Energy Levels of $^{50}$Ca Nucleus as a Function of the Classical Coupling Angle within MSDI

Ali Khalaf Hasan¹*, Dalal Naji Hameed²

Abstract

In the construction of this kind of shell model, we take the residual interaction to be modified surface delta interaction MSDI. We have studied the excitation energies of the $^{50}$Ca nucleus, which contain two neutrons outside closed shell of the $^{48}$Ca. Neutrons are in the model space $p_f p_g$. The energy levels and angular momentum of all possible cases were investigated. Thereby, we have effectively utilized a theoretical process to find a link among the traditional coupling angle and energy levels at different orbital within neutron-neutron interaction. We observe the energy stages appear to follow two overall functions which depend on the classical coupling angles but are unconstrained of angular momentum I. We find out that our results agree with the experimental data.

Key Words: Shell Model, Modified Surface Delta Interaction, $^{50}$Ca.

Introduction

Lately, intense mass calcium isotopes (N > 28) have been the main topic of theoretical attention and refreshed experimental in view of the fact that they stretch out distant from the steadiness valley, as a result permitting to recognize the development of the shell configuration when moving towards the neutron drip line. This has result in a foremost amount of experimentations in this, part (Liddick et al., 2004; Dinca et al., 2005; Perrot et al., 2006; Rejmun et al., 2007; Fornal et al., 2008; Maierbeck et al., 2009) directing explore the development of the single particle orbitals theoretically describing the spectroscopic characteristics of these nuclei, the shell model with an assortment of two-body efficient interactions has been widely employed recently. Through the recent times, shell model effective interactions devoid of any experimental adjustments have demonstrated to be capable of portraying with notable accuracy of the spectroscopic characteristics of nuclei in several mass regions (Coraggio et al., 2006; Coraggio et al., 2007; Coraggio et al., 2009). Having simple shell model in mind, Talmi used the surface delta interaction to evaluate properties of nuclear states with few' nucleons' on a magic core (Hameed et al., 2018). It employs assumption: at first there subsists an inert core model of close shell, which take actions with central forces upon valence nucleons; second, there subsists a residual interaction caused as a result of two-body forces acting flanked by the valence nucleons. Schiffer (Molinari et al., 1975) considered only those nuclei in which two (particle or hole) and (1p and 1h) are present together with the closed shell, as well as travel in the orbits $j_a$ and $j_b$ of a self-reliable field. It must be noted that Schiffer found out the collective activities of the effective interaction in accordance with the angle flanked by the angular momenta of the interacting nucleons, characteristic which was later exposed to be associated with the short-range character of the effective interaction (Isacker, 2014; Isacker, 2018).

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The angle $\theta_{a,b}$ flanked by the proton (hole or particle) and neutron (hole or particle) angular momentum vectors $j_a$ and $j_b$ was effectively explained in Ref. (Caurier et al., 2005; Schiffer, 1971). There are a number of theoretical works discussing nuclear shell model, by means MSDI (Heusler et al., 2016; Mraheem et al., 2019; Majeed et al., 2019). Poves et al., (Poves et al., 2001) performed nuclear shell model calculation of the $A = 50$. J. Kostensalo and J. Suhonen (Jouhet et al., 2017) performed a calculation of the properties pairing interaction for (even-even) reference nuclei in the mass region $A = 50$ to $102$. J.G.Li, et al. (Li et al., 2020) studied the spectra $^{52,54,56,57,58,59}Ca$. In Ref. (Piekarewicz et al., 2017) explain low-energy monopole strength along $^{40}Ca$ to $^{60}Ca$ (even even). While N.N. Arsenyev, et al. (Arsenyev et al., 2017) studied energies and B(E2) states in $^{46,48,50}Ca$. R. F. Garcia Ruiz, et al. (Ruiz et al., 2016) performed a calculation of the charge radii of $^{49,51,52}Ca$. In the recent years, we studied the energy levels for state hole –hole (Hameed et al., 2020), particle –hole (Hameed et al., 2018; Hameed et al., 2019) and particle–particle (Hasan et al., 2013) by using MSDI and surface delta interaction. Earlier investigations give confidence the aspiration for the current study by means of application of MSDI forecasting low-lying levels structure of $^{50}Ca$ nuclei. Here, the authors have effectively employed a theoretical progression for the purpose of finding link flanked by the traditional coupling angle and energy levels at different orbital.

### Theoretical Model

Schrodinger equation has been indispensable steps to a particular adequate Hamiltonian, so that a traditional shell-model of effective Hamiltonian can be written as (Hasan et al., 2013; Lawson, 1980)

$$H = \sum_{k=1}^{l} H_0 + \sum_{k \leq l} V_{kl}$$

(1)

Where $\sum_{k \leq l} V_{kl}$ is expressed on residual 2-body interactions and can be re-written as:

$$\langle H \rangle_{12} = \langle H \rangle_{21} = V_{abcd}$$

(6)

To calculate the matrix element by using MSDI potential for the residual nucleon-nucleon interaction (Brussaard et al., 1977)

$$V_{a,b} = -4\pi A_T \delta \Omega_{a,b} \delta (r(a) - R_0)\delta (r(b) - R_0) + B \tau_a \tau_b$$

(7)

where $\hat{f}(a), \hat{f}(b)$ indicates the position vectors of interacting particles, $R_0$ represents the nuclear radius (Fortune et al., 2002; Faessler et al., 1967) the strength of interaction $A_T$. The correction term $B \tau_a \tau_b$ is introduced to account for the splitting between the groups of levels with different isospin. Such a form of interaction is called MSDI. The antisymmetrized matrix element of $V^{IT}_{abcd}$ is given by (Brussaard et al., 1977; Jasielska et al., 1976; Glaudemans et al., 1967)

$$\langle H \rangle_{11} = \rho j_c + \rho j_d + V^{IT}_{cdcd}$$

(4)

$$\langle H \rangle_{22} = \rho j_c + \rho j_d + V^{IT}_{cdcd}$$

(5)
\[ V_{ab,cd}^{IT} = -\frac{A_T}{2(I+1)} \times \sqrt{(2j_a + 1)(2j_b + 1)(2j_c + 1)(2j_d + 1)} \times (1 + \delta_{ab})(1 + \delta_{cd}) \times \]

\[ (-1)^{l_a + l_b + j_c + j_d} h_I(j_a j_b) h_I(j_c j_d) \left[ \begin{array}{c} 1 - (-1)^{l_c + l_d + I + T} \end{array} \right] - \]

\[ \left[k_I(j_a j_b) k_I(j_c j_d) \right] \left[ 1 + (-1)^T \right] + \left\{ 2T(T + 1) - 3 \right\} B + C \delta_{a,c} \delta_{b,d} \]

Where it is \( h_I(j_a j_b) = \left\{ j_b, \frac{1}{2}, j_a, \frac{1}{2} \right\} \); Where \( \langle \rangle \) is the Clebsh - Gordon coefficients

The comportment of the diagonal 2 - body matrix element as a function of the spin I of (particle - particle) state is extremely distinctive when their value are plotted in a proper way. Consider (neutron - neutron) in orbits \( j_a \) and \( j_b \) with \( I = j_a + j_b \) one can write then (Brussaard et al., 1977; Issacker et al., 2013)

\[ I^2 = (j_a + j_b)^2 = j_a^2 + j_b^2 + 2\times(j_a j_b)\cos \theta_{a,b} \] (9)

Where \( \theta_{a,b} \) is the angle between the vectors \( j_a \) and \( j_b \). Since the length of vector \( j \) is specified by \( j(j + 1) \) one gets hold of from eq (10) in a conventional picture (Faessler et al., 1967; Heyde, 1994)

\[ \cos \theta_{a,b} = \frac{I(I+1) - j_a(j_a + 1) - j_b(j_b + 1)}{2\sqrt{j_a(j_a + 1)j_b(j_b + 1)}} \] (10)

The 1-dependence of the matrix element \( V_{abcd}^{IT} \) can thus be plotted the same as a function of the angle \( \theta_{a,b} \). The radial overlaps of the particle orbits for light nuclei differ from those for heavy nuclei. The proton - neutron configurations correspond to nucleon pair having mixed isospin and one find (Brussaard et al., 1977)

\[ E_I(p, n) = \frac{1}{2} \left\{ (V_{abcd}^{IT})_{T=1,I} + (V_{abcd}^{IT})_{T=0,I} \right\} \] (11)

Plotting the excitation energy of these states the same as a function of the corresponding angle \( \theta_{a,b} \) determined as specified by Eq. (10). For neutron and proton in various orbits the absolute value of average two body energy is given by (Brussaard et al., 1977; Heyde, 1994):

\[ E = \sum_I (2I + 1) E_I \left\{ \sum_I (2I + 1) \right\}^{1/2} \] (12)

With \( E_I \) defined by Eq. (11).

### Results and Discussion

In this work, some properties of nuclear structure for ground bands of \( ^{50}\text{Ca} \) nuclei have been calculated using MSDI. Valence nucleons of these nucleus are distributed in pfpg (1p3/2 0f5/2 1p1/2 and 0g9/2) model space. In these computations MSDI have been utilized to estimate the energy levels and classical coupling angles \( \theta_{a,b} \) for two neutrons. Thus, \( ^{50}\text{Ca} \) nuclei are discussed the following. While considering this scenario, there are 2 neutrons exterior to the closed shell \( N = 28 \). The inert core employed at this point in this computation is \( ^{46}\text{Ca} \), in addition to the 2 valance neutrons are disseminated over the model space specifically bound through the orbits from pfpg model. The wave function of the excited states encloses typically neutron's. While considering this scenario, the original MSDI Hamiltonian is regulated to properly the ground state energy. It is to be noted that the acceptance with the investigational value is extremely better. With the intention of involving the neutrons contribution, configurations mixing among the orbit are applied. The spectrum of this nucleus was calculated by using Eq. (4,5,6 and 8). Energy levels can be obtained by using the single particle energy (Burrowsa, 2008). Where: \( \rho_{1p3/2} = -5.146 \text{ MeV} \), \( \rho_{0f5/2} = -1.561 \text{ MeV} \), \( \rho_{1p1/2} = -3.123 \text{ MeV} \) and \( \rho_{0g9/2} = -1.133 \text{ MeV} \). Table 1 show a comparison between a theoretical \& experimental: Exp. Res. \& excitation energies \& MeV: for \( ^{50}\text{Ca} \) nucleus by using MSDI. The MSDI
calculations of the energies, and parity are in superior concord with the investigational values (Chen et al., 2019). There is uncertain parity and the spin of certain energy levels experimental. So, the same energy is predicted with parity and spin value such as: the levels with energies of (3.0021, 4.4758, 4.8702, 5.1098 and 8.38) MeV have a respectively angular momentum of (2⁺, 1⁺, 2⁻, 5⁻ and 7⁻).

Experimentally, the energy level (5.043, 4.8306 and 5.5169) MeV was uncertain at the state (1⁺, 4 and 5). Theoretically, the energy of the states (3 2⁺, 1 2⁻, and 4 2⁻), respectively, with the interaction appeared close to the recent experimental value. Also, there is improbability in the spin of certain levels while through investigational approach, the identical energy is calculated with the spin value where the levels with energies of (3.5317, 3.997) MeV with a whole angular momentum of 1 and 3 with positive parity. The new energy levels, which are expected for this nucleus in the states (4 1⁺, 2 2⁻, 4 2⁻; 0 1⁺, 2 2⁻, 5 2⁻, 6 2⁻; 8 1⁺, 2 2⁻, 6 2⁻, 4 2⁻, 3 2⁻) were not well established experimentally.

Plotting excitation energy of these states as a function of the identical angle determined according to Eq. (10). Table 2 show (according to states of angular momentum I, all possible cases of the classical coupling angle data) one can draw the curve shown in Fig. 1 (D and E) show the behaviour for even states \(j_a + j_b + I = \text{even}\) and Fig. 1 (A, Band C) that for odd states \(j_a + j_b + I = \text{odd}\) of effective interaction deduced from the value. Curve is a kind of computation of a small range

<table>
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<th>(I^\pi)</th>
<th>Energy (MeV)</th>
<th>Energy (MeV)</th>
<th>(I^\pi)</th>
<th>Energy (MeV)</th>
<th>Energy (MeV)</th>
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Table 1. A comparison between a theoretical - Theor. Res. and experimental - Exp. Res. - excitation energies - MeV - for \(^{40}\)Ca nucleus by using MSDI

The form of the curve in Fig. 1 (A, B, C, D and E) for \(\theta_{a,b} = 90\), the particle ranges have a diminutive overlap, that effect in a feeble interaction. For \(\theta_{a,b} = 180\). The ranges of the (particle-particle) interaction, shifting in contradictory track, encompass a huge overlap. In view of the fact that the nuclear force is of diminutive range the interaction will be completely sturdy 'large and attractive'. It is observed that this interaction makes noticeably why the curve in Fig. 1 (A, B, C, D and E) have apposite slope for unreliable from 180 to 90. In case of smaller angle, the Pauli exception principle become significant. While considering \(\theta_{a,b} = 0\) and \(j_a = j_b\), it is essential to differentiate the two possibilities of isospin combination.

During the \(\theta_{a,b} = 180\) scenario, the particle occupy a spatially symmetric particle-particle state which as a result of the strong diminutive range attraction, produce a huge negative matrix component. In the scenario, T=1 case the particle generate a spatially antisymmetric state and consequently their comparative distance increases for decreasing angle \(\theta_{a,b}\) to 0.
Table 2. According to states of angular momentum $I$, all possible cases of the classical coupling angle data

<table>
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<tr>
<th>$I$</th>
<th>$\pi$</th>
<th>Configuration $^{50}\text{Ca}$</th>
<th>state</th>
<th>$I$</th>
<th>$\pi$</th>
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<th>state</th>
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(A) $\text{Ca-50}$

(B) $\text{Ca-50}$
Fig. 1. The value of theoretical energy intensities for particle-particle states as a function of the classical coupling angles $\theta_{ab}$.

Fig. 2. The relationship between classical coupling angles of an even and an odd cases, with the angular momentum of all possible cases.

**Conclusion**

The agreement between theoretical and experimental levels is satisfactory for excitation energies. There are many uncertain experimental energy levels certain by our calculations and new data for energy levels which were not specified in the experimental value. The theoretical calculations for MSDI reasonably concur with the provided experimental data. The minimum I values stand for utmost angle $\theta_{u,b}$ and vice versa. This indicates that the MSDI is very good to illustrate the nuclear structure for $^{50}$Ca nuclei.

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