On the non-Existing Problem of Order-Effect in Quantum Cognitive Studies and Judgment

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ABSTRACT

We examine the results that were recently obtained by some authors relating quantum cognitive studies and judgment. We evidence the inconsistency of these results in the light of the foundations of quantum mechanics as well as when we submit them to a methodological examination.

Key Words: quantum cognition, Conte model of quantum cognition, order effect in quantum-like models of cognition, time symmetry in quantum mechanics, dispersion free ensembles

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Introduction

Recently, some authors (Kassem *et al.,* 2016) published a paper having as basic theme the quantum models of judgment and considered in detail the so called question of the order effect in studies on quantum cognition. In particular, they elaborated about the model of the present author (Conte *et al.,* 2009) and derived a well-known result of quantum mechanics that, in the case under examination, may be formulated in the following manner. If x_i and y_i are the answers that subjects give to two subsequent tasks A and B, in one order or in the other, that is to say or first A and after B or instead first B and after A, the probabilities, predicted by the quantum mechanical formalism, $p(x_i / y_i)$ and $p(y_i / x_i)$, satisfy the general rule $p(x_i / y_i) = p(y_i / x_i)$ under the condition that the eigenvalues are not degenerate. At this point, they used some equations that they called the Grand Reciprocity (GR) equations and subsequently, analyzing by this GR the experimental data obtained by Conte et al. (2009), they found the manner to question about the experimental results of Conte et al. (2009).

We need to elaborate about some fundamental problems. We will divide them in questions relating the basic foundations of quantum mechanics and questions relating instead the methodological profile of application when using quantum theory

a) Questions relating the basic foundations of quantum mechanics

These authors derived the basic general rule $p(x_i / y_i) = p(y_i / x_i)$ of quantum mechanics as usually

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it is done in ordinary textbooks of quantum mechanics. As correctly it is necessary when doing such derivation, in page 36 line 25 they report a mathematical relation that in fact is crucial in the derivation that they perform. The relation is the following

$$< y_i / x_i >=< x_i / y_i >*$$

As seen, it implies directly the mathematical operation of complex conjugate of a complex number. We have here the first important reservation. When doing research in a field and particularly when it relates a sphere of application that relates actual possible advances of knowledge, it is not acceptable that one takes the first formula that goes under his/her eyes and, in his/her mental construction, elaborates a text of verification. One should evaluate different features that are: first of all, quantum mechanics is a so complex theory, so distant from our common every day manner of thinking; still, it has a mathematical support but also a physical interpretation that holds after years and years of efforts from physicists. All we are aware that a long debate signed the advent of quantum mechanics and presently it is still no more concluded. In such so much complex situation if I limit to take a little of mathematics and using it I assume to be scientifically in the conditions to elaborate conclusions, I am in a profound error. Mathematics without physical interpretation is nothing. It is nothing particularly in physics and out of any doubt when we are considering quantum mechanics. Said in other terms, I cannot think to take some particular mathematical feature of a so complex theory and use it as a simple instrumental method to perform deductions and conclusions. This is to damage seriously the serious tentative that different authors are attempting to perform in the tentative to explore if quantum mechanics has possibly a role during human perceptive and cognitive performance.

Let us examine the consequences.

The authors escape to outline that behind this obvious mathematical relationship, quantum mechanics really delineates, as conceptual and formal counterpart, one of its greatest pillars. It is represented from the intrinsic and irreducible feature that is the time symmetry in this theory. This symmetry enters in the fundamental construction of quantum mechanics and is basically linked to the CPT theorem that is another pillar of the theory. Consequently, the previous mathematical relation

$$< y_i / x_i >=< x_i / y_i >*$$

seen but itself is a trivial mathematical relation if I use it as a simple instrumental mathematical evidence but really it links a so profound physical problem that is, no less than, the question of time direction in our reality.

In addition, as everyone may immediately acknowledge, this feature of time symmetry is of basic importance just in the analysis of the question of the order effect that the previously mentioned authors attempt to consider in their mentioned paper (Kassem *et al.,* 2016).

Consequently, we go to tap one of the thorniest issues, more full of mines, of all physics and with a little formula we develop conclusions.

Generally, scholars on quantum mechanics are aware of the basic role of time symmetry of quantum mechanics and correctly they often manipulate expressions of this kind: given the state ψ_0 at the initial time t_0, and $\psi(t)$ at time $t > t_0$, and $U(t)$ the time evolution operator with $U(t)U^*(t) = 1$, it is

$$a_{t,j} = \langle \psi(t) | \omega_j \rangle = \langle \psi(0) | U^+(t) \omega_j \rangle$$

since a complete orthonormal basis $\{\omega_j\}$ $(j = 1, 2, ...)$ has been identified.

According to the foundations of quantum physics, the intrinsic time symmetry of this theory is the cause of then obtain $p(x_i / y_i) = p(y_i / x_i)$. Owing to the importance of such foundation, we add still some further considerations. The traditional wave function in use in quantum mechanics is represented by retarded and advanced waves, the retarded wave passing from an initial event, i, to a future event, a, and the advanced wave, the complex conjugate of the retarded wave, passing from a back in time toward i.

These features relating time symmetry in quantum mechanics are well established matter of this theory from the early days of its advent. The founding fathers of quantum mechanics as Wheeler (Wheeler, 1978; Feynman and Wheeler, 1945; Wigner, 1959; Costa de Beauregard, 2006; Cramer, 1988), and also the present author (Conte, 1981a; 1981b; 2010a; 2010b; 2015a; b; c) also if with few contributions, just to quote only the smallest list of authors, have spent years of their activity to explain this foundation of quantum mechanics in the framework of retarded and advanced actions. Costa de Beauregard considered a standard quantum measurement that always consists in a quantum measurement operation of a preparation of the system and a final performed operation of actual measurement performed in laboratory. In the so called collapse of the state that is considered as basic process of quantum mechanics characterizing the measurement, really one must account for the collapse and for the retro-collapse. This last is to say a time symmetric phenomenon that goes back in time until the preparation from the measurement and both are realized by the universal known mechanism of the retarded and advanced actions. The first induces information as knowledge in the observer performing the measurement and the second instead realizes back in time information as order at the preparation of the system. Cramer has produced celebrated papers and, in order to illustrate the matter, has repeatedly represented the situation by using the terminology of an emitter and an absorber. In an attempt to illustrate an issue that is actually very complex, we can use an example of physics. On the general physical plane, we may consider a single space dimension x in a one-time dimension t. This is a wave-on-a-string situation in which the light cones become a diagonal Minkowski line connecting an emitter to an observer. The emitter must simultaneously send the retarded wave function $F_1 = \psi$ and the advanced wave function $G_1 = \psi *$ in the two time and space directions. The absorber at a later time receives the retarded wave $F_1 = \psi$ and terminates it by producing the cancelling wave $F_2 = -\psi$. Being in a time symmetric context, the absorber also produces advanced wave $G_2 = -\psi *$ which travels back along the Minkowski line until it reaches the emitter. At the emitter it exactly cancels $G_1 = \psi *$ that the emitter had produced in the negative time direction. The net result is a superposition of $F_1 = \psi$ and $G_2 = -\psi *$ connecting emitter with the absorber. Let us look at the important conclusion: an observer remains unaware of the time symmetric process involved and says that a forward going wave was emitted and subsequently absorbed. This elaboration has a name: it is called quantum mechanics and has one of its profound link with the trivial expression

$$< y_i / x_i >=< x_i / y_i > *$$

Before to use it, do we want ask to ourselves how the connected matter of such symmetric time phenomenon is related in the case of my performed experiments?

We have given a simple physical example. The same mechanism emitter absorber is involved in my experiments involving two groups of subjects during perception-cognition of ambiguous figures.

There is still more.

By using this approach, we have also explanation of the probabilistic nature of the quantum events. The previous formulation explains why a quantum event, described by a wave function ψ (that we have identified in our experiments) has a density probability of occurrence given by $\psi\psi *$. It really measures the strengths of the advanced - wave echoes arriving back at the site of emission at the instant of emission. (Cramer, 1988). As previously said, linking emitter and absorber with the dynamics that involved the subjects in our perceptive cognitive experiments in which they observed first an ambiguous figure and subsequently the second ambiguous figure, one may take consideration of the so complex time symmetric mechanisms that were involved in those experiments at perceptive and cognitive experiment. How may one expect to identify or to belie them?

Here is the crucial point that we should take in mind and may also be expressed in a lightly modified manner by using a different terminology. Time symmetry holds rigorously in quantum mechanics as *law -like* and it is fixed rigorously in the body of the theory. Instead *irreversibility* follows as FACT-like. As consequence, time symmetry is the basic law that we find at each step in the basic mathematical and conceptual framework of quantum mechanics and instead as a FACT-Like we observe irreversibility. Owing to irreversibility as fact-like, if we expect to find in an experimental situation the result $p(x_i / y_i)$=.., this means that we hope in pure expectation of a matter of the case.

Still there is more. We have elaborated this question of time reversibility and irreversibility considering the example in a pure physical plane. If we consider instead that in our experiments we used Human being having their individual and subjective mental functions, the seriousness to have attempted to carry out a discriminative test in these conditions of experimentation, become even more and more serious Consequently, the posed question of order effect in quantum cognitive studies becomes only a trivial question that does not deserve consideration since it is the irreversibility that, depending in a contextualized manner on the experimental conditions acting during the experiment, can mask the effects of time symmetry and let us get that $p(a/b)$ results different from $p(b/a)$.

In conclusion, according to the rules of quantum mechanics, the probability distribution (probability per unit volume) for an event to occur, is given by $\psi^*\psi$ whose meaning has been previously illustrated. The advanced wave modulates the retarded wave thus producing the required $\psi^*\psi$ probability pattern. The probability, $\psi^*\psi$, which then results in a probability for a kind of transaction (Cramer, 1988), a correlation between the two events, arises as a probability field at the initial event. The question that reversibility is mixed to irreversibility is crucial and, on the other hand, is well known in quantum mechanics from years.

Our conclusion is net. In studies on quantum cognition, as well as in all studies that consider quantum mechanics, one cannot use the basic formalism of quantum mechanics as an instrumental method, right there, like a quantum- like adventure and just because at first glance the use of this formalism seems to give better results than the classic one. To be more specific. The formalism of quantum mechanics involves the theory of the linear operators in Hilbert space and the Schrödinger equation or, equivalently, the Heisenberg representation and still more. However, these formulations do not constitute quantum mechanics. This last is obtained by establishing appropriate links between the mathematical formalism, interpretation of the formalism and experiment. Only the formalism supplemented by robust interpretational explanation, constitutes quantum mechanics. The apparently beneficial use of only a formalism, used as instrumental method and protected from any possible criticism from a term coined specifically and ad hoc as a quantum-like, cannot be admitted since the formalism and its interpretation form a natural whole: physical theory applied to cognition.

b) Questions relating the methodological profile

b_1) Preparation - Measurement in Quantum Mechanics

We have previously delineated a question relating the foundations of the theory but also attempting to overlook these aspects obviously not negligible, other basic limitations remain also under the methodological profile. As previously said, the notion of quantum measurement runs about two basic concepts. One is that one of the final measurement that actually is performed in laboratory from the operator. The other, the most important, is the notion of preparation for the measurement. When one performs the demonstration that $p(x_i/y_i)=p(y_i/x_i)$ and uses the standard derivation that is found of course in all textbooks of quantum mechanics, obviously is admitting that the notion of preparation and of final measurement are those robustly established in the theory of quantum mechanics. In this case the notion of preparation is related to an ensemble of systems that are admitted to have been all rigorously prepared in the same identical manner. The notion of preparation of the system to be submitted to a measurement is crucial. The systems must respond to the requirements to have all been identically prepared before of the measurement. The preparation has been conceived to consist in the application called usually filter, admitting, as example in the physical case, only particles (or, generally speaking, systems) with some rigorously fixed, common, and specific characteristic, and rejecting all others. One has to take care that the foundation here is that the desired characteristic must prevail immediately after having finished the preparation of the state so as to be able to use it for predictions of results of future measurements. In the case of the final measurement, one has to take care not to spoil or to change or to influence the properties which are going to be measured before the act of the measurement has been accomplished. The complex of such notion seems to be still possible for physical objects where we may also admit that a priori do not exist substantial differences as example in an ensemble of identically prepared particles as electrons. When the demonstration on $p(x_i/y_i)=p(y_i/x_i)$ is performed, and subsequently and subsequent tests are realized in order to analyze experimental data, one should be sure that all the previously mentioned requirements are respected. Obviously, in the case of quantum cognition studies, no one of the required statements on the preparation to the measurement is really respected. In our case the human subjects are submitted to tasks and each subject has his/her marked subjectivity. The mind functions of the subjects involved in the experiments are subjective and differentiated, subject by subject and time by time. The hope of considering systems that are all equally prepared is practically nothing and then, since it is instead a basic requirement for the subsequent demonstration, one can never, even remotely, dream of applying it as a test of the experimental observed data.

b2) We have now to consider what really it happens by application of Conte model in the experimental case.

As known, tasks are given to groups of subjects. Consider the experimental case conducted on a large number of respondents as it was in the different experimental studies that were conducted. Given the task to a group of subjects, each of them will bring into it the uniqueness of his (her) mind, each one with a specific conceptual network forming his (her) inner memory structure. This means that participants in the experiment will have a different way of choosing his (her) answer among those that are available to them to be selected. There is still more. The time answers of each subject cannot be accurately defined as in an ordinary experiment of physics depending it instead from his (her) Reaction Time that is a psychological and subjective variable that literature has shown to be linked to a lot of psychological and neurological factors. The conceptual consequence of such existing situation at experimental level is that we have what P. T. Landsberg called (1964) an incompletely specified system at the experimental level. In quantum mechanical terms, this is to say that, given a complete orthonormal basis $\{\omega_j\}$, the probability that arise experimentally no more must be considered to be

$$p(t) = \sum_{j=1}^{n} \left| a_{t,j} \right|^2$$

but, as it is due to the profound uncertainties in the subjects in their mental conditions and time, owing to the presence of an incompletely specified system, we observe probabilities that are due to profound fluctuations so that finally a mean value is given as result of the observation. This is to say

$$< p(t) >= \sum_{j=1}^{n} < \left| a_{t,j} \right|^2 >$$

In order to illustrate the situation, let us go on with an example. Consider the case of our experiments relating a dichotomic task. We have a two-dimensional space where ω_1 and ω_2 are the basis vectors. Let us assume as example that amplitudes are real, and this is to say that $a_1 = \cos\alpha$ and $a_2 = sen\alpha$. The condition that specifies our question is then the angle α. In the usual quantum mechanical case a value of α is given. To be clear we have $a_1 = \cos\alpha_1$ and $a_2 = sen\alpha_1$ where α_1 is a precise value in the possible range $0 \le \alpha \le 2\pi$. In our case of quantum cognition, when instead we have the condition of an incompletely specified experiment that is due to the different mental features of the subjects participating to the experiment and to their subjective variable of response time, we no more may conclude that each subject will perform the task giving one and only one value of α. On the contrary each subject will perform the task on the basis of his (her) mental and neurological attitudes and thus with a particular value of α In conclusion α will fluctuate from subject to subject in the range $0 \le \alpha \le 2\pi$. These values of α will remain obviously all strongly correlated but we will be in a condition of impossibility to express a complete specification in the initial condition and in the time of response for the experimental group of subjects. The only remaining feature is that one of expressing a probability for α to be between α and $\alpha + d\alpha$. For example, for a probability of a subject of giving a value of alpha between α and $\alpha + d\alpha$ we could in principle assume a function of distribution

$$f(\alpha) = A sen^b (2\alpha) d\alpha$$

with $\int f(\alpha) d\alpha = 1$ and A constant of normalization and b parameter ($b > -1$) ($b = 0$ responding as example to the case of an strictly uniform distribution and $b = 2$ responding as example to the case of a weakly uniform distribution) and arriving to estimate $< \cos^2\alpha >$ and $< sen^2\alpha >$ in order to approach what we observe experimentally and that, we repeat, is given by

$$< p(t) >= \sum_{j=1}^{n} < \left| a_{t,j} \right|^2 > .$$

No more we can do (for details on such estimations see Conte, 2012). The conclusion is that, owing to the presence of fluctuations in the mental conditions of the subject submitted to experimental tasks, we really may arrive to observe mean values of probabilities. As consequence it is evident that we cannot apply any tests of the kind suggested from the authors. The only enabled test is the estimation of the spread of the individual values of the subject respect to the values exhibited from the other subjects by using the second moment

$$Z = \frac{< (p(t) - < p(t) >)^2 >}{(< p(t) >)^2} = \frac{< p(t)^2 >}{< p(t) >^2} - 1$$

The use of this test immediately enables us to experience if we are or are not in presence of a dispersion free ensemble as of course it was obtained by von Neumann for dispersion free ensembles (Conte, 2012).

References

Boyer-Kassem T, Duchene S, Guerci E. Testing quantum - like models of judgment for question order effect. Mathematical Social Sciences 2016; 80: 33-46

Conte E. On psi Retrocollapse in Quantum Mechanics. Lettere Nuovo Cimento 1981; 31(11): 380-382

Conte E. A Predictive Model of psi Collapse Retrocollapse of Quantum Mechanics. Lettere Nuovo Cimento 1981; 32(9): 286-288.

Conte E, Khrennikov A, Todarello O, Fedrici A, Mendolicchio L, Zbiult JP. Mental States Follow Quantum Mechanics During Perception and Cognition of Ambiguous Figures. Open Systems and Information Dynamics 2009; 15(1): 1-17.

Conte E. On the Possibility That We Think in a Quantum Probabilistic Manner. NeuroQuantology 2010a; 8(4): S3-S47

Conte E. A Reformulation of von Neumann's postulates On Quantum Measurement by Using the Clifford Algebra. International Journal of Theoretical Physics 2010b; 49(3): 587-614

Conte E. May We Verify Non-Existing Dispersion Free Ensembles by Application of Quantum Mechanics in Experiments at Perceptive and Cognitive Level? NeuroQuantology 2012; 10(1): 14-19

Conte E. Additional Comments to a Recent Answer to G. Ghirardi. Journal of Modern Physics 2015a; 6(1): 12-15.

Conte E. On Some Explanations and Analysis of the Basic Foundations of Quantum Cognition: Comments on a Paper by Pothos, Busemeyer and Trueblood. NeuroQuantology 2015b; 13(13): 371-383.

Conte E. A Brief Comment on Some Recent Evaluations by Basieva and Khrennikov, Wang *et al.,* Boyer-Kassem *et al.,* on Order Effects in Quantum Cognition. NeuroQuantology 2015c; 13(2): 250-252

Costa de Beauregard O. Two Principles of the Science of Time. Annals of the New York Academy of Sciences 2006; 138: 407-421.

Cramer JG. An Overview of the Transactional Interpretation. International Journal of Theoretical Physics 1988; 27: 227-231.

Landsberg PT. Does Quantum Mechanics Exclude life? Nature 1964; 203: 928-930.

Wheeler JA. The 'Past' and the 'Delayed-Choice' Double-Slit Experiment. A.R. Marlow, editor, Mathematical Foundations of Quantum Theory, 1978; Academic Press, 9–48

Wheeler JA, Feynman R. Interaction with the Absorber as the Mechanism of Radiation. Reviews of Modern Physics 1945; 17(2–3): 157–161.

Wigner EP. Group Theory and its Application to the Quantum Mechanics of Atomic Spectra, translated from the German by J. J. Griffin, Expanded and Improved Edition Academic Press, New York, 1959.