What is The Reason to Use Clifford Algebra in Quantum Cognition?

Part II:
Information, Cognition and the Principle of Existence are Intrinsically Structured in the Quantum Model of Reality

Elio Conte

ABSTRACT
The thesis of this paper is that Information, Cognition and a Principle of Existence are intrinsically structured in the quantum model of reality. There is an intrinsic “factor of knowledge” that supports its structure. We reach such evidence by using the Clifford algebra. In detail we analyze quantization in some traditional cases of interest in quantum mechanics and, in particular, in quantum harmonic oscillator, orbital angular momentum and hydrogen atom. We adopt the basic von Neumann results that projection operators represent logical statements and we demonstrate that are intrinsically structured in the cases of quantization that are taken in consideration.

Key Words: Information, quantum cognition, principle of existence, quantum mechanics, quantization, Clifford algebra

Preliminary Remarks
The present paper represents the second part of a paper that we published months ago on NeuroQuantology, entitled “What is the reason to use Clifford algebra in quantum cognition? Part I: It from qubit (Conte, 2012).”

The conclusion was that we have basic biological unities, biomolecules as the amino acids, whose interfaced counterpart may be represented by the basic Clifford mathematical and thus computational entities that of course enable us to represent the basic foundations of quantum mechanics, also if in a rough bare bone skeleton, and qubit.

There is still another result that we outlined in section I and we repeat here again anticipating some features that will discuss later with obvious literature reference. In these years we have shown that there are stages of our reality in which it results impossible to unconditionally defining the truth. Logic, language and thus cognition enter with a so fundamental role in quantum mechanics because there are levels of our reality in which the fundamental features of cognition and thus of logic and language, and thus the conceptual entities, acquire the same importance as the features of what is being described. At this level of reality we no more may separate the features of matter per se from the features of the cognition, of the logic and of the language that we use to describe it. Conceptual entities non more are separated from the object of cognitive performance. As correctly Yuri Orlov (Orlov 1994) outlined in his several papers, the truths of logical statements about dynamic variables relating
matter structure become dynamic variables themselves in quantum mechanics, and thus the cognition becomes in itself an immanent feature that operates symbiotically with the matter phenomenology that traditional physics aims to represent.

This was the conclusion of the previous paper. The aim of such second part of the work is to give support to such thesis introducing some new results on this matter.

In particular we demonstrate that Information, Cognition and a Principle of Existence are Intrinsically Structured in the Quantum Model of Reality. We advise the reader that the basic mathematical and physical developments have been submitted by us elsewhere (Conte, 2013). Therefore, in order to facilitate the approach of the reader we did not mention here in detail such detailed mathematical developments reporting the essential contents in some appendices at the end of the paper.

Introduction
Quantum mechanics was formulated in the first decade of the 20th century following about the same time the basic discoveries of physics as the atomic theory and the Einstein corpuscular theory of light. Quantum theory was significantly reformulated in the mid-1920s by Werner Heisenberg, Max Born and Pascual Jordan, who created matrix mechanics (Mehra and Rechenberg, 1982), Louis de Broglie and Erwin Schrödinger who introduced wave mechanics, and Wolfgang Pauli and Satyendra Nath Bose who introduced the statistics of subatomic particles. Finally, the Copenhagen interpretation became widely accepted but with profound reservations of some distinguished scientists In particular, Einstein prospected the alternative view point of the hidden variables. A large debate about the conceptual foundations of the theory followed and it received renewed strengthening interest with the advent of Bell theorem (Bell, 1964). Currently, such debate still continues in the present days also if, starting with 1927, quantum mechanics has only received continuous verifications, controls and constant confirmations. By 1930, quantum mechanics was further unified and formalized by the work of David Hilbert, Paul Dirac and John von Neumann (von Neumann, 1932).

Conventionally formulated quantum mechanics starts always with the combined standard mathematical, well known, description from one hand and the use of classical physical analogies on the other hand.

Quantum mechanics is a crushing theory able to demolish our standard and empirical conceptual foundations. Our position is that any tentative to represent quantum mechanics by using a large amount of classical analogies enables us to risk to negate the fundamental nature of quantum reality that is fixed on some basic and unclassical features. They are the integer quanta, the non-commutation, the intrinsic-irreducible indetermination and quantum interference. It is possible to demonstrate that quantization, non-commutation, intrinsic and irreducible indetermination, and quantum interference may be also obtained in a rough scheme due to the outset of the basic axioms of Clifford algebra.

First, let us follow the illuminating thinking of P. Dirac. As previously said, P. A. M. Dirac contributed at the highest level to the final formulation of quantum mechanics. In his “The Development of Quantum Theory” (Dirac, 1977) and “History of Twentieth Century Physics” (Dirac 1971), he wrote:

“I saw that non commutation was really the dominant characteristic of Heisenberg’s new theory. It was really more important than Heisenberg’s idea of building up the theory in terms of quantities closely connected with experimental results. So I was led to concentrate on the idea of non-commutation. I was dealing with these new variables, the quantum variables, and they seemed to be some very mysterious physical quantities and I invented a new word to describe them. I called them q-numbers and the ordinary variables of mathematics I called c-numbers to distinguish them....Then I proceed to build up a theory of these q-numbers. Now, I did not know anything about the real nature of these q-numbers. Heisenberg’s matrices, I thought, were just an example of q-numbers, may be q-numbers were really something more general. All that I knew about q-numbers was that they obeyed an algebra satisfying the ordinary axioms except for the commutative axiom of multiplication. I did not bother at all about finding a precise mathematical nature of q-numbers”.

http://www.neuroquantology.com
Our approach may be reassumed as it follows. Initiating with 2010 (Conte, 2009; 2010) we started giving proof of two existing Clifford algebras, the \( S_i \) that has isomorphism with that one of Pauli matrices and the \( N_{i,1} \), where \( N \) stands for the dihedral Clifford algebra.

We showed that the \( N_{i,1} \) may be obtained from the \( S_i \) algebra when we attribute a numerical value (+1 or −1) to one of the basic elements \( (e_1, e_2, e_3) \) of the \( S_i \). We utilized such result to advance a criterium under which the \( S_i \) algebra has as counterpart the description of quantum systems that in standard quantum mechanics are considered in absence of observation and quantum measurement while the \( N_{i,1} \) attend when a quantum measurement is performed on such system with advent of wave function collapse.

The physical content of the criterium is that the quantum measurement and wave function collapse induce the passage in the considered quantum system from the \( S_i \) to \( N_{i,1} \) or to the \( N_{i,2} \) algebras. On this basis we re-examined the von Neumann postulate on quantum measurement, and we gave a proper justification of such postulate by using the \( S_i \) algebra. We also studied some direct applications of the above mentioned criterium to some cases of interest in standard quantum mechanics. In each of such cases examined, we found that the passage from the algebra \( S_i \) to \( N_{i,1} \), considered during the quantum measurement of the system, actually describes the collapse of the wave function. Therefore we concluded that the actual quantum measurement has as counterpart in the Clifford algebraic description, the passage from the \( S \) to the \( N_{i,1} \) Clifford algebras, reaching in this manner the objective to reformulate von Neumann postulate on quantum measurement and proposing a self-consistent formulation of quantum theory. We reached also another objective. The combined use of the \( S_i \) Clifford algebra and the \( N_{i,1} \) dihedral Clifford algebra, also accomplishes to another basic requirement that the advent of quantum mechanics strongly outlined. Heisenberg initial view point was to modify substantially our manner to look at the reality. He replaced numbers by actions as also outlined by Stapp (Stapp, 2010); a number represents the manner in which the dynamics of a given object has happened. Heisenberg replaced such standard viewpoint requiring instead that we have to explicit the mathematical action (let us remember that the notion of operator will be subsequently adopted), and this action becomes the mathematical counterpart of the physical corresponding action whose outcome will give a number as final determination. Such double features of standard quantum mechanics represent of course a basic and conceptually profound innovation in our manner to conceive reality and the methodology to investigate it. It is clearly synthesized in our Clifford algebraic formulation by using from one hand the Clifford \( S \) and, as counterpart, the \( N_{i,1} \) dihedral Clifford algebra.

The general question may be posed now in the following manner: quantization, non-commutation, intrinsic-irreducible indetermination and quantum variables as new “mysterious physical quantities”, also if in a rough scheme, may be actually described and due to the outset of the basic axioms of Clifford algebra. This is the reason because we started in 1972 to attempt to formulate a bare bone skeleton of quantum mechanics by using Clifford algebra and on this basis we have obtained also some other interesting results. Rather recently, as example, we have obtained a very interesting result. J. von Neumann (von Neumann, 1932) constructed matrix logic on the basis of quantum mechanics. In (Conte, 2011a; 2011b; 2011c) we inverted the demonstration we showed that quantum mechanics may be constructed from logic. This feature may represent a turning point. In fact, the evidence is that we have indication about the logical origin of quantum mechanics and by this way we are induced to conclude that quantum reality has intrinsically a new feature that we are not accustomed to attribute to it. Quantum reality starts admitting a role for logic, thus for cognition, language, semantic not in a foreseen sense. As we outlined also in the first part of the present paper and re-outlined in the Preliminary Remarks of the present papers, there is a principle in quantum reality that we are addressed to evidence in the following manner: there are stages of our reality (those engaged from quantum theory, precisely) in which matter no more may be conceived by itself, it no more may be conceived independently from the cognition.
that we have about it. This is a new viewpoint that involves mind like entities, modulating matter with cognition ab initio in our quantum reality. Therefore it opens a new way to acknowledge a role of quantum mechanics in cognitive sciences (Conte, 2012; Conte et al., 2003)

In previous papers we have investigated such our approach considering indeterminism and quantum interference. The aim of the present paper is to add here new results to such thesis. We select to consider here the problem of the quantization.

We know that quantum mechanics started about the concept of quantization. Therefore it explains a central role in quantum mechanics. Cognition and a principle of existence are intrinsically structured in the inner scheme of quantum mechanical reality.

It is well known that in quantum mechanics some physical quantities may be expressed in the following manner

\[ A = f(N, a, b, c, ...) \]  \hspace{1cm} (2.1)

where \(a, b, c, \ldots\) may be constants and \(N\) assumes only discrete, integer values, say \(0, 1, 2, 3, 4, 5, \ldots\). \(N\) may be conceived to be the following Clifford manner

\[ N = \sum_{n=0}^{k} n <q_n> \]  \hspace{1cm} (2.2)

where \(q_n\) are specific Clifford members having some specific properties.

Let us consider the case \(n = 0, 1\).

In this case \(N\) is given in the following manner

\[ N = 0 <q_0> + 1 <q_1> \]  \hspace{1cm} (2.3)

where \(q_0\) and \(q_1\) are the following idempotent elements

\[ q_0 = \frac{1}{2}(1 + e_3); \quad q_1 = \frac{1}{2}(1 - e_3) \]  \hspace{1cm} (2.4)

We have

\[ <q_0> = \frac{1}{2} + \frac{1}{2} <e_3> \quad \text{and} \quad <q_1> = \frac{1}{2} - \frac{1}{2} <e_3> \]  \hspace{1cm} (2.5)

Let us write the mean value of \(e_3\). It is

\[ <e_3> = p(+1) + (-1) p(-1) \]  \hspace{1cm} (2.6)

being \(p(+1)\) and \(p(-1)\) the corresponding probabilities for the abstract entity \(e_3\) to assume or the numerical value \((+1)\) or the numerical value \((-1)\). We must now adopt the following basic statement. \(e_3\) is a cognitive entity. Of course we know that, according to von Neumann (von Neumann, 1932) density operators as well as correspondingly, idempotent elements may be considered logic statements.

Let us admit that a primitive form of cognitive entity is represented by \(e_3\) and it is in the condition of absolute certainty that the represented system \(S\) to which \(N\) is connected has the value \((+1)\). This means in the (2.6) that \(p(+1) = 1\) and \(p(-1) = 0\). Consequently \(N\) will be characterized from the discrete integer value \(n = 0\). In the other possible case, \(N\) will be characterized from the discrete integer value \(n = 1\).

The crucial question is that we have connected the integer quantized condition of the physical observable that relates a given observable of matter, and identified here to contain \(N\) and the cognitive task that must be performed. In order to ascertain the quantized integer value of \(N\), a cognitive task must be performed in the sense that a semantic act is here clearly involved. Of course Orlov (Orlov, 1994) was the first to identify \(e_3\) as the basic and universal logic operator.

In conclusion we have given an initial proof of a necessary and sufficient link existing between \(N\) and \(e_3\).

We should write

\[ A = f(N(e_3), a, b, c, \ldots) \]  \hspace{1cm} (2.9)

with

\[ q_0 q_1 = q_1 q_0 = 0, \quad q_0 + q_1 = 1, \quad q_0^2 = q_0, \quad q_1^2 = q_1 \]  \hspace{1cm} (2.10)

We may also consider the case in which \(N\) assumes four possible integer values.

In this case we need a Clifford algebraic structure given at the order \(n = 4\). The four possible combinations of the basic primitive idempotent elements are

\[ q_0 = \frac{1}{4}[(E_{oo} + E_{o3})(E_{oo} + E_{o3})]; \]

\[ q_1 = \frac{1}{4}[(E_{oo} - E_{o3})(E_{oo} + E_{o3})]; \]

www.neuroquantology.com
\[ q_2 = \frac{1}{4} \left[ \left( E_{oo} + E_{o3} \right) \left( E_{oo} - E_{30} \right) \right]; \]
\[ q_3 = \frac{1}{4} \left[ \left( E_{oo} - E_{o3} \right) \left( E_{oo} - E_{30} \right) \right]. \quad (2.11) \]

Note that in this case we invoke the basic and universal logic operators \((E_{o3} \text{ and } E_{oo})\) and the coupling (conjunction) \(E_{33} = E_{o3} \cdot E_{30} = E_{30} \cdot E_{o3}\). Obviously, also the relations like the (2.10) hold in this extended case.

\[ \sum q_i = 1; q_i = q_j q_j = 0; i \neq j; i = 0, 1, 2, 3; \]
\[ j = 0, 1, 2, 3 \quad (2.12) \]

Let us apply now the previous criteria \((S, N_{cn})\) that we considered previously. Let us admit now that \(E_{o3}, E_{30}, E_{33}\) are cognitive entities. As previously said, we know that, according to von Neumann (von Neumann, 1932), density operators as well a correspondingly, idempotent elements are logic statements.

Let us admit that the cognitive entity, represented by \(E_{o3}\) is in the condition of absolute certainty that the represented system \(S\) to which \(N\) is connected, has the value (+1). This means (see Appendix A) that \(p(+1) = 1\) and \(p(-1) = 0\). The same reasoning may be developed for \(E_{30}\), and for \(E_{33}\).

It results evident that we are obtaining indication of a new arising scheme of reality. In substance the cognitive entities that we invoke here relate the same concept of existence. Is this existing condition of reality actually existing? The concept of Existence becomes here itself a variable that assumes therefore two possible values, indicating yes/not cognitive condition. Existence and cognition result therefore profoundly linked in the scheme of reality that we are delineating. The arising indication is that in the basic foundation of our reality ab initio there are elements of existence defined, not in terms of some hazy metaphysical concept of existence, but in the sense that existence is represented by a symbol that contains only two possibilities, existence or non-existence. Just as in our treatment by using Clifford algebra, these authors assumed that the structural concept of existence is represented by an idempotent of some appropriate algebra and satisfying the conditions previously given.

Let us verify that the situation is posed actually in these terms.

Let us admit that the abstract logic entities go to assume numerical values as it follows
\[ E_{o3} \rightarrow +1, \quad E_{30} \rightarrow +1. \]

We have in the (2.2) \(<q_o>=1, <q_l><q_j><q_3>=0\) \((2.16)\) and the first integer value that is selected is obtained.

If instead the cognitive performance ascertains that
\[ E_{o3} \rightarrow -1, \quad E_{30} \rightarrow -1 \quad \text{we have in the (2.2)} \]
\[ <q_3>=1, <q_j><q_3><q_o>=0 \] \((2.17)\) and the second integer is obtained.

In the case in which
\[ E_{o3} \rightarrow +1, \quad E_{30} \rightarrow -1 \quad \text{we have in the (2.2)} \]
\[ <q_3>=1, <q_j><q_3><q_o>=0 \] \((2.18)\) and the third integer is obtained.

Finally, with
\[ E_{o3} \rightarrow -1, \quad E_{30} \rightarrow +1 \quad \text{we have in the (2.2)} \]
\[ <q_3>=1, <q_j><q_3><q_o>=0 \] \((2.19)\) and the fourth integer is obtained.

The case \(n = 8\) proceeds in the same manner.

We need Clifford algebraic elements:
\[ E_{o00}, E_{o30}, E_{300}, E_{o33}, E_{303}, E_{330}, E_{333} \] \((2.20)\)

We may be sure that our Clifford algebraic structure at the order \(n = 8\) will be
\[ (E_{o01}, E_{o02}, E_{o03}), (E_{o10}, E_{o20}, E_{o30}), (E_{100}, E_{200}, E_{300}) \]
\[ (2.21) \]
and related sets providing coupling. In this case they give origin to the following basic Clifford elements
\[ q_o = \frac{1}{8}[(1 + E_{oo3})(1 + E_{oo3})(1 + E_{oo3})] \]
(2.22)
\[ q_i = \frac{1}{8}[(1 - E_{oo3})(1 + E_{oo3})(1 + E_{oo3})] \]
(2.23)
\[ q_2 = \frac{1}{8}[(1 + E_{oo3})(1 - E_{oo3})(1 + E_{oo3})] \]
(2.24)
\[ q_3 = \frac{1}{8}[(1 - E_{oo3})(1 - E_{oo3})(1 + E_{oo3})] \]
(2.25)
\[ q_4 = \frac{1}{8}[(1 + E_{oo3})(1 + E_{oo3})(1 - E_{oo3})] \]
(2.26)
\[ q_5 = \frac{1}{8}[(1 - E_{oo3})(1 + E_{oo3})(1 - E_{oo3})] \]
(2.27)
\[ q_6 = \frac{1}{8}[(1 + E_{oo3})(1 - E_{oo3})(1 - E_{oo3})] \]
(2.28)
\[ q_7 = \frac{1}{8}[(1 - E_{oo3})(1 - E_{oo3})(1 - E_{oo3})] \]
(2.29)

Note the particular alternation in the signs of the idempotent elements arising for each \( q_i \) with \( i = 0, 1, ..., 7 \).

We have \((+,+,+),(-,+,+),(+,+,+),(-,+,-),(+,-,+),(-,-,+)\). A combination of all the possible alternatives: a clear semantic message is contained and it is intrinsic to the inner structure of such arising integer quanta mechanism.

For \( E_{oo3} \rightarrow +1, E_{oo3} \rightarrow +1, E_{oo3} \rightarrow +1 \), we have 
\[ q_o = 1, q_i = q_2 = q_5 = q_7 = 0 \] 
(2.30)
and the first integer value is obtained.

For \( E_{oo3} \rightarrow +1, E_{oo3} \rightarrow +1, E_{oo3} \rightarrow +1 \), we have 
\[ q_o = 1, q_i = q_3 = q_5 = q_6 = q_7 = 0 \] 
(2.31)
and the first integer value is obtained.

For \( E_{oo3} \rightarrow +1, E_{oo3} \rightarrow -1, E_{oo3} \rightarrow +1 \), we have 
\[ q_o = 1, q_3 = q_5 = q_6 = q_7 = 0 \] 
(2.32)
and the second integer value is obtained.

For \( E_{oo3} \rightarrow +1, E_{oo3} \rightarrow -1, E_{oo3} \rightarrow +1 \), we have 
\[ q_o = 1, q_4 = q_5 = q_6 = q_7 = 0 \] 
(2.33)
and the third integer value is obtained.

For \( E_{oo3} \rightarrow -1, E_{oo3} \rightarrow -1, E_{oo3} \rightarrow +1 \), we have 
\[ q_i = 1, q_2 = q_3 = q_4 = q_5 = q_6 = q_7 = 0 \] 
(2.34)
and the fourth integer value is obtained.

For \( E_{oo3} \rightarrow +1, E_{oo3} \rightarrow +1, E_{oo3} \rightarrow -1 \), we have 
\[ q_o = 1, q_i = q_2 = q_3 = q_4 = q_5 = q_6 = q_7 = 0 \] 
(2.35)
and the fifth integer value is obtained.

For \( E_{oo3} \rightarrow -1, E_{oo3} \rightarrow +1, E_{oo3} \rightarrow -1 \), we have 
\[ q_o = 1, q_i = q_3 = q_4 = q_5 = q_6 = q_7 = 0 \] 
(2.36)
and the sixth integer value is obtained.

For \( E_{oo3} \rightarrow +1, E_{oo3} \rightarrow -1, E_{oo3} \rightarrow -1 \), we have 
\[ q_o = 1, q_i = q_3 = q_4 = q_5 = q_6 = q_7 = 0 \] 
(2.37)
and the seventh integer value is obtained.

For \( E_{oo3} \rightarrow -1, E_{oo3} \rightarrow -1, E_{oo3} \rightarrow -1 \), we have 
\[ q_o = 1, q_i = q_3 = q_4 = q_5 = q_6 = q_7 = 0 \] 
(2.38)
and the eighth integer value is obtained.

Clifford algebra also admits nilpotent elements. For details see Appendix B.

Let us consider now the following two basic nilpotent elements
\[ R = a(e_{12} + i e_{23}) \] 
and \[ S = a(e_{12} + e_{23}) \],
\[ R^2 = S^2 = 0, \] 
\[ RS \neq 0; \] 
\[ R^{n}S^n = 0; \] 
\[ R^{n-1}S^{n-1} \neq 0 \] 
(at the order \( n = 2 \) in our case).

\( a \) is some prefixed real constant.

Note that, in spite of being \( R^nS^n = 0 \) (absurdum) \( n = 2 \) in the present case,
\[ RS = a^2(\frac{1}{2} - \frac{1}{2}i e_3); \] 
\[ SR = a^2(\frac{1}{2} + \frac{1}{2}i e_3); \] 
\[ RS - SR = a^2e_3 \] 
(2.43)

an idempotent element is instead obtained promptly at the order \( n = 1 \).

Let us admit to construct now some variables starting with \( R \) and \( S \). In detail, let us introduce the variables \( Q \) and \( P \) (Clifford algebraic elements) in the following manner
\[ Q = \frac{1}{2}(R + S); \] 
\[ P = \frac{b}{2}(R - S) \] 
(2.44)
with \( b \) some given real constant.

\[ QP - PQ = -\frac{b}{2}(RS - SR) = a^2b \] 
\[ Q^2 = \frac{1}{4}(RS + SR) = a^2 \] 
\[ P^2 = -\frac{b^2}{4}(RS + SR) = -\frac{b^2a^2}{4} \] 
(2.45)

Let us now examine \( R^nS^n = 0 \). On the basis of Appendix C we obtain that

www.neuroquantology.com
We have to outline here some important results.
The first is that have been reduced again to idempotent elements (logic statements). The second is that we have obtained a tautology. The (2.47) is always true in itself, when we consider \( e_i \sim 1 \) as well as when we consider \( e_i \sim -1 \).

The argument proceeds in the same manner in the case \( n=4 \). Looking at the appendix D we finally obtain

\[
R^n S^n = 0; R^{n-1} S^{n-1} \neq 0 \quad \text{with} \ (n = 4).
\]

In each case nilpotent elements are finally reduced to idempotent elements indicating logical statements.

Also in this case the substantial result is that such cognitive features are linked to matter as it is in the thesis of our papers. To convince ourselves that this is the case let us take

\[
a = \left( \frac{2h}{m \omega} \right)^{1/2} \quad \text{in the starting (2.42)} \text{ and }
\]

\[
b = \frac{im \omega}{2} \quad \text{in the starting (2.44)}.
\]

Consider the Clifford elements \( Q \) and \( P \) to represent the position and the momentum of a particle signed by the Hamiltonian

\[
H = \frac{1}{2m} p^2 + \frac{1}{2} m \omega^2 Q^2 \quad \text{(2.50)}
\]

It results that in this condition we are examining the well-known case of the harmonic oscillator in standard quantum mechanics.

As it is well known, the quantized oscillator energy is given by

\[
E = \hbar \omega (N + \frac{1}{2}). \quad \text{(2.51)}
\]

In our case it results

\[
N = \frac{m \omega}{2 \hbar} RS \quad \text{(2.52)}
\]

and the quantized levels are obtained from the (2.46) at order \( n=2 \). The following energy levels are obtained at the order \( n=4 \), \( n=8 \), and so on.

Also in this case we have a direct connection between logic statements, semantics, cognition from one hand and a material object as a quantum harmonic oscillator on the other hand. Remember of course that the harmonic oscillator was developed in the whole profile of quantum mechanics starting with the original and initial results of Heisenberg and arriving also to the most recent applications of the harmonic oscillators in the current days of quantum mechanics. We have spoken thus of an object that is at the basic of the quantum model of reality.

We may now take in consideration another fundamental quantum variable. We speak of orbital angular momentum. We do not add other comments on this variable since it is well known.

Relating orbital angular momentum, it is well known that

\[
L_i = Q_i P_3 - Q_3 P_i; \quad L_2 = Q_2 P_3 - Q_3 P_2; \quad L_3 = Q_3 P_2 - Q_2 P_3 \quad \text{(2.53)}
\]

with

\[
L_i L_j - L_j L_i = i \hbar L_k; \quad i = 1, 2, 3; \quad j = 1, 2, 3; \quad k = 1, 2, 3; \quad i \neq j \neq k. \quad \text{(2.54)}
\]

At just derived previously, at the order \( n = 2 \), we have the basic Clifford elements previously discussed for quantization

\[
J_z = \frac{1}{2} (e_1 + i e_2); \quad J_+ = \frac{1}{2} (e_1 - i e_2); \quad J_- = \frac{1}{2} e_3; \quad J^2 = \frac{3}{4} e_0 \quad \text{(2.55)}
\]

All the usual commutation relations of standard quantum mechanics are verified.

At the order \( n = 4 \), we have

\[
J_z = \frac{\sqrt{3}}{2} (E_{00} + i E_{00}) \\
+ \frac{1}{2} E_{00} (E_{00} - i E_{00}) \\
+ \frac{i}{2} E_{00} (E_{00} - i E_{00}) = \\
= \frac{\sqrt{3}}{2} (E_{00} + i E_{00}) + \frac{1}{2} (E_{00} - i E_{00}) (E_{00} + i E_{00}) \quad \text{(2.56)}
\]
The basic step was performed by Pauli who used the well-known. The further step, realized by Pauli (Pauli, 1926), was the analysis of the Hydrogen atom by Pauli using the well-known Lorentz-Runge Lentz vector (Laplace, 1799; Lenz, 1924).

In substance he used three matrices

\[
A_i = \frac{1}{mZ^2} \left( L_z P_3 + P_3 L_z - L_3 P_z - P_z L_3 \right) + Q R^{-i}
\]

The details of our elaboration are given in Appendix F. Since we have found that

\[
M^2 = M_1^2 + M_2^2 + M_3^2 = -\frac{1}{4} \frac{mZ^2 e^4}{8\hbar^2 E}
\]

under the condition \( E < 0 \), we write that

\[
-\frac{mZ^2 e^4}{8\hbar^2 E} = \frac{1}{4} + j (j + 1)
\]

or

\[
-\frac{mZ^2 e^4}{2\hbar^2 E} = 1 + 4j (j + 1) = n^2
\]

with \( n = 2j + 1 \).

In conclusion, we have that

\[
E = -\frac{mZ^2 e^4}{2n^2 \hbar^2} \quad ; \quad n = 1, 2, 3
\]

that is just the usual formula of the energy levels for the hydrogen atom as it is obtained in the standard case of the usual quantum mechanics. Again we have found that is given by idempotent elements. They are logical statements. We are now in the condition to draw our conclusions.

Discussion

Let us start taking in consideration the position of Clifford as philosopher. As a philosopher, Clifford’s name is associated with two phrases of his elaboration. The "mind-stuff" and the "tribal self". We will consider here the first, the “Mind Stuff”. His conception is that mind is the one ultimate reality; not mind as we know it in the complex forms of
conscious feeling and thought, but the simpler elements out of which thought and feeling are built up. The hypothetical ultimate element of mind is an “atom” of mind-stuff. In the position of such so celebrated and distinguished scientist, mind stuff corresponds to the hypothetical atom of matter, being the ultimate fact of which the material atom is the phenomenon. Matter and the sensible universe are the relations between particular organisms, that is, mind organized into consciousness, and the rest of the world. Technically this position resembles some kind of idealistic monism. In his paper (Clifford, 1878) he seems to clear the sense of his elaboration. A moving molecule of inorganic matter does not possess mind or consciousness; but it possesses a small piece of mind-stuff. When molecules are so combined together as to form the film on the underside of a jelly-fish, the elements of mind-stuff which go along with them are so combined as to form the faint beginnings of Sentience. When the molecules are so combined as to form the brain and nervous system of a vertebrate, the corresponding elements of mind-stuff are so combined as to form some kind of consciousness; that is to say, changes in the complex which take place at the same time get so linked together that the repetition of one implies the repetition of the other. When matter takes the complex form of a living human brain, the corresponding mind-stuff takes the form of a human consciousness, having intelligence and volition.

We will not enter here in a discussion about Clifford basic elaboration. We collocate our research out of philosophy and in the proper framework of science. We have mentioned here Clifford first of all because we use his algebra from several years in our research activity and soon after to evidence in some manner that our scientific results do not appear to leave in an isolated context. Clifford recalls some analogies to our basic formulation that of course is based on a rigid mathematical and physical derivation

As previously said, we move in the framework of science. Therefore our conclusion while from one hand does not give some direct explanation, on the other hand seems to respond to all the basic requirements of the scientific approach.

We have derived quantization as general approach to quantum systems. After we have discussed the general case of the classical quantum harmonic oscillator. Soon after we have also discussed the case of the angular momentum. Still, using the Lorentz-Runge Lenz vector that of course was used also by Pauli, we have performed the analysis of hydrogen atom energy levels. According to standard formulation of quantum mechanics, we have covered a rather large spectrum of interest in this discipline. Always we have found the same result. Idempotent elements are involved. Since, as previously said, idempotent elements are representative of logical statements and thus of cognition and semantics, we conclude that in the basic foundation of our quantized basic reality ab initio there are elements of existence defined, not in terms of some hazy metaphysical concept of existence, but in the sense that existence, related to the cognitive act, is represented by abstract entities of the Clifford algebra, and it contains only two possibilities: existence or non-existence. A pure dichotomic cognitive variable structured ab initio in the inner architecture of our reality. There is ab initio in quantum reality a variable, we could call it “the factor of knowledge and existence” that travels with more traditional physical variables that identify matter per se and that we are accustomed to use in the traditional approach to reality that we formulate in classical physics. There are stages of our reality in which we no more may separate matter per se from the cognition and the principle of existence that we have to attribute to it.

Let us take a step back. As repeatedly outlined, J. von Neumann (von Neumann, 1932) showed that projection operators $\Lambda$, satisfying as it is well known that $\Lambda(\Lambda-1)=0$, and quantum density matrices can be interpreted as logical statements.

Let us consider a quantum system $S$ and its quantum observable $K$. $|k\rangle$ is a state vector for the quantum state in which the observable $K$ is equal to $k$. The density matrix $\Lambda_\pm$ with $\Lambda_\pm = |k\rangle\langle k|$ represents the logical statement $\Lambda_\pm$. It says “$K = k$”. This is of course the basic argument that was developed from Yuri F. Orlov just in 1994 (Orlov, 1994). As it is well known, generalizing we arrive to write the most general relation of quantum mechanics

$$K = \sum_\pm k_\pm \Lambda_\pm$$  \hspace{1cm} (2.97)
Tr $\Lambda_k = 1$; $\sum_i \Lambda_i = 1$ \hspace{1cm} (2.98)

In the (2.97) $K$ is an operator –observable, connected directly to observable features of matter. $\Lambda_{\pm}$ are instead logic statements, thus connected to cognition. The (2.97) clearly explains that such two basic features, matter from one hand and cognition from the other hand, and are indissolubly connected from its starting in the theory. Matter cannot be conceived per se but in relation to the cognition that it is possible to have about it. Logic statements, i.e. cognitive elements $\Lambda_{\pm}$ are quantum observables themselves, nonlocal by nature, variables themselves in the dynamics of our reality and commuting with the corresponding quantum physical observables. In this manner a new framework of quantum reality arises in which ab initio existence and to cognition. Matter does no go on by only in its dynamics but it is constantly coupled to an actual principle of existence and to cognition.

**Appendix A**

Let us write the mean values of $E_{\alpha \beta}$ and of $E_{\beta \alpha}$ and $E_{\beta \beta}$. It is

\[
< E_{\alpha \beta} > = (+1) p(+1) + (-1) p(-1);
\]

\[
< E_{\beta \alpha} > = (+1) p(+1) + (-1) p(-1);
\]

\[
(2.14) < E_{\beta \beta} > = (+1) p(+1) + (-1) p(-1);
\]

\[
(2.15)
\]

being $p(+1)$ and $p(-1)$ the corresponding probabilities for the abstract entities to assume or the numerical value $(+1)$ or the numerical value $(-1)$.

**Appendix B**

It is well known that the Clifford $A(S_i)$, in addition to admits idempotent, also contains nilpotent. Generally speaking, it is known that an element $x$ of a ring $R$ is called nilpotent if there exists some positive integer $n$ such that $x^n = 0$.

Previously we have considered two idempotent in $S_i$ written as $(1 + e_i)/2$ and $(1 - e_i)/2$. In the same algebra two nilpotent can be written as $(e_i + ie_j)/2$ and $(e_i - ie_j)/2$. This is at the order $n = 2$ but we may easily generalize them at higher orders.

The important thing is to observe here that the two nilpotent elements may be rewritten linked to idempotent:

\[
(e_i + ie_j)/2 = e_i(1 - e_j)/2
\]

\[
(e_i - ie_j)/2 = e_i(1 + e_j)/2
\]

(2.39)

where we have used the Clifford representation of the imaginary unity $i = e_1e_2e_3$.

These nilpotent elements are the same as the idempotent elements multiplied by $e_i$.

Still it is instructive to observe that

\[
e_i(1 - e_j)/2 = (1 + e_j)e_i/2;
\]

\[
e_i(1 + e_j)/2 = (1 - e_j)e_i/2
\]

(2.40)

and

\[
e_i(1 - e_j)/2 = [(1 + e_j)/2)e_i[(1 - e_j)/2];
\]

\[
e_i(1 + e_j)/2 = [(1 - e_j)/2)e_i[(1 + e_j)/2]
\]

(2.41)

What is the reason to have introduced here the notion of nilpotent that of course is well known in Clifford algebra? The reason is that on the basis of the previously discussed link existing in our view point between idempotent elements, logic, semantic, information, and cognitive abstract entities, also on the other hand the existing link between idempotent and nilpotent elements, must be defined also under the profile of the logic, semantic, information, and cognition delineating what is the meaning of nilpotent. In our view point, the condition that there exists some positive integer $n$ such that $x^n = 0$, under the logic, semantic, and cognitive profile, means that at this order $n$ we reach an absurdum that our reality cannot admit.

**Appendix C**

It is $R(SR)S = R(SR - a^2e_3)a(SR - a^2) = 0$ \hspace{1cm} (2.46)

Let us write it explicitly. We obtain that

\[
R(SR)S = R(SR - a^4e_3)S =
\]

\[
RS(RS - a^2) = -a^4(1/2 - 1/2)e_3(1/2 + 1/2e_3) = 0
\]

(2.47)
Appendix D
We have
\[
R = a \left[ \frac{1 + \sqrt{3}}{4} (E_{o1} - iE_{o2}) + \frac{1 - \sqrt{3}}{4} (E_{o3} - iE_{o2}) \right]
+ \frac{\sqrt{3}}{2} (E_{o1} + iE_{o2})(E_{o3} - iE_{o2})
\]
\[
= a \left[ \frac{1 + \sqrt{3}}{4} (E_{o1} + iE_{o2}) + \frac{1 - \sqrt{3}}{4} (E_{o3} + iE_{o2}) \right]
+ \frac{\sqrt{3}}{4} (E_{o1} - iE_{o2})(E_{o3} + iE_{o2})
\]
(2.48)

Appendix E
At the order \( n = 2 \) as well as at the order \( n = 4 \) we obtain the basic relation
\[
J_{\pm} = J_{\pm}' = 0 \quad \text{and} \quad J_{\mp} \neq 0, J_{\mp}' \neq 0 \quad (2.60)
\]
that gives origin to the quantization.

We have that
\[
J_x = J_x + iJ_y; \quad J_- = J_x - iJ_y;
\]
\[
J_y = \frac{\sqrt{3}}{2} E_{o1} + \frac{1}{2} E_{o2} E_{o3} + \frac{1}{2} E_{o3} E_{o2};
\]
\[
J_y = \frac{\sqrt{3}}{2} E_{o2} + \frac{1}{2} E_{o3} E_{o1} - \frac{1}{2} E_{o1} E_{o3};
\]
\[
J_z = \frac{1}{2} E_{o1} + E_{o2};
\]
(2.64)

with
\[
J_x J_y - J_y J_x = iJ_y;
\]
\[
J_y J_x - J_x J_y = iJ_x;
\]
\[
J_y J_y - J_y J_y = iJ_y;
\]
(2.65)

Appendix F
As required in our formulation we have that
\[
J_{\pm}^n = J_{\pm}' = 0; \quad J_{\mp}^n \neq 0; \quad J_{\mp}' = 0 \quad (2.67)
\]
Therefore our basic formulation fixed on nilpotent and idempotent Clifford algebraic elements is again recalled.

It remains only a feature that needs to be explained. When considering \( J_x, J_y, J_z \), as said in the (2.65), we obtain
\[
J_x J_y - J_y J_x = iJ_y;
\]
\[
J_y J_x - J_x J_y = iJ_x;
\]
\[
J_y J_y - J_y J_y = iJ_y \quad (2.68)
\]
that do not correspond to the standard basic Clifford algebra \( \mathcal{A}(S) \) where in fact we have that
\[
[e_i, e_j] = 2i\epsilon_{ijk} e_k \quad \text{being the difference by a factor} \quad 2.
\]

We gave detailed proof on the existence of the \( \mathcal{A}(S) \). The new algebra connected to the (2.68) may be demonstrated following the same procedure (see the [3,4]) and obtaining in this case the new basic elements
\[
\hat{e}_1 = \frac{1}{2} e_1; \quad \hat{e}_2 = e_2 = e_3 = \frac{2}{3}; \quad \hat{e}_i = -\epsilon_i \hat{e}_i = \frac{1}{2} e_i \quad \text{and}
\]
cyclic permutation of \((i,j,k)\), (2.69)
\[
i=1,2,3; j=1,2,3; k=1,2,3.
\]

Idempotent elements become in this case
\[
\left\{ \frac{1}{2} \pm \hat{e}_i \right\}.
\]

Appendix G
With \( R^2 = Q^2 + Q_z^2 + Q_y^2 \) (2.74)

They satisfy the following basic properties:
\[
L_A A_1 + L_B A_2 + L_C A_3 = 0
\]
and
\[
A_1^2 + A_2^2 + A_3^2 - 1 = \frac{2}{m^2} e^i e^j E (L_i^2 + L_j^2 + L_k^2 + \hbar^2) \quad (2.75)
\]
where it results that
\[
E = H = \frac{1}{2m} \left( P_x^2 + P_y^2 + P_z^2 \right) - Z e^2 R^{-1} \quad (2.76)
\]
It is trivial to acknowledge the basic meaning of \( E \).

Still we find that the following relations hold.
The second important property is that

\[ [A_i, H] = 0 \ ; [A_i, L_i] = 0 \ ; L_i A_2 - A_2 L_i = i h \ A_3 \ ; \]

\[
L_3 A_4 - A_4 L_3 = -i h \ A_3 \ ; L_3 A_2 - A_2 L_3 = i h \ A_1 \ ;
\]

\[
L_3 A_3 - A_3 L_3 = -i h \ A_1 \ ; L_3 A_1 - A_1 L_3 = i h \ A_2 \ ;
\]

Finally, it results that

\[ L_3 A_3 - A_3 L_3 = -i \frac{2 h}{m Z^2 e^4} HL_3 ; \]

\[ A_2 A_3 - A_3 A_2 = -i \frac{2 h}{m Z^2 e^4} HL_1 \ ; A_3 A_1 - A_1 A_3 = -i \frac{2 h}{m Z^2 e^4} HL_2 \]

Let us attempt to write Clifford basic elements in \( A(S_i) \).

Consider the following elements

\[
K_i = \left( -\frac{m Z^2 e^4}{2\epsilon} \right)^{1/2} A_i ; \]

\[
K_2 = \left( -\frac{m Z^2 e^4}{2\epsilon} \right)^{1/2} A_2 ; \]

\[
K_3 = \left( -\frac{m Z^2 e^4}{2\epsilon} \right)^{1/2} A_3 \]

We will obtain that

\[ L_1 K_1 + L_2 K_2 + L_3 K_3 = 0 ; \]

\[ K_1^2 + K_2^2 + K_3^2 + \frac{m Z^2 e^4}{2\epsilon} = -h^2 - L_1^2 - L_2^2 - L_3^2 \]

and finally it results that

\[ K_1 K_2 - K_2 K_1 = i h \ L_1 ; K_2 K_3 - K_3 K_2 = i h \ L_1 ; K_3 K_1 - K_1 K_3 = i h \ L_2 \]

Let us introduce still the following basic elements

\[ M_1 = \frac{1}{2h} (L_1 + K_1) ; N_1 = \frac{1}{2h} (L_1 - K_1) ; M_2 = \frac{1}{2h} (L_2 + K_2) ; N_2 = \frac{1}{2h} (L_2 - K_2) ; M_3 = \frac{1}{2h} (L_3 + K_3) ; N_3 = \frac{1}{2h} (L_3 - K_3) \]

We have that

\[ M_1^2 + M_2^2 + M_3^2 - N_1^2 - N_2^2 - N_3^2 = 0 \]

The second important property is that

\[ -1 - \frac{m Z^2 e^4}{2h^2} \]

The basic property that we need to be sure to be in the Clifford algebraic structure \( S_i \) is that we now have

\[ M_1 M_2 - M_2 M_1 = i M_3 ; \]

\[ N_1 N_2 - N_2 N_1 = i N_3 ; \]

\[ M_2 M_3 - M_3 M_2 = i M_1 ; \]

\[ N_2 N_3 - N_3 N_2 = i N_1 ; \]

\[ M_3 M_1 - M_1 M_3 = i M_2 ; \]

\[ N_3 N_1 - N_1 N_3 = i N_2 \]

as we obtained previously in (2.68) and in (2.69).

We have now given proof that we are in \( S_i \). We have

\[ M_1^2 + M_2^2 + M_3^2 = N_1^2 + N_2^2 + N_3^2 \]

and

\[ M^2 = M_1^2 + M_2^2 + M_3^2 . \]

We may again realize the Clifford algebraic elements

\[ M_1 = M_1 + i M_2 , \text{ and} \]

\[ M_1 = M_1 - i M_2, \text{ (2.86)} \]

and

\[ M_1 M_2 = M_1^2 - i M_2 M_1 + i M_4 M_1 + M_2^2 = M_1^2 - M_2^2 - M_3 \]

and

\[ M_2 M_3 = M_2^2 - i M_3 M_2 + i M_4 M_2 + M_3^2 = M_2^2 - M_3 M_3 - M_2 \]

Since we have found that

\[ M^2 = M_1^2 + M_2^2 + M_3^2 = -\frac{m Z^2 e^4}{2h^2 E} \]

under the condition \( E < 0 \), we write that

\[ -\frac{m Z^2 e^4}{8h^2 E} = \frac{1}{4} + j (j + 1) \]

or
\[ \frac{-mZe^4}{2\hbar^2 E} = 1 + 4j(j + 1) = n^2 \]  

(2.91)

with \( n = 2j + 1 \).

In conclusion, we have that

\[ E = \frac{mZ^2e^4}{2n^2\hbar^2}; \quad n = 1, 2, 3 \]  

(2.92)

that is just the usual formula of the energy levels for the hydrogen atom as it is obtained in the standard case of the usual quantum mechanics. It is instructive to observe that the (2.92) arises from the (2.89) that we have obtained by using the (2.82), the (2.83), and, in particular the (2.88). Again idempotent elements are contained in such basic formulation since, looking at the new basic Clifford scheme given in the (2.69) we have expressions as

\[ M_i + iM_\sigma = 2M_i(\frac{1}{2} - M_\sigma) \]  

and

\[ M_i - iM_\sigma = 2M_i(\frac{1}{2} + M_\sigma) \]  

(2.93)

where

\[ \frac{1}{2} - M_3 \text{ and } \frac{1}{2} + M_3 \]  

(2.94)

are still idempotent elements according to the (2.69).

References


Laplace PS. Traité de mécanique celeste. Tome I, Premiere Partie, Livre II, 1799, 165


