclusively to ensembles but not to individual systems. Still again this was a consequence of an approximate knowledge of quantum mechanics. These authors frequently mixed up two notions: the concept of measurement of observables applying to single physical systems with the different question of possible estimation of a wave function which is expected to represent the system but is unknown to the researcher which is going to perform the estimation. The reconstruction is a posteriori. Note that, according to quantum mechanics, the concept of measurement of observables applied to single physical systems must be regarded as a preparation of a situation represented by an *eigenfunction* of the observable which actually has been measured. As said, many authors moved to induce confusion between two different notions. To clear: the function psi is not an observable in the traditional sense of this word. It is an abstract entity as of course a mental state is. Since it is not observable, it cannot be measured. However, being it subjected to the well-known probabilistic intrinsic features linked to intrinsic indetermination, it may be estimated statistically, provided a large number of systems (in our case subjects submitted to the tasks) is available, each represented by a similar psi wave function. In other words, there are two questions, the most fundamental here being about which function psi would enable one to make statistical predictions in agreement with a given statistical distribution of some measurement result. Obviously this is a question that may be answered only with the help and use of an ensemble. However, this problem must not be confused with another question relating which function psi results for a system in consequence of submitting it to a complete measurement. Both such questions are related with determination of a psi function but in a substantial different sense. A confusion of these two problems repeatedly induced the opinion that quantum mechanics applies only to ensembles. In our case, these further comments evidence still more as quantum cognition is a so difficult discipline in which it is so easy to enter with misunderstanding giving consequently approximate and often bad indications or comments or conclusions.

Finally, we have to mention the question of *time evolution* of mental entities according to quantum cognition. We have formulated in detail time evolution of abstract entities (mental entities) identified by the basic elements of Clifford Algebra (Conte *et al.,* 2007) and we have given direct experimental verifications as example by using the priming effect that is of extraordinary importance in psychology (Conte *et al.,* 2012). In this formulation, we did not use also only one addition of physics to realize such formulation but only abstract entities assimilated to mental entities. The elaboration is of extraordinary elegance and correctness and enters in the whole body of a rigorous formulation of quantum cognition.

These last comments conclude our exposition. We have to add only some final comments on a question that is often recalled in studies on quantum cognition and relating the so called order effect.

Fortunately, psychology has introduced some physical observable that may be related to mental state, to studies on its dynamics and to cognition. This is the notion of Response Time (RT) that is of so much interest in psychology. Let us give some indication.

Consider the following task: ask to a subject to memorize the following five letters XFLTH. Soon after such frame disappears and thus, as example, the letter F appears and it is contained in the previous group. In a subsequent frame the letter S appears that instead it is not contained. Each time the subject is asked to answer if the shown letter was or not contained in the initial group of letters (positive or negative probe). Here we may have well a statistical distribution about correct and incorrect answers as well as we may measure RT at each step. In substance we could again apply quantum cognition as well as we may examine RT at each step. It is an evaluation in short-term memory task.

According to Cooney and Traver and to our previous studies (Cooney *et al.,* 1994; Conte *et al.,* 2011) let us write the expression of the RT at each time. We have that

$$RT_{t+1} = k_1\ RT_t - k_2 RT_t^2 + k_3$$

We are not interested to discuss here this experiment under the profile of quantum cognition. We are instead interested to discuss in detail the meaning of the coefficients k_1 , k_2 ., and k_3. k_1 is usually called the SI, the control parameter relating the susceptibility to interference Here is one of the central features indicating complexity of studies on quantum cognition. k_1 relates the psychological condition that the simple fact of responding of the subject is considered to reactivate memory traces which in turn may result in his /her recording. According to our quantum scheme previously discussed, the subject is still in a superposition of states like

$$|\psi\rangle = c_1|+\rangle + c_2|-\rangle .$$

since he has only two alternatives but the act of responding means that he/she will activate the state $|+\rangle$ or $|-\rangle$ and only one of these, reactivating contextually memory traces . k_2 represent EA processes. Trace strength affects the rate at which episodic memory traces are activated by the task stimuli. Stronger traces determine faster rate of integrating aspects of the task stimuli with the permanent memorial representations. Finally k_3 represents a carry – over effect of probe positive or negative. All such influences will be contextual and will determine the nature of the final mental state that will determine the answer.

Owing to the presence of the factors k_1, k_2 and k_3, we will have the subsequent superposition chain

$|\psi\rangle = c_1|+1\rangle + c_2|-1\rangle \rightarrow |+1\rangle$ (or $|-1\rangle$) ; $|+1\rangle \rightarrow d_1|+2\rangle + d_2|+2\rangle$; $|-1\rangle \rightarrow g_1|+2\rangle + g_2|+2\rangle \rightarrow |+2\rangle$ or $|-2\rangle$, …

There is still another feature , discussing the order effect: probabilities obeying the basic feature p(A/B) = p(B/A) represent certainly a basic rule in quantum mechanics and of course it has profound implications based on the fundamental symmetries in our reality and that we have discussed in detail elsewhere. Owing to the presence of quantum collapse and of a lot of effects only partially represented with our example in k_1, k_2, k_3 and owing to the presence still of a lot of mental contributions , p(A/B) ≠ p(B/A) follows as fact like as we have discussed in detail in another paper that appeared on this journal (Conte, 2015).

3. Conclusion

The previous elaboration has given theoretical basic evidence that we need to highlight the importance of studies on quantum cognition and of the results that have obtained. We have attempted to explain that a satisfactory presentation of the theory encourages the constant increasing interest in such new born field of quantum cognitive modelling.

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