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Gravitational Brainwaves, Quantum Fluctuations and Stochastic Quantization
Part-I

Daniel Bar

Abstract
It is known that the biological activity of the brain involves radiation of electric waves. These waves result from ionic currents and charges traveling among the brain's neurons. However, it is obvious that these ions and charges are carried by their relevant masses that should give rise, according to the gravitational theory, to extremely weak gravitational waves. We use in Part I of this work the stochastic quantization (SQ) theory to calculate the probability to find a large ensemble of brains radiating similar gravitational waves. We also use this SQ theory in Part II of this work to derive the equilibrium state of the quantum fluctuations related to the known Lamb shift.

Key Words: brainwaves, gravitational waves, stochastic quantization
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I. Introduction
As known, the human brain radiates, during its biological activity, several kinds of electric waves (EW) which are generally classified as the α, β, δ and θ waves (Freeman, 1975; Tran, 2001). These EW, which differ in their frequencies (Hz) and amplitudes (μV) and are detected by electrodes attached to the scalp, are tracked to the human states (Freeman, 1975) such as relaxation (related to the α waves), alertness (related to the β waves) and sleep which gives rise to the δ and θ waves. The source of these EW are the neurons in the cerebral cortex which are transactional cells which receive and transmit among them inputs and outputs in the form of ionic electric currents over short and long distances within the brain (Chapter 1 in Freeman, 1975). These ionic electric currents are, of course, electric charges in motion which may be calculated through the known Gauss law (Halliday and Resnick, 1978). That is, assuming the brain is surrounded by some hypothetical surface S one may measure, using the mentioned electrodes, the electric field which crosses that surface so that he can calculate, using Gauss law (Halliday and Resnick, 1978)

$$\oint E_c \cdot ds = C_c q = \frac{q}{\varepsilon_0}$$

the charge q inside the brain which is related to the measured EW. The E_c in the former Gauss's law is the electric field vector and ε_0 is the permittivity constant. But as known, any ion and any charge q has a mass m which actually carries it so that one may use the corresponding Gauss's law for gravitation (P. 618 in Halliday and Resnick, 1978)

$$\oint E_g \cdot ds = C_g m = -4\pi G \cdot m$$

to relate the mass m to the gravitational field vector E_g which is identified at the neighbourhood of the earth surface with the

Corresponding author: Daniel Bar
Address: P. O. B: 1076, Ashdod, Israel
e-mail: bardan2006@nana.co.il

gravitational acceleration g, i. e., $E_g = g$. The constant G is the universal gravitational constant and the gravitational field vector at the earth surface E_g is a specific case of the generalized gravitational waves (GW) which have tensorial properties (Misner et al, 1973; Hartle, 2003;Thorn, 1980a; Thorn, 1980b). These GW are very much weak compared to the corresponding EW as may be seen by comparing (in the MKS system) the constants which multiply the mass m and charge q in the former two Gauss's laws, e.g,

$$|\frac{C_g}{C_c}| = \frac{4\pi G}{\frac{1}{\varepsilon_0}} = 4\pi \cdot 6.672 \cdot 10^{-11} \frac{Nm^2}{kg^2}$$
$$\cdot 8.854 \cdot 10^{-12} \frac{C^2}{Nm^2} = 7.4234 \cdot 10^{-25} \frac{C^2}{kg^2}.$$

One may, however, consider the real situation in which the mentioned GW's originate not from one human brain but from a large ensemble of them. Thus, if these waves have the same wavelength and phase they may constructively interfere (Bar, 2007a; Bar, 2007b) with each other to produce a resultant significant GW. It has been shown (Bar, 2007a; Bar, 2007b), comparing gravitational waves with the electromagnetic ones, that the former may also display constructive or destructive interference as well as holographic properties.

We emphasize here before anything else that this work is not about consciousness, mind or thinking at all (the way discussed, for example, by Roger Penrose in his books (Penrose, 1989; Penrose, 1994) or in (Arx, 2001)) but use only the assumption that the mass, associated with the ionic currents and charges in the brain, should be involved with gravitational field as all masses do. But, in contrast to the electromagnetic waves, no GW of any kind and form were directly detected up to now, except through indirect methods,[2] even with the large terrestrial interferometric Ligo (Abbott et al, 2004), Virgo (Acernese et al, 2002), Geo (Danzmann, 1995) and Tama (Ando, 2002) detectors. Morover, in contrast to other physical waves (for example, the electromagnetic waves), GW's do not propagate as three-dimensional (3D) oscillations in the background of the stationary four-dimensional (4D) spacetime but are themselves perturbations of this spacetime itself (Misner et al, 1973; Hartle, 2003; Thorn, 1980a; Thorn, 1980b). That is, the geometry of spacetime curves and oscillates in consequence of the presence of the passing GW so that, in case it is strong enough, it may even impose its own geometry upon the traversed spacetime (Beig and Murchadha, 1991; Abrahams and Evans, 1992). Thus, the GW is an inherent part of the involved 4D spacetime in the sense that its geometry is reflected in the related metric form ds^2. This is seen, for example, in the metric form of the cylindrical spacetime (Einstein and Rosen, 1937; Kuchar, 1971) or in the linearized version of general relativity where one uses the flat Minkowsky metric form to which a small perturbation is added which denotes the appropriate weak passing GW (Misner et al, 1973).

No one asks in such cases if these 4D perturbations, which propagate as GW's, occur in the background of some stationary higher dimensional neighbourhood. One may, however, argue that as other physical waves, such as the electromagnetic ones, are considered as 3D oscillations in the background of the stationary 4D spacetime so the GW's may also be discussed as 4D oscillations in the background of a stationary 5D neighbourhood. This point of view was taken in the known Kaluza's 5D theory and in the projective field formulations of general relativity (unified field theories, Chapter XVII in (Bergmann, 1976)) where it was shown that the related expressions in the 5D spacetime were decomposed not only to the known Einstein field equations but also to the not less known Maxwell equations.

In this work, which comprises two parts, we discuss in the first part of it GW from this point of view and use the stochastic quantization (SQ) of Paris-Wu-Namiki (Namiki, 1992; Parisi and Wu, 1981) which is known to yield by a unique limiting process the equilibrium state of many classical and quantum phenomena (Namiki, 1992). An important and central element of the SQ is the assumption of an extra dimension termed in (Namiki, 1992) fictitious

[2] Gravitational waves were indirectly proved by Taylor and Hulse (which receive the Nobel price in 1993 for this discovery) through astronomical observations which measure the spiraling rate of two neighbouring neutron stars.

time in which some stochastic process, governed by either the Langevin (Coffey, 1996) or the Fokker-Plank (Risken, 1984) equations, is performed. Thus, one may begin from either one of the two mentioned equations, which govern the assumed stochastic process in the extra dimension, and ends, by a limiting process in which all the different values of the relevant extra variable (denoted s) are equated to each other and taken to infinity (Namiki, 1992), in the equilibrium state. The main purpose of the SQ theory (Namiki, 1992) is to obtain the expectation value of some random quantity or the correlation function of its variables.

In Part I of this work we consider, as an example of stochastic process which may be discussed in the framework of the Parisi-Wu-Namiki SQ, the mentioned activity of the human brain. That is, as it is possible to calculate the correlation between a large ensemble of brains in the sense of finding them radiating similar EW's so one may, theoretically, discuss the probability to find them radiating similar GW's. We show that although, as mentioned, the GW radiated by one brain is negligible compared to the related EW the correlation between the GW's radiated from a large number of them may not be small. But in order to be able to properly calculate this correlation we should discuss some specific kind, from a possible large number of kinds, of GW's. Thus, we particularize to the cylindrical one and calculate the probability (correlation) to find an ensemble of n human brains radiating cylindrical GW's. We do this by calculating this correlation in the extra dimension and show that once it is equated to unity one finds that in the stationary state (where the extra variable is eliminated) *all the ensemble of brains radiate similar cylindrical GW's*. As mentioned, no one has directly detected, up to now, any kind of GW so all our discussion is strictly theoretical in the hope that some day in the future these GW may at last be directly detected.

As mentioned, the SQ theory is suitable for discussing stochastic and unpredictable phenomena which should be analyzed by correlation terminology and probability terms. Thus, we found it convenient to discuss in Part II of this work the electron-photon interaction which originates from quantum fluctuations (Nelson, 1985) and results in the known Lamb shift (Lamb, 2001; Lamb and Sargent, 1974; Hansch et al, 1972a; Hansch et al, 1972b) by the SQ methods. Note that these quantum fluctuations stand at the basis of Quantum Gravity which is believed to be the mechanism which operates also in the activity of human consciousness and thinking (Penrose, 1989; Penrose, 1994; Arx, 2001). We first calculate (in Appendix C of Part II) the states of the electron and photon and the interaction between them in the extra dimension and then show that in the limit of eliminating the extra variable one obtains the known expressions which characterize the Lamb shift (Lamb, 2001; Lamb and Sargent, 1974; Hansch et al, 1972a; Hansch et al 1972b; Haken, 1981).

The formalism and the main expressions of the Parisi-Wu-Namiki SQ theory are detaily represented in Appendix A of Part II of this work. We, especially, introduce the expressions for the correlation among an ensemble of variables along given intervals of the time t and the extra variable s. In our discussion here of the cylindrical GW we use the fact emphasized in (Kuchar, 1971) that *the ADM canonical formalism for the cylindrical GW is completely equivalent to the parametrized canonical formalism for the cylindrically symmetric massless scalar field on a Minkowskian spacetime background*. Moreover, as also emphasized in (Kuchar, 1971), one may use the half-parametrized formalism of the mentioned canonical formalism without losing any important content. Thus, in Section II we introduce a short review of this half parametrized cylindrical massless scalar field in the background of the Minkowsky spacetime where use is made of the results in (Kuchar, 1971). In Section III we represent and discuss the cylindrical GW in the framework of the SQ formalism and introduce the probability that a large ensemble of brains are found to radiate cylindrical GW's. This probability is calculated in a detailed manner in Appendix B of Part II. In Section IV we realize that the somewhat complex expression of the calculated probability in the extra dimension is greatly simplified at the mentioned stationary limit so that one may clearly see that for a unity value of it all the n-brain ensemble radiate the same cylindrical GW's. In Section V we summarize the discussion.

II. The massless cylindrical wave in the Minkowskian background

As discussed in Appendix A in Part II of this work the stochastic process in the extra dimension s is described by the n variables $\psi(s,t)=(\psi_0(s,t),\psi_1(s,t),\ldots\psi_{(n-2)}(s,t),\psi_{(n-1)}(s,t))$ where the finite intervals $(s_{(0)},s)$, $(t_{(0)},t)$ of s and t during which the former process "evolutes" are assumed each to be subdivided into N subintervals $(t_{(0)},t_1),(t_1,t_2),\ldots(t_{(N-1)},t)$ and $(s_{(0)},s_1),(s_1,s_2),\ldots(s_{(N-1)},s)$. In the application of the SQ formalism for the ensemble of brains we identify the mentioned ensemble of n variables $\psi_i(s,t)$, $0\le i\le(n-1)$, which describe the stochastic process in the extra dimension s, with the ensemble of brains. This ensemble of variables (brains) is related, as is customary in the SQ theory, to the corresponding ensemble of random forces $\eta(s,t)=(\eta_0(s,t),\eta_1(s,t),\ldots\eta_{(n-2)}(s,t),\eta_{(n-1)}(s,t))$.

As mentioned, our aim is to calculate the correlation between the n-member ensemble of brains with respect to the cylindrical GW. That is, according to the results of Appendix B, we calculate the conditional probability to find this ensemble of brains radiating at t and s the cylindrical GW's $\psi(s,t)$ if they were found at $t_{(N-1)}$ and $s_{(N-1)}$ radiating the cylindrical GW's $\psi(s_{(N)},t_{(N)})$ .... and at $t_{(N-3)}$ and $s_{(N-3)}$ they were found radiating the cylindrical GW's $\psi(s_{(N-2)},t_{(N-2)})$ .... and at $t_{(0)}$ and $s_{(0)}$ they were radiating the cylindrical GW's $\psi(s_{(1)},t_{(1)})$ (see the discussion after Eqs (B_10), (B_13) and (B_14) in Appendix B of Part II). As mentioned, the cylindrical GW, in its ADM canonical formalism (Arnowitt et al, 1962; Misner et al, 1973), is completetly equivalent (Kuchar, 1971) to the parametrized canonical formalism for the cylindrically symmetric massless scalar field in a Minkowskian background. Thus, for introducing the relevant expressions related to the cylindrical GW (Kuchar, 1971) we write the action functional S for the massless cylindrical wave in the Minkowskian background (Kuchar, 1971; Misner et al, 1973; Arnowitt et al, 1962)

$$S=2\pi\int_{-\infty}^{\infty}dT\int_{(0)}^{\infty}dRL, \tag{1}$$

where L is the Lagragian density (Kuchar, 1971)

$$L=\frac{1}{2}R((\psi_{,T})^2-(\psi_{,R})^2) \tag{2}$$

The T denotes the Minkowskian time and R is the radial distance from the symmetry axis in flat space (Kuchar, 1971). The expressions $\psi_{,T}$, and $\psi_{,R}$ denote the respective derivatives of ψ with respect to T and R. In the parametrized canonical formalism in a Minkowskian background one have to introduce (Kuchar, 1971) curvilinear coordinates t and r in flat space

$$\begin{aligned} t&=t(T,R), \quad r=r(T,R) \\ T&=T(t,r), \quad R=R(t,r) \end{aligned} \tag{3}$$

As shown in (Kuchar, 1971) one may discuss the cylindrical scalar waves in a half-parametrized canonical formalism without losing any physical content except for the spatial covariance of the scalar wave formalism (Kuchar, 1971). In this half-parametrized canonical formalism one use the following coordinates

$$r=R, \quad t=t(T,R) \tag{4}$$

It was shown in (Kuchar, 1971), using Eqs (1)-(2) and (4), that the action S assumes the simplified form

$$\begin{aligned} S&=2\pi\int_{-\infty}^{\infty}dt\int_{(0)}^{\infty}dRL= \\ &2\pi\int_{-\infty}^{\infty}dt\int_{(0)}^{\infty}dR(\Pi_T T_{,t}+\pi_\psi\psi_{,t}-NH), \end{aligned} \tag{5}$$

where $T_{,t}$ and $\psi_{,t}$ denote derivatives of T and ψ with respect to t. The N is a Lagrange multiplier and H is (Kuchar, 1971)

$$H=\Pi_T+\underline{H}, \tag{6}$$

where $\underline{H}$ and Π_T are related as (Kuchar, 1971)

$$\begin{aligned} &\underline{H}=-\Pi_T=\frac{1}{2}(1-T_{,R}^2(R,t))^{-1} \\ &(-iR^{-\frac{1}{2}}\frac{\delta}{\delta\psi(R,t)}-R^{\frac{1}{2}}T_{,R}(R,t)\psi_{,R}(R,t))^2+ \\ &+\frac{1}{2}R\psi_{,R}^2(R,t)=\frac{1}{2(1-T_{,R}^2(R,t))}\cdot \\ &\cdot(R^{-1}\pi_\psi^2(R,t)-2T_{,R}(R,t)\pi_\psi(R,t)\psi_{,R}(R,t)+ \\ &+R\psi_{,R}^2(R,t)) \end{aligned} \tag{7}$$

The last result were obtained by using the following definition of the momentum operator $\pi_\psi(R,t)$

$$\pi_\psi(R,t) = -i\frac{\delta}{\delta(\psi(R,t))} \quad \textbf{(8)}$$

From Eqs (6)-(7) one realizes that H satisfies the constraint (Kuchar, 1971)

$$H = 0 \quad \textbf{(9)}$$

Note that we do not discuss yet the SQ theory with the extra dimension which will be discussed in the following section. Eqs-(9) ensure that in the framework of the half parametrized canonical formalism the following variational principle is satisfied (Kuchar, 1971)

$$\begin{aligned}\delta S &= \delta\{2\pi\int_{-\infty}^{\infty}dt\int_{(0)}^{\infty}dR(\Pi_T T_{,t}(R,t) \\ &+\pi_\psi(R,t)\psi_{,t}(R,t) - NH)\} = 0,\end{aligned} \quad \textbf{(10)}$$

where all variables T, Π_T, $\psi(R,t)$, $\pi_\psi(R,t)$, and N may be varied freely (Kuchar, 1971). Note that the function Π_T may be represented as the operator (Kuchar, 1971) $\Pi_T = -i\frac{\delta}{\delta(T(R,t))}$. Also, it should be remarked that the commutation relation between $\pi_\psi(R,t)$ and $\psi_{,R}(R,t)$ is zero at the same point, i.g.,

$$\begin{aligned}[\psi_{,R}(R,t),\pi_\psi(R',t)] &= i\frac{\delta(\psi_{,R}(R,t))}{\delta\psi(R',t)} \\ &= i\frac{d}{dR}(\frac{\delta(\psi(R,t))}{\delta\psi(R',t)}) = i\frac{d\delta(R-R')}{dR} = 0\end{aligned}$$

since the δ function is antisymmetric so that one have $\frac{d\delta(0)}{dR} = 0$. The wave function $\psi(R,T)$ (not in the half-parametrized formalism), which is obtained as a solution of the Einstein field equations for the cylindrical line element, is generally represented as an integral over all modes k

$$\psi(R,T) = \int_{(0)}^{\infty}dkJ_0(kR)(A(k)e^{(ikT)} + A^*(k)e^{-(ikT)}) \quad \textbf{(11)}$$

where $j_0(kR)$ is the bessel function of order zero (Abramowitz and Steegun, 1970). The quantities $A(k), A^*(k)$ denote the amplitude and its complex conjugate for some specific mode k. Note that here one assumes, as done in the literature, $c = \hbar = 1$ so that $w = \tilde{k} = p$ where w is the frequency, $\tilde{k}$ the wave number and p the momentum of some mode. The momentum $\pi_\psi(T,R)$, canonically conjugate to $\psi(R,T)$, may be obtained (Kuchar, 1971) by solving the Hamilton equation

$$\frac{\partial\psi(R,T)}{\partial t} = \{\psi(R,T),H\}, \quad \textbf{(12)}$$

where $\psi(R,T)$ is from Eq (11) and the curly brackets at the right denote the Poisson brackets. The Hamilton function H is (Kuchar, 1971)

$$H = \int_{(0)}^{\infty}dr(\tilde{N}\tilde{H} + \tilde{N}^1\tilde{H}_1) \quad \textbf{(13)}$$

where $\tilde{H}$ and $\tilde{H}_1$ are respectively the rescaled superHamiltonian and supermomentum which where given in (Kuchar, 1971) (Eqs (93)-(97) and (106)-(108) in (Kuchar, 1971)) as

$$\begin{aligned}\tilde{H} &= R_{,r}\Pi_T + T_{,r}\Pi_R + \frac{1}{2}R^{-1}\pi_\psi^2(R,T) + \frac{1}{2}R\psi_{,r}^2(R,T) \\ \tilde{H}_1 &= T_{,r}\Pi_T + R_{,r}\Pi_R + \psi_{,r}(R,T)\pi_\psi(R,T)\end{aligned} \quad \textbf{(14)}$$

The quantities $\psi_{,r}(R,T)$, $T_{,r}$ $R_{,r}$ denote differentiation of $\psi(R,T)$, T, R with respect to r (where in the half-parametrized formalism $R_{,r} = 1$ as realized from the first of Eqs (4)) and Π_T, Π_R are the respective momenta canonically conjugate to T and R. The $\tilde{N}$ and $\tilde{N}^1$ from Eq (13) respectively denote the rescaled lapse and shift function N, N^1 (Eqs (96) in (Kuchar, 1971)). Thus, the $\pi_\psi(T,R)$ in the half-parametrized formalism were shown (Kuchar, 1971) to be

$$\begin{aligned}\pi_\psi(T,R) &= R(\frac{(1-T_{,R}^2)}{T_{,t}}\psi_{,t}(R,T) \\ &+T_{,R}\psi_{,R}(R,T)) = iR(1-T_{,R}^2)\int_{(0)}^{\infty}dkkJ_0(kR)\cdot \\ &\cdot(A(k)e^{(ikT)} - A^*(k)e^{-(ikT)}) - \\ &-RT_{,R}\int_{(0)}^{\infty}dkkJ_1(kR)(A(k)e^{(ikT)} + \\ &+A^*(k)e^{-(ikT)}) + iR\int_{(0)}^{\infty}dkkJ_0(kR)(T_{,R})^2(A(k)e^{(ikT)} \\ &-A^*(k)e^{-(ikT)})\end{aligned} \quad \textbf{(15)}$$

where $j_1(kR)$ is the first order Bessel function (Abramowitz and Steegun, 1970) obtained by differentiating $j_0(kR)$ with respect to R, e.g., $j_0(kR)_{,R} = -kj_1(kR)$. Note that one may express, using the expression (Abramowitz and Steegun, 1970)

$$\int_{(0)}^{\infty} dr'r' \int_{(0)}^{\infty} dkkJ_n(kr)J_n(kr')f(r') = f(r),$$

the observables $A(k)$ and $A^*(k)$ in terms of $\psi(R,T)$ and $\pi_\psi(R,T)$ as

$$A(k) = \frac{1}{2}\int_{(0)}^{\infty} dRe^{-ikT}\{Rk[\psi(R,T)(J_0(kR) - -iT_{,R}J_1(kR))] - iJ_0(kR)\pi_\psi(R,T)\}$$

$$A^*(k) = \frac{1}{2}\int_{(0)}^{\infty} dRe^{ikT}\{Rk[\psi(R,T)(J_0(kR) + +iT_{,R}J_1(kR))] + iJ_0(kR)\pi_\psi(R,T)\} \quad (16)$$

III. The cylindrical GW in the SQ formalism

We, now, discuss the cylindrical GW from the SQ point of view and begin by writing the Langevin equation (A_1) of Appendix A in Part II for the subintervals $(t_{(k-1)}, t_k)$ and $(s_{(k-1)}, s_k)$ in the following form (Namiki, 1992)

$$\frac{\psi_i^k(s) - \psi_i^{k-1}(s)}{(s_k - s_{(k-1)})} - K_i(\psi^{k-1}(s)) = \eta_i^k(s), \quad (17)$$

where $\frac{d\psi_i}{ds_k} \approx \frac{\psi_i^k - \psi_i^{(k-1)}}{s_k - s_{(k-1)}}$ and the $\eta_i(s)$ are conditioned as (Namiki, 1992)

$$<\eta_i(s)>= 0, \quad <\eta_i(s)\eta_j(s`)>= \begin{cases} 0 & for\ s \neq s` \\ 2\alpha\delta_{ij} & for\ s = s` \end{cases} \quad (18)$$

Note that although the s dependence is emphasized in the last two equations one should remember that there exist also spatial and time dependence (see the following discussion and Eq 19). The α in Eq (23) is as discussed after Eq (A_3) of Appendix A of part II. The appropriate K_i for the massless cylindrical scalar wave in the Minkowskian background may be obtained by using Eq (A_2) in Appendix A of part II and Eq-5 from which one realizes that the Lagrangian L depends upon two independent variables t, R and five dependent varables $\psi(R,t)$, $\pi_\psi(R,t)$, $T(R,t)$, Π_T, N. Note that in the following we represent ψ and π_ψ by the expressions from Eqs (11) and (15) as mentioned after the following Eq (27). Thus, although the functions ψ and π_ψ should be denoted, because of that, as $\psi(R,T)$ and $\pi_\psi(R,T)$ we denote them as $\psi(R,t)$ and $\pi_\psi(R,t)$ and take, of course, into account the dependence of T upon r and t as realized, for example, in the following Eqs (27). The mentioned dependence of L upon the dependent variables include in our case, as seen from Eqs (6)-(7) and (15), dependence of L also upon some derivatives of them, i.e., $\psi_{,t}$, $\psi_{,R}$, $T_{,t}$, $T_{,R}$. Thus, the involved variation of δS is given by

$$\delta S = 2\pi\int_{-\infty}^{\infty} dt \int_{(0)}^{\infty} dR\delta L$$
$$= 2\pi\int_{-\infty}^{\infty} dT \int_{(0)}^{\infty} dR(\frac{\partial L}{\partial \psi}\delta\psi + \frac{\partial L}{\partial \psi_{,R}}\delta\psi_{,R}$$
$$+\frac{\partial L}{\partial \psi_{,t}}\delta\psi_{,t} + +\frac{\partial L}{\partial T}\delta T + \frac{\partial L}{\partial T_{,R}}\delta T_{,R} + \frac{\partial L}{\partial T_{,t}}\delta T_{,t} \quad (19)$$
$$+\frac{\partial L}{\partial \pi_\psi}\delta\pi_\psi + \frac{\partial L}{\partial \Pi_T}\delta\Pi_T + \frac{\partial L}{\partial N}\delta N)$$

As seen from Eq (17) we are interested in calculating the function K_i which is given by Eqs (23) and (A_2) in Appendix A of Part II as

$$K_i(\psi^{k-1}(s)) = -(\frac{\delta S_i[\psi]}{\delta\psi})_{\psi=\psi(s,t,x)}$$

where the function ψ as function of s is introduced only after varying the action S_i as functional of ψ. Also, in order to deal with compact and simplified expressions, as done, for example, in Eqs (19 and 24), we do not always write the various functions such as ψ, π_ψ, T etc in their full dependence upon R and T.

We, now, should realize that the integrand in the last equation (19) is the total differential δL, whereas we are interested in $K_i(\psi^{k-1}(t_k, s_k))$ which is seen from Eqs (23) and (A_2) in Appendix A of Part II to be equal to the negative variation of the action S_i with respect to ψ. Thus, according to the definition of S from Eq (1) $K_i(\psi^{k-1}(t_k, s_k))$ should involve the R and t integration of the negative variation of the Lagrangian L with respect to ψ. That is, we should consider only the first three terms of Eq (19) which are related to ψ and its derivatives. Thus, for calculating the variations of these derivatives we note that $\delta\psi_{,t}$, $\delta\psi_{,R}$ are the respective differences between the original and varied $\psi_{,t}$, $\psi_{,R}$ and, therefore, they may be written as (P. 493 in (Schiff, 1968))

$$\delta\psi_{,t} = \frac{\partial(\delta\psi)}{\partial t},\ \delta\psi_{,R} = \frac{\partial(\delta\psi)}{\partial R} \qquad \textbf{(20)}$$

Using the former discussion and the last equations (20) one may write the appropriate expression for δS as

$$\delta S = \frac{\delta S}{\delta\psi}\delta\psi = 2\pi\int_{-\infty}^{\infty}dt\int_{(0)}^{\infty}dR(\frac{\partial L}{\partial\psi}\delta\psi + \frac{\partial L}{\partial(\psi_{,R})}\frac{\partial(\delta\psi)}{\partial R} + \frac{\partial L}{\partial(\psi_{,t})}\frac{\partial(\delta\psi)}{\partial t}) \qquad \textbf{(21)}$$

The second term at the right hand side of the last equation may be integrated by parts with respect to R where the resulting surface terms are assumed to vanish because ψ tends to zero at infinite distances (Kuchar, 1971). The third term at the right hand side of Eq (21) may also be integrated by parts with respect to t where the boundary terms vanish because of the following assumed conditions of the variational principle (Weinstock, 1974)

$\delta\psi(R,-\infty) = \delta\psi(R,+\infty) = 0$.
Thus, Eq (21) becomes

$$\delta S = 2\pi\int_{-\infty}^{\infty}\int_{(0)}^{\infty}(\frac{\partial L}{\partial\psi} - \frac{\partial}{\partial R}(\frac{\partial L}{\partial(\psi_{,R})}) - \frac{\partial}{\partial t}(\frac{\partial L}{\partial(\psi_{,t})}))\delta\psi\, dtdR \qquad \textbf{(22)}$$

We note that analogous discussion regarding the quantization of wave fields may be found at pages 492-493 in (Schiff, 1968). Thus, using the former discussion and Eq (22) one may write the following expression for $K_i(\psi^{k-1}(t_k,s_k))$

$$\begin{aligned} K_i(\psi^{k-1}(t_k,s_k)) &= -(\frac{\delta S_i[\psi]}{\delta\psi})_{\psi=\psi(s,t,x)} = \\ &-2\pi\int_{-\infty}^{\infty}dt\int_{(0)}^{\infty}dR\frac{\delta L}{\delta\psi} = \\ &= -2\pi\int_{-\infty}^{\infty}\int_{(0)}^{\infty}(\frac{\partial L}{\partial\psi} - \frac{\partial}{\partial R}(\frac{\partial L}{\partial(\psi_{,R})}) \\ &- \frac{\partial}{\partial t}(\frac{\partial L}{\partial(\psi_{,t})}))dtdR \end{aligned} \qquad \textbf{(23)}$$

In order to obtain final calculable results we use, as mentioned, for ψ and π_ψ the respective expressions of Eqs (11) and (15). Also, noting that π_ψ from Eq (15) depends upon the derivatives $\psi_{,R}$, $\psi_{,t}$ one may use Eqs (5)-(7) and (9) to calculate the three expressions in the integrand of the last equation (23) as follows

$$\begin{aligned} &\frac{\partial L}{\partial\psi} = 0 \\ &\frac{\partial L}{\partial(\psi_{,R})} = T_{,t}\frac{\partial\Pi_T}{\partial(\psi_{,R})} + \psi_{,t}\frac{\partial\pi_\psi}{\partial(\psi_{,R})} \\ &= -\frac{T_{,t}}{2(1-T_{,R}^2)}(2R^{-1}\pi_\psi\frac{\partial\pi_\psi}{\partial(\psi_{,R})} - 2T_{,R}\pi_\psi - \\ &-2T_{,R}\psi_{,R}\frac{\partial\pi_\psi}{\partial(\psi_{,R})} + 2R\psi_{,R}) + \psi_{,t}\frac{\partial\pi_\psi}{\partial(\psi_{,R})} \\ &= -\frac{T_{,t}}{(1-T_{,R}^2)}(R\psi_{,R} - R\psi_{,R}(T_{,R})^2) + \\ &\quad +\psi_{,t}RT_{,R} = R(\psi_{,t}T_{,R} - T_{,t}\psi_{,R}) \\ &\quad \frac{\partial L}{\partial(\psi_{,t})} = T_{,t}\frac{\partial\Pi_T}{\partial(\psi_{,t})} + \pi_\psi + \psi_{,t}\frac{\partial\pi_\psi}{\partial(\psi_{,t})} \\ &= -\frac{T_{,t}}{2(1-T_{,R}^2)}(2R^{-1}\pi_\psi\frac{\partial\pi_\psi}{\partial(\psi_{,t})} - \\ &\quad -2T_{,R}\psi_{,R}\frac{\partial\pi_\psi}{\partial(\psi_{,t})}) + \pi_\psi + \psi_{,t}\frac{\partial\pi_\psi}{\partial(\psi_{,t})} \\ &= -(R(\frac{(1-T_{,R}^2)}{T_{,t}}\psi_{,t} + T_{,R}\psi_{,R}) - \\ &\quad -RT_{,R}\psi_{,R}) + 2\frac{R(1-T_{,R}^2)}{T_{,t}}\psi_{,t} \\ &+RT_{,R}\psi_{,R} = \frac{R(1-T_{,R}^2)}{T_{,t}}\psi_{,t} + RT_{,R}\psi_{,R} \end{aligned} \qquad \textbf{(24)}$$

As seen from Eq (23) the expressions

$$\frac{\partial L}{\partial(\psi_{,R})} \text{ and } \frac{\partial L}{\partial(\psi_{,t})}$$

should be respectively differentiated with respect to R and t. Thus, taking into account that these derivatives serve as integrands of integrals over R and t and using Eqs (24) one may write Eq (23) as

$$\begin{aligned}K_i(\psi^{k-1}(t_k,s_k)) &= 2\pi\{\int_{-\infty}^{\infty}(\int_{(0)}^{\infty}\frac{\partial}{\partial R}(\frac{\partial L}{\partial(\psi_{,R})})dR)dt \\ &+\int_0^{\infty}(\int_{-\infty}^{\infty}\frac{\partial}{\partial t}(\frac{\partial L}{\partial(\psi_{,t})})dt)dR\} = \\ &= 2\pi\{\int_{-\infty}^{\infty}dt(\frac{\partial L}{\partial(\psi_{,R})})|_{R=0}^{R=\infty} + \int_0^{\infty}dR(\frac{\partial L}{\partial(\psi_{,t})})|_{t=-\infty}^{t=\infty}\} \\ &= 2\pi\{\int_{-\infty}^{\infty}dt(R(\psi_{,t}T_{,R} - \\ &-T_{,t}\psi_{,R}))|_{R=0}^{R=\infty} + \int_{R=0}^{R=\infty}dR(\frac{R(1-T_{,R}^2)}{T_{,t}}\psi_{,t} \\ &+RT_{,R}\psi_{,R})|_{t=-\infty}^{t=\infty}\}\end{aligned} \quad \textbf{(25)}$$

In the following we use the boundary values related to the function T (Section III in (Kuchar, 1971))

$$\lim_{t\to\pm\infty} T = t, \quad \lim_{r\to\infty} T = t \quad \textbf{(26)}$$

Also, because of representing ψ through the expression (11), one may use the relation (Abramowitz and Steegun, 1970) $j_0(kR)_{,R} = -kj_1(kR)$ in order to write the derivatives of ψ with respect to t and R as

$$\begin{aligned}\frac{\partial\psi(R,t)}{\partial R} &= -\int_{(0)}^{\infty}dkkJ_1(kR)(A(k)e^{(ikT)} + A^*(k)e^{-(ikT)}) + \\ &+i\int_{(0)}^{\infty}dkkT_{,R}J_0(kR)(A(k)e^{(ikT)} + A^*(k)e^{-(ikT)}) \\ \frac{\partial\psi(R,t)}{\partial t} &= iT_{,t}\int_{(0)}^{\infty}dkkJ_0(kR)(A(k)e^{(ikT)} - A^*(k)e^{-(ikT)})\end{aligned} \quad \textbf{(27)}$$

Note that the leading terms of the Bessel's functions of integer orders in the limits of very small and very large arguments are (Abramowitz and Steegun, 1970; Schiff, 1968)

$$\begin{aligned}\lim_{R\to 0} J_n(R) &= \frac{R^n}{(2n+1)!!}, \\ \lim_{R\to\infty} J_n(R) &= \frac{1}{R}\cos(R - \frac{(n+1)\pi}{2}),\end{aligned} \quad \textbf{(28)}$$

where $(2n+1)!! = 1\cdot 3\cdot 5\cdots(2n+1)$. From the last limiting relations one obtains for $J_0(kR)$ and $J_1(kR)$

$$\begin{aligned}\lim_{kR\to 0} J_0(kR) &= 1, \\ \lim_{kR\to\infty} J_0(kR) &= \frac{1}{kR}\cos(kR - \frac{\pi}{2}) \\ \lim_{kR\to 0} J_1(kR) &= \frac{kR}{1\cdot 3}, \\ \lim_{kR\to\infty} J_1(kR) &= \frac{1}{kR}\cos(kR - \pi)\end{aligned} \quad \textbf{(29)}$$

Taking into account Eqs (27) and the derivative $j_0(kR)_{,R} = -kj_1(kR)$ one may realize that the right hand side of Eq (25) becomes

$$\begin{aligned}K_i(\psi^{k-1}(t_k,s_k)) &= 2\pi\int_{-\infty}^{\infty}dt(RT_{,R}\psi_{,t} - RT_{,t}\psi_{,R})|_{R=0}^{R=\infty} \\ &+2\pi\int_{R=0}^{R=\infty}dR(\frac{R(1-T_{,R}^2)}{T_{,t}}\psi_{,t} + \\ &+RT_{,R}\psi_{,R})|_{t=-\infty}^{t=\infty} \\ &= 2\pi\int_{-\infty}^{\infty}dt[\int_{(0)}^{\infty}dkkRT_{,t}J_1(kR)(A(k)e^{(ikT)} + \\ &+A^*(k)e^{-(ikT)})]|_{R=0}^{R=\infty} \\ &+2\pi\int_0^{\infty}dR[i\int_{(0)}^{\infty}dkkRJ_0(kR)(A(k)e^{(ikT)} - \\ &-A^*(k)e^{-(ikT)}) \\ &-RT_{,R}\int_0^{\infty}dkkJ_1(kR)(A(k)e^{(ikT)} \\ &+A^*(k)e^{-(ikT)})]|_{t=-\infty}^{t=\infty}\end{aligned} \quad \textbf{(30)}$$

Using, now, (1) the limiting relations from Eqs (26) and (28-29 and 2) the basic complex relation $i^2 = -1$, (3) the trigonometric identity $2i\sin(\phi) = (e^{i\phi} - e^{-i\phi})$ and (4) the general property of Bessel's functions of integer orders (Abramowitz and Steegun, 1970)

$$\frac{d(x^nJ_n(x))}{dx} = x^nJ_{n-1}(x),$$

which reduces, for $n=1$, to

$$\frac{d(xJ_1(x))}{dx} = xJ_0(x)$$

it is possible to show that the first two terms at the right hand side of Eq (30) cancel each other

$$\begin{aligned}
&2\pi\int_{-\infty}^{\infty}dt[\int_{(0)}^{\infty}dkkRT_{,t}J_1(kR)(A(k)e^{(ikT)}\\
&+A^*(k)e^{-(ikT)})]|_{R=0}^{R=\infty}+\\
&+2\pi\int_0^{\infty}dR[i\int_{(0)}^{\infty}dkkRJ_0(kR)(A(k)e^{(ikT)}\\
&-A^*(k)e^{-(ikT)})]|_{t=-\infty}^{t=\infty}=\\
&=4\pi\int_0^{\infty}dk\frac{\cos(kR-\pi)}{k}\sin(kt)(A(k)\\
&+A^*(k))-4\pi\int_0^{\infty}dk\sin(kt)\cdot\\
&\cdot\frac{(A(k)+A^*(k))}{k}\int_{(0)}^{\infty}d(kR)\frac{d((kR)J_1(kR))}{d(kR)}\\
&=4\pi\int_0^{\infty}dk\frac{\sin(kt)}{k}cos(kR-\pi)\cdot\\
&\cdot(A(k)+A^*(k))-4\pi\int_0^{\infty}dk\frac{\sin(kt)}{k}(A(k)\\
&+A^*(k))\cos(kR-\pi)=0,
\end{aligned} \tag{31}$$

where we have passed in the last result from the integral variable R to kR and use the relation from Eqs (29)

$$J_1(kR)|_{kR=0}=\lim_{kR\to 0}J_1(kR)=\lim_{kR\to 0}\frac{kR}{1\cdot 3}\approx 0.$$

Thus, one remains with only the last term at the right hand side of Eq (30) which, using Eqs (11), (26) (29) and the integrals (Abramowitz and Steegun, 1970)

$$\int_{(0)}^{\infty}xJ_1(x)dx=-xJ_0\,|_0^{\infty}+\int_{(0)}^{\infty}J_0(x)dx$$

and

$\int_{(0)}^{\infty}J_0(x)dx=1$, may be reduced to

$$\begin{aligned}
&K_i(\psi^{k-1}(t_k,s_k))\\
&=-2\pi\int_{(0)}^{\infty}dk\int_{(0)}^{\infty}d(kR)kR\frac{J_1(kR)}{k}T_{,R}(A(k)e^{ikT}\\
&+A^*(k)e^{-ikT})|_{t=-\infty}^{t=\infty}=-2\pi\int_{(0)}^{\infty}dk(A(k)e^{ikt}\\
&+A^*(k)e^{-ikt}-A(k)e^{-ikt}\\
&-A^*(k)e^{ikt})T_{,R}(\int_{(0)}^{\infty}d(kR)J_0(kR)-\\
&-kRJ_0(kR)|_{kR=0}^{kR=\infty})=-4i\pi\int_{(0)}^{\infty}dk\sin(kt)(A(k)\\
&-A^*(k))T_{,R}\quad+2\pi\int_{(0)}^{\infty}dk(A(k)e^{ikt}\\
&+A^*(k)e^{-ikt}-A(k)e^{-ikt}\\
&-A^*(k)e^{ikt})T_{,R}kRJ_0(kR)|_{kR=0}^{kR=\infty}=\\
&=-4i\pi\int_{(0)}^{\infty}dk\sin(kt)(A(k)\\
&-A^*(k))T_{,R}+2\pi\lim_{kR\to\infty}kR\psi(t,R)T_{,R}-\\
&-\int_{(0)}^{\infty}dk\cos(kR-\frac{\pi}{2})(A(k)e^{-ikt}\\
&+A^*(k)e^{ikt})T_{,R}
\end{aligned} \tag{32}$$

We note, as emphasized in (Kuchar, 1971), that a hypersurface of constant time t is not assumed to have conical singularity on the axis of symmetry $R=0$. This requires the condition (Kuchar, 1971) $T_{,R}=0, for\ R=0$. But spacetime is assumed to be locally Euclidean at spatial infinity (Kuchar, 1971) which means that the hypersurface of constant time t have no conical singularity also at infinity so that $\lim_{R\to\infty}T_{,R}\approx 0$. Thus, one may suppose that the relation $\lim_{kR\to\infty}kRT_{,R}$ in Eq (32) tends to finite value so that the prefix of $\lim_{kR\to\infty}$ may be omitted. It may be realized in this respect from the definition of T and its r derivative, i.e., $T(r)=T(\infty)+\int_{\infty}^{r}(-\pi_{\gamma})dr$, $T_{,r}=\pi_{\gamma}$ (Eqs (98) and (100) in (Kuchar, 1971)) that the r dependence of T is especially through the r at the upper end of the integral interval. Thus, the $T_{,R}$ may be taken outside the integral over kR. Also, one may note that the boundary value of $kRJ_0(kR)$ at $kR=0$ is ignored since, as seen from Eqs (29), it obviously vanishes. Substituting from Eq (32) into the Langevin equation (17) one obtains

$$\begin{aligned}
&\frac{\psi_{(i)}^{(k)}(s_k,R,t_k)-\psi_{(i)}^{(k-1)}(s_{(k-1)},R,t_{(k-1)})}{(s_k-s_{(k-1)})}\\
&+4i\pi\int_{(0)}^{\infty}dk\sin(kt)(A(k)-A^*(k))T_{,R}-\\
&-2\pi[(\lim_{kR\to\infty}kRT_{,R})\psi(t,R)\\
&-\int_{(0)}^{\infty}dk\cos(kR-\frac{\pi}{2})(A(k)e^{-ikt}\\
&-A^*(k)e^{ikt})T_{,R}]=\eta_i^k
\end{aligned} \tag{33}$$

Thus, the probability from Eq (A_10) of Appendix A in Part II for the subintervals $(t_{(k-1)},t_k)$, $(s_{(k-1)},s_k)$ assumes the following form for the cylindrical gravitational wave

$$P(\psi^{(k)}_{(n-1)},t_k,s_k\,|\,\psi^{(k-1)}_{(0)},t_{(k-1)},s_{(k-1)}) = (\frac{1}{\sqrt{2\pi(2\alpha)}})^n \exp\{-\sum_i [\frac{1}{2(2\alpha)}\{\frac{(\psi^{(k)}_{(i)}-\psi^{(k-1)}_{(i)})}{(s_k-s_{(k-1)})}+ + 4i\pi\int_{(0)}^{\infty}dk\sin(kt)(A(k)-A^*(k))T_{,R} -2\pi[(\lim_{kR\to\infty}kRT_{,R})\psi(t,R)- -\int_{(0)}^{\infty}dk\cos(kR-\frac{\pi}{2})(A(k)e^{-ikt} -A^*(k)e^{ikt})T_R]\}^2]\}, \quad \textbf{(34)}$$

which is the probability that the η_i^k from the right hand side of Eq (33) takes the value at its left hand side (Namiki, 1992) and the index i runs over the n members of the ensemble. Here, we relate the variable s to the possible geometries of the gravitational wave in the sense that different values of s refer to different geometries of the radiated GW's. This is the meaning of saying that the right hand side of Eq (33), which represents the unpredictability of the stochastic forces, should reflects the left hand side of it which represents the variable character of the waves radiated by the brain. A Markov process (Kannan, 1979; Rogers and Williams, 1987; Doob, 1953) in which $\eta(s)$ does not correlate with its history is always assumed for these correlations. Eq (34) is, actually, a conditional probability which is detaily discussed in the following section and, especially, in Appendix B of Part II.

IV. The probability that the large ensemble of brains radiates cylindrical gravitational waves

The correlation for the n-ensemble of variables ψ_i, $(n-1)\geq i\geq 0$ over the entire N subintervals into which each of the $(s_{(0)},s)$ and $(t_{(0)},t)$ intervals are subdivided may be taken from either Eq (A_11) or the equivalent Eq (A_12) of Appendix A of Part II which is (Namiki, 1992)

$$P(\psi_{(n-1)},t,s\,|\,\psi_0,t_{(0)},s_0)=\int\cdots\int\cdots\int\cdots \cdots P(\psi^{(N)}_{(n-1)},t_N,s_N\,|\,\psi^{(N-1)}_{(0)},t_{(N-1)},s_{(N-1)}) \cdots P(\psi^{(k)}_{(n-1)},t_k,s_k\,|\,\psi^{(k-1)}_{0},t_{(k-1)},s_{(k-1)}) \cdots(\psi^{(1)}_{(n-1)},t_1,s_1\,|\,\psi^{(0)}_{0},t_{(0)},s_0)d\psi^{(N)} \cdots d\psi^{(k)}\cdots d\psi^{(0)}, \quad \textbf{(35)}$$

where each P at the right hand side of the last equation is essentially given by Eq (34). In order to be able to solve the integrals in the last equation we should substitute from Eq (34) for the P's. But we should remark that in Appendix B in Part II and in this section the relevant probability is calculated by performing the relevant summations first over the n variables denoted by the suffix i and then over the N subintervals denoted by the superscript k. That is, as emphasized after Eq (B_1) in Appendix B in Part II, the sum over i in the exponent of that equation, in contrast to Eq (A_11) in Appendix A of that Part II, precedes that over k and, therefore, the squared expression involves the variables $\psi^{(k)}_{(i)}$, $\psi^{(k)}_{(i-1)}$ etc (instead of $\psi^{(k)}_{(i)}$, $\psi^{(k-1)}_{(i)}$ of (A_11) in Part II and Eq (34). Now, before proceeding we define the following expressions

$$B_1(R,t)=2\pi kRT_{,R}$$
$$B_2(R,t)=2\pi\int_{(0)}^{\infty}dk\cos(kR-\frac{\pi}{2})(A(k)e^{-ikt} -A^*(k)e^{ikt})T_{,R} \quad \textbf{(36)}$$
$$B_3(R,t)=i4\pi\int_{(0)}^{\infty}dk\sin(kt)(A(k)-A^*(k))T_{,R},$$

where, as remarked after Eq (32), the prefix of $\lim_{kR\to\infty}$ were omitted from the definition of $B_1(R,t)$. Thus, Eq (33) may be written as

$$\frac{\partial\psi(s_k,R,t)}{\partial s}= B_1(R,t)\psi(R,t)-B_2(R,t)-iB_3(R,t)+\eta_i^k, \quad \textbf{(37)}$$

where the $\eta_i^{(k)}$ satisfies the Gaussian constraints from Eq (18). Solving Eq (37) for $\psi_i^{(k)}(s_k,R,t)$ one obtains

$$\psi_i^k(s_k,R,t)=\psi_{(0)}\exp(2\pi s_k B_1(R,t)) +2\pi\int_0^{s_k} ds_k'\{\exp(B_1(R,t)(s_k-s_k'))\cdot \cdot(\eta_i^k-B_2(R,t)-iB_3(R,t))\}, \quad \textbf{(38)}$$

for initial condition $\psi(0)=\psi_{(0)}$ at $s_k=0$. Note that differentiating Eq (38) with respect to s_k, using the rules for evaluating integrals dependent on a parameter (Pipes, 1958), one obtains Eq (37). In Appendix *B* in Part II we have derived in a detailed manner the appropriate expressions for the correlations of the ensemble of n variables over the given subintervals. We note, as emphasized at the beginning of Section II, that these variables are related with the involved ensemble of brains. Thus, the correlation of these n brains over the N subinterval $(s_{(1)}-s_{(0)})\ldots(s_{(N)}-s_{(N-1)})$ is given by Eq (B_20) in Appendix *B* in Part II as

$$P_{i,j,l,\ldots\ldots}(\psi_{(n)}^{(N)},s_{(N)},t_{(N)}|\psi_{(0)}^{(1)},s_{(0)},t_{(0)}) =(\frac{N}{4\pi\alpha(\Delta s)^2\sum_{k=0}^{k=(n-1)}a_1^k})^{\frac{1}{2}}\cdot \cdot\exp\{-\frac{N}{4\alpha(\Delta s)^2\sum_{k=0}^{k=(n-1)}a_1^k}(\psi_{(n)}^{(N)} -(\sqrt{a_1})^{n+1}\psi_{(0)}^{(N)}+a_2\sum_{r=0}^{r=n+1}(\sqrt{a_1})^r)^2\}, \quad \textbf{(39)}$$

where a_1 and a_2 are given in Eqs (B_5) of Appendix *B* in Part II as $a_1=(1+2\pi B_1\Delta s)^2$, $a_2=2\pi\Delta s(B_2+iB_3)$ and Δs is a representative s subinterval from the N available which are all assumed to have the same length. The correlation of Eq (39) means, as remarked in Appendix *B* in Part II, the conditional probability to find at $s=s_{(N)}$ and $t=t_{(N)}$ the variables $\psi_{(n-1)}$, $\psi_{(n-2)},\ldots\psi_{(1)}$ at the respective states of $\psi_{(n)}^{(N)}$, $\psi_{(n-1)}^{(N)},\ldots\psi_{(2)}^{(N)}$ if at $s=s_{(N-1)}$ and $t=t_{(N-1)}$ they were found at $\psi_{(n-2)}^{(N)}$, $\psi_{(n-3)}^{(N)},\ldots\psi_{(0)}^{(N)}$ …… and at $s=s_{(N-3)}$ and $t=t_{(N-3)}$ they were found at $\psi_{(n-2)}^{(N-2)}$, $\psi_{(n-3)}^{(N-2)},\ldots\psi_{(0)}^{(N-2)}$ …… and at $s=s_{(0)}$ and $t=t_{(0)}$ they were at $\psi_{(n-2)}^{(1)}$, $\psi_{(n-3)}^{(1)},\ldots\psi_{(0)}^{(1)}$. That is, the conditional probability here includes a condition for each of the N subintervals $(s_{(0)},s_{(1)}),(s_{(1)},s_{(2)}),\ldots(s_{(N-1)},s_{(N)})$ so that the superscripts of the variables ψ at the beginnings of all these subintervals are the same as at the ends of them as remarked after Eqs (B_10), (B_13), (B_14) and (B_15) in Appendix *B* of Part II.

From the last equation (39) one may realize that for assigning to $P_{i,j,l,\ldots\ldots}(\psi_{(n)}^{(N)},s_{(N)},t_{(N)}|\psi_{(0)}^{(1)},s_{(0)},t_{(0)})$ a probability meaning which have values only in the range (0, 1) the following inequality should be satisfied

$$(\frac{4\pi\alpha(\Delta s)^2\sum_{k=0}^{k=(n-1)}a_1^k}{N})^{\frac{1}{2}}\geq \exp\{\frac{N}{4\alpha(\Delta s)^2\sum_{k=0}^{k=(n-1)}a_1^k}((\sqrt{a_1})^{n+1}\psi_{(0)}^{(N)} -\psi_{(n)}^{(N)}-a_2\sum_{r=0}^{r=n+1}(\sqrt{a_1})^r)^2\} \quad \textbf{(40)}$$

Taking the ln of the two sides of the last inequality and solving for $\psi_{(n)}^{(N)}$ one obtains

$$\psi_{(n)}^{(N)}\geq(\sqrt{a_1})^{n+1}\psi_{(0)}^{(N)}-a_2\sum_{r=0}^{r=n+1}(\sqrt{a_1})^r- -[(\frac{2\alpha(\Delta s)^2\sum_{k=0}^{k=(n-1)}a_1^k}{N})\ln(\frac{4\pi\alpha(\Delta s)^2\sum_{k=0}^{k=(n-1)}a_1^k}{N})]^{\frac{1}{2}}, \quad \textbf{(41)}$$

where for a unity probability one should consider the equality sign of the last inequality. That is, if the variables $\psi_{(n)}^{(N)}$ and $\psi_{(0)}^{(N)}$ are related to each other in the extra dimension according to the equality sign of (41) then the probability to find at the equilibrium state (where the variable s is eliminated) the whole ensemble of variables all related to the same gravitational geometry is unity. And since, as remarked, these variables are identified with the discussed ensemble of brains this means that they are all radiating cylindrical GW's. This may be shown when one equates all the different values of s to each other and taking the infinity limit as should be done in the stationary configuration. In such case one have $\Delta s=0$ and therefore it may be realized from Eqs (B_5) in Appendix *B* of Part II that the following relations are valid

$$a_{1_{\Delta s=0}} = \sqrt{a_{1_{\Delta s=0}}} = (\sqrt{a_{1_{\Delta s=0}}})^{(n+1)} = 1,$$
$$\sum_{r=0}^{r=n+1} (\sqrt{a_{1_{\Delta s=0}}})^r = (n+2) \quad \textbf{(42)}$$
$$\sum_{k=0}^{k=(n-1)} a_{1_{\Delta s=0}}^k = n, \quad a_{2_{\Delta s=0}} = 0$$

That is, using the last relations and noting that the ln function satisfies the limiting relation (Pipes, 1958) $\lim_{x\to 0} x^2 \ln(x^2) = 0$ one obtains from Eq (41) the expected stationary state

$$\psi^{(N)}_{(n)_{st}} = \psi^{(N)}_{(0)_{st}} \quad \textbf{(43)}$$

Noting the way by which the conditional probability from Eq (B_20) in Appendix B in Part II was derived and the fact that N and n denote general numbers it may be realized that the last result from Eq (43) ensures that at $t = t_{(N)}$ in the equilibrium situation all the variables $\psi^{(N)}_{i_{st}}$, $0 \le i \le n$ are equal to each other. This means that the probability to find the related ensemble of brains all radiating at $t_{(N)}$ cylindrical GW $\psi^{(N)}_{(i)_{st}}$ is unity.

Note from the discussion in Appendix B in Part II that the stationary state from Eq (43) have been obtained by inserting the cylindrical GW Langevin expression from Eqs (33 and 34) into the action S_k for each subinterval $(s_{(k-1)}, s_k)$, $1 \le k \le N$ of each member of the ensemble of variables as realized from Eqs (B_1)-(B_3) in Appendix B in Part II. This kind of substitution is clearly seen in Eq (34) which includes the Langevin relation from (33) in each variable $\psi_i, 0 \le i \le n$ and for each subinterval $(s_{(k)} - s_{(k-1)}), 1 \le k \le N$. As one may realize from Eqs (41 and 43) the substituted expressions differ by s and only at the limit that these expressions have the same s that one finds the same cylindrical GW pattern shared by all the ensemble members. Thus, when these differences in s are eliminated by equating, in the stationary state, all the s values to each other one may obtain the situation in which all the members of the ensemble of brains radiate cylindrical GW and, therefore, the correlation is maximum.

V. Concluding Remarks

For Part I of this work we have used and generalized the indisputable fact that the ionic currents and charges in cerebral system radiate electric waves as may be realized by attaching electrodes to the scalp. That is, one may physically and logically assume that just as these ionic currents and charges in the brain give rise to electric waves so the masses related to these ions and charges should give rise, according to the Einstein's field equations, to weak GW's. From this we have proceeded to calculate the correlation among an n brain ensemble in the sense of finding them at some time radiating similar gravitational waves if they were found at an earlier time radiating other GW's. We have used as a specific example of gravitational wave the cylindrical one which have been investigated in a thorough and intensive way (Kuchar, 1971).

The applied mathematical model, used for calculating the mentioned correlation, was the Parisi-Wu-Namiki SQ theory (Namiki, 1992) which assumes a stochastic process performed in an extra dimension so that at the limit of eliminating the relevant extra variable one obtains the physical stationary state. The hypothetical stochastic process, which is governed by either the Langevin equation or the Fokker-Plank one, allows a large ensemble of n different variables ψ which describes this process (Namiki, 1992; Parisi and Wu, 1981; Nelson, 1985) and represent the mentioned gravitational brainwaves radiated by the n brain ensemble. Thus, we have calculated *the correlation in the extra dimension* among the n brain ensemble and show that at the limits of (1) eliminating the relevant extra variable and (2) maximum correlation one obtains the expected result of finding all of them radiating the same cylindrical GW.

A similar and parallel discussion of the electron-photon interaction, which results in the known Lamb shift, was carried in Part II of this work. This physical example is known to have originated from quantum fluctuations and is in effect one of the first phenomena which were found to be related to these fluctuations. Moreover, strong quantum fluctuations are believed to stand at the basis of quantum gravity which, as argued in the literature (Penrose, 1989; Penrose, 1994), controls also the mechanism of human consciousness and thinking. Thus, it seems natural to discuss it in terms of the SQ theory in which, as mentioned, some stochastic random forces at an extra dimension generate at the equilibrium stage the known physical stationary state.

As mentioned, the mechanism which allows the reduction of the random stochastic process in the extra dimension to the known physical stationary state is the introduction of this same state in all the N subintervals of all the n variables. This means that once all the different s values are eliminated for all the subintervals of all the variables one remains with the same introduced physical stationary state for all of them. The same mechanism may be shown to take effect not only for the assumed weak cylindrical GW's radiated by the brain and the quantum fluctuations of the Lamb shift but also for any other physical phenomena which may be discussed by variational methods.

References

Abbott B *and the LIGO team*. Analysis of LIGO science data for stochastic gravitational waves. Phys Rev D 2004; 69: 122004

Abrahams AM, Evans CR. Trapping a geon: black hole formation by an imploding gravitational wave. Phys Rev D 1992; 46. R4117-R4121.

Abramowitz M, Steegun IA. (eds). Handbook of mathematical functions. Dover New-York, 1970.

Acernese F *and the VIRGO team*. Status of VIRGO. Class Quantum Grav 2002; 19:1421-1428.

Ando M and the TAMA collaboration. Current status of TAMA . Class Quantum Grav 2002; 19:1409-1419.

Arnowitt R, Deser S, Misner CW. The dynamics of General Relativity. in Witten L. (ed). Gravitation: An Introduction to current research. Wiley New-York, 1962.

Arx WS. On the biophysics of consciousness and thought and characteristics of the human mind and intelect. Medical Hypotheses 2001; 56:302-313.

Bar D. Gravitational holography and trapped surfaces. Int J Theor Phys 2007; 46:664-687.

Bar D. Gravitational wave holography. Int J Theor Phys 2007; 46:503-517.

Beig R, Murchadha NO. Trapped surfaces due to concentration of gravitational radiation. Phys Rev Lett 1991;66:2421.

Bergmann PG. Introduction to the theory of relativity. Dover, New-York, 1976.

Coffey W. The Langevin Equation. World Scientific, Singapore, 1996.

Danzmann K. GEO-600 a 600-m laser interferometric gravitational wave antenna. In Coccia E *et al*. (eds). First Edoardo Amaldi conference on gravitational wave experiments. World Scientific, Singapore, 1995.

Doob JL. Stochastic Processes. Wiley, New York, 1953.

Einstein A, Rosen N. On gravitational waves. J Franklin Inst 1937; 223: 43.

Freeman WJ. Mass action in the nervous system. Academic Press, New York, 1975.

Haken H. Light. Vol 1. Elsevier, 1981.

Halliday D, Resnick R. Physics. third Edition. Wiley, New York, 1978.

Hansch TW, Schawlow AK, Toschek AL. Ultrasonic response of a CW dye laser to Selective extinction. IEEE J Quant Electr 1972; QE-8: 802-804.

Hansch TW, Shahin IS, Schawlow AL. Optical resolution of the Lamb shift in atomic hydrogen by laser saturation spectroscopy. Nature (London) Phys Sci 1972; 235: 63-65.

Hartle JB. Gravity: An introduction to Einstein's general relativity. Addison-Wesley, San Fracisco, 2003.

Kannan D. An Introduction to Stochastic Processes. Elsevier, 1979.

Kuchar K. Canonical quantization of cylindrical gravitational waves. Phys Rev D 1971; 4:955-986.

Lamb WE Jr. Sargent M. Laser Physics. Addison-Wesley, 1974.

Lamb WE Jr. The Interpretation of Quantum Mechanics. Rinton Press, 2001.

Misner CW, Thorne KS, Wheeler J. Gravitation. Freeman, San Francisco, 1973.

Namiki M. Stochastic Quantization. Springer Verlag, Berlin, 1992.

Nelson E. Quantum Fluctuation. Princeton, New Jersey, 1985.

Parisi G, Wu Y. Perturbation theory without gauge fixing. Sci Sin 1981; 24:483-496.

Penrose R. Shadows of the mind. Oxford University Press, 1994.

Penrose R. The Emperor's New Mind. Oxford University Press, 1989.

Pipes AL. Applied Mathematics for Engineers and Physicists. 2nd edition. McGraw-Hill, 1958.

Risken H. The Fokker-Plank Equation. Springer, 1984.

Rogers LC, Williams D. Diffusions, Markov Processes and Martingales. 2nd edition. Wiley, 1987.

Schiff LI. Quantum Mechanics. 3-rd Edition. McGraw-Hill, 1968.

Thorne KS. Gravitational wave research: Current status and future prospect. Rev Mod Phys 1980; 52:285-297.

Thorne KS. Multipole expansions of gravitational radiation. Rev Mod Phys 1980; 52: 299-339.

Tran Y, Craig A, McIsaac P. Extraversion-introversion and 8-13 Hz waves in frontal cortical regions. Personality and Individual Differences 2001; 30:205-215.

Weinstock R. Calculus of variations. Dover, New-York, 1974.