Optimal and Approximate Analysis of an Information Network

Daegene Song

ABSTRACT

One of the important outcomes of recent studies of quantum theory is developing useful and practical applications from often controversial and philosophical debates such as the simultaneous existence of multiple states or superluminal influencing, which appears to violate the locality imposed by relativity. Establishing a long-distance correlation may be effective in a number of applications in quantum information technology. In this paper, an entanglement swapping scheme for three 2-level states is examined using numerical methods. In particular, it is studied that there is a class of non-maximal states that approximate the optimal outcome, namely, the weakest link. The approximate class is shown to be limited rather than optimal, yet nevertheless substantially distributed. Both the finding and analysis of approximate class may be helpful in establishing an entanglement network, since the process would inevitably contain imperfect situations.

Key Words: Numerical simulation, Entanglement, Network

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Introduction

Quantum information theory has continued to develop, particularly in tandem with the introduction of quantum computer in the 1980s (see (Nielsen et al., 2000) for a review). It has advanced numerous important concepts involving the fundamental role of information based on quantum theory. In particular, quantum information science introduced a simple way of viewing an extremely complicated nature, i.e., the theory treats complex phenomena with degrees of freedom represented by simple bits, or quantum bits.

While quantum mechanics has involved measurement from the very beginning, many textbooks introducing quantum theory try to minimize the role of observation or the observer, and limited discussion is provided involving measurement, often implying that it is a philosophical issue. However, the quantum information scientific approach presents the observer or the subject from the outset and establishes it on an equal footing with the object. Rather than trying to hide the subjectivity of the theory, it embraces it.

Together with the simplification achieved through representing the degrees of freedom of physical systems in terms of bits, the treatment of the subject and object on an equal footing places the scientific investigation as a relation, or an interaction, between the consciousness of the subject and the object of the universe as shown in Fig 1. Why consciousness of the observer? The state vector that represents objects and the observables that correspond to the mathematical representation of the reference frame of the subject are defined in complex vector space rather than in classical space.

There is an ongoing debate over whether the objects of quantum nature are real. Since they are defined in an imaginary complex space that is

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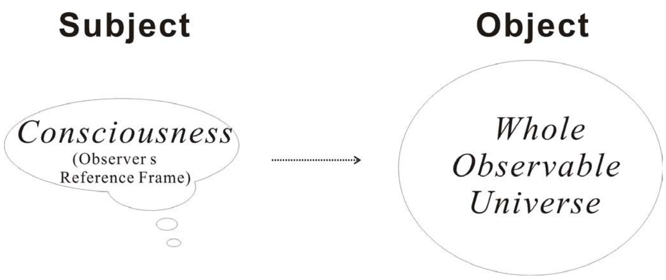

Figure 1. A scientific investigation may be considered a relationship between the consciousness of the observer and the object of the whole universe.

not directly observed, there remains doubt about whether the quantum objects are real as we currently understand them. John Bell mentioned an imaginary quantum world, one not directly observed in classical space (Bell, 1971):

"Theoretical physicists live in a classical world, looking out into a quantum-mechanical world. The latter we describe only subjectively, in terms of procedures and results in our classical domain."

In the 17th century, the philosopher and mathematician René Descartes considered a similar dilemma. He attempted to cast doubt on every possibility in order to come up with an absolutely undoubtable truth, as the following suggests (Descartes, 1644):

"If you would be a real seeker after truth, it is necessary that at least once in your life you doubt, as far as possible, all things."

One may argue that the existence of objects may be illusional and that their existence may not be as convincing as often thought. However, the thought of the subject cannot be doubted and the existence of the thought itself should correspond to an undoubtable truth. This line of thought led Descartes to conclude with the well-known phrase: "I think, therefore I am."

In a way similar to Descartes' argument, the existence of quantum objects can be questioned. However, the capacity of the subject to imagine quantum objects is certain to exist. Since the quantum reference frame of a subject does not belong to the classical domain, observables should correspond to the subject's mental state. If we expand objects to be the whole observable universe, the observables ought to correspond to the subject's consciousness, as shown in Fig. 2.

Although quantum theory is often portrayed as strange and counterintuitive, its unique aspects have

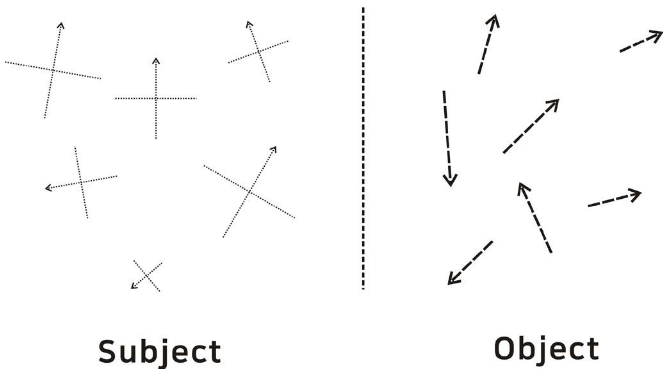

Figure 2. The quantum information approach provides a simple way of representing the subject and the object in terms of mathematics as observables of the reference frame and state vectors of the observed universe, respectively.

led to some remarkable applications in the field of computation (Deutsch, 1985; Kok *et al.*, 2007; Ladd *et al.*, 2010) and cryptography (Bennett *et al.*, 1984; Ekert, 1991). In the following, we wish to examine entanglement swapping protocols (Zukowski *et al.*, 1993; Boulant *et al.*, 2003; Kaltenbaek *et al.*, 2009) for three 2-level correlated states. In (Bose *et al.*, 1998; Shi *et al.*, 2000), it was shown that two 2-level states yield a weaker link when entanglement swapping is applied. However, for three or more 2-level states, this is generally not the case.

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Materials and Methods

Previously, numerical approaches have been provided to examine the cases of two 3- and 4-level correlations (Song, 2018a; Song, 2018b). In this note, we examine the entanglement swapping protocol for three 2-level non-maximal states. In the case of two 2-level states, Bell measurement yields the optimal average correlation. Let us consider the following two 2-level entanglements with ordered Schmidt coefficients,

$$\left|\phi^{(1)}\right\rangle_{12} = \sum_{i=0}^{1} \sqrt{\alpha_i}\left|i,i\right\rangle \qquad (1)$$

$$\left|\phi^{(2)}\right\rangle_{34} = \sum_{j=0}^{1} \sqrt{\beta_j}\left|j,j\right\rangle \qquad (2)$$

When states 2 and 3 are measured with Bell states, states 1 and 4 are projected to establish a correlation. If we assume $\left|\phi^{(1)}\right\rangle_{12}$ has weaker entanglement, then the average maximal states between 1 and 4 yields

$$2\alpha_1\beta_0 + 2\alpha_1\beta_1 = 2\alpha_1 \qquad (3)$$

This corresponds to the average maximal entanglement of $\left|\phi^{(1)}\right\rangle_{12}$ in (1), which therefore is optimal. While the entanglement swapping protocol yields the best possible scenario when there are two states, this is no longer accurate for more than two non-maximal 2-level states. To investigate this further, let us consider an additional entanglement as follows:

$$\left|\phi^{(3)}\right\rangle_{56} = \sum_{k=0}^{1}\sqrt{\gamma_k}\left|k,k\right\rangle \qquad (4)$$

Given these three 2-level states, ($\left|\phi^{(1)}\right\rangle_{12}$ is assumed to have the weakest entanglement), that is, (1), (2), and (4), when Bell-type measurements are made on 2 and 3, and 4 and 5, a new correlation is established between 1 and 6. In particular, let us consider the following conditions on the coefficients of the resulting entanglement between 1 and 6:

$$\alpha_0\beta_0\gamma_0 \geq \alpha_1\beta_1\gamma_1 \qquad (5)$$

$$\alpha_0\beta_0\gamma_1 \geq \alpha_1\beta_1\gamma_0 \qquad (6)$$

$$\alpha_0\beta_1\gamma_0 \geq \alpha_1\beta_0\gamma_1 \qquad (7)$$

$$\alpha_0\beta_1\gamma_1 \geq \alpha_1\beta_0\gamma_0 \qquad (8)$$

It may be easily checked that when (5-8) are satisfied, the average maximal entanglement between 1 and 6 corresponds to $2\alpha_1$, that is, the maximum average entanglement for an initial state between 1 and 2 (Joanthan *et al.*, 1999; Hardy, 1999; Hardy *et al.*, 1999). As shown in Fig. 3, the non-maximal states that satisfy (5-8) are widely distributed.

Results and Discussion

While it is good to have these coefficients, it is desirable to have more states that approximate the weakest link. Let us consider the following condition:

$$\alpha_0\beta_1\gamma_1 < \alpha_1\beta_0\gamma_0 \qquad (9)$$

Using numerical method, it can be checked that there is a class of non-maximal states that satisfy (5,6,7) with (9) replacing (8), yielding an average maximal entanglement close to the optimal value, namely, the weakest link, with margin <0.01. For example, when $\alpha_0 = 0.945$, $\alpha_1 = 0.055$, β_0=0.93, β_1=0.07 $\gamma_0 = 0.565$, and $\gamma_1 = 0.435$, the average maximum entanglement, $E_{\max} = 0.102066\ldots$, is obtained, which approximates the weakest link, $E_{\max}^{Weakest}$ = 0.11 (also see Fig. 4).

In Fig. 5, the new class of coefficients that satisfy (5,6,7,9) (indicated by *X*'s) is compared with the previous coefficients with (5-8) (shown with *O*'s). It can be seen that while the new class for α_0, α_1 (i) is not as widely distributed as the previous case shown with *O*'s, the coefficients that approximate the optimal result become as widely distributed as the previous non-maximal states for γ_0, γ_1 (iii).

Quantum theory has revolutionized the way people think about nature (Peres, 1997). Before quantum theory, people often sought an objective reality, that is, something that does not change dependent on the observer or whether there is an observer. In fact, this approach has been so successful in the history of science that researchers were able to predict the way nature evolves with great precision. 99

Indeed, quantum theory has changed the way people pursue scientific endeavors in the sense that the traditional approach of discovering objective facts or rules has shifted to finding relationships

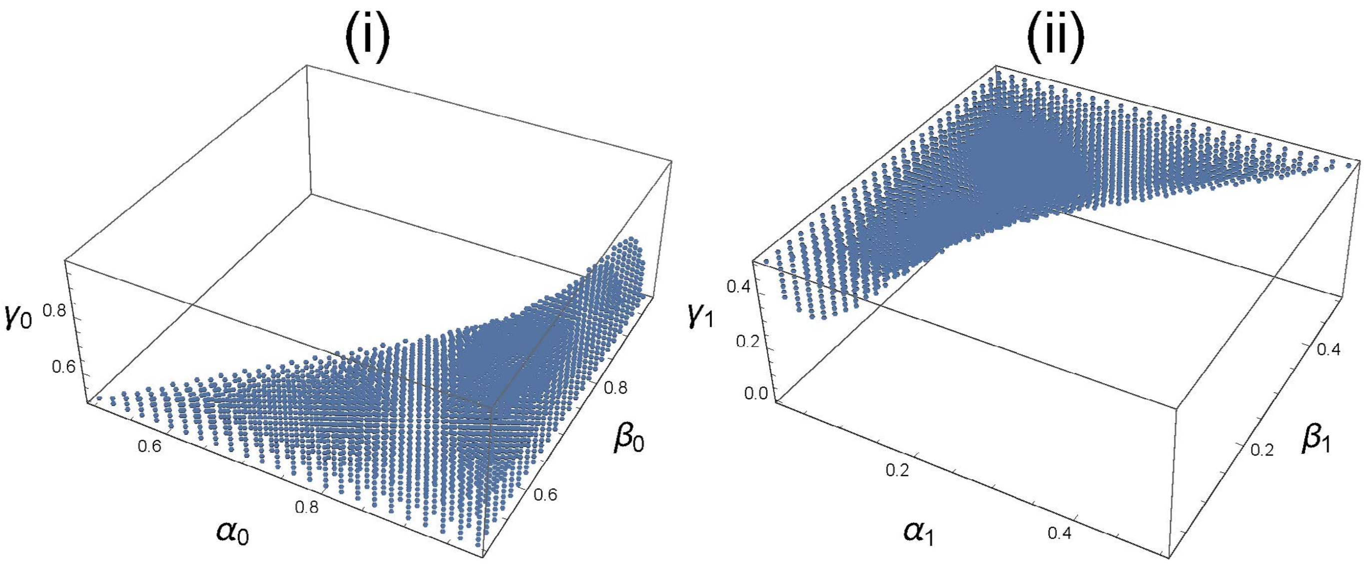

Figure 3. The distribution of non-maximal states that satisfy (5-8). The entanglement swapping protocol with these coefficients in fact yields the optimal result, that is, the weakest link.

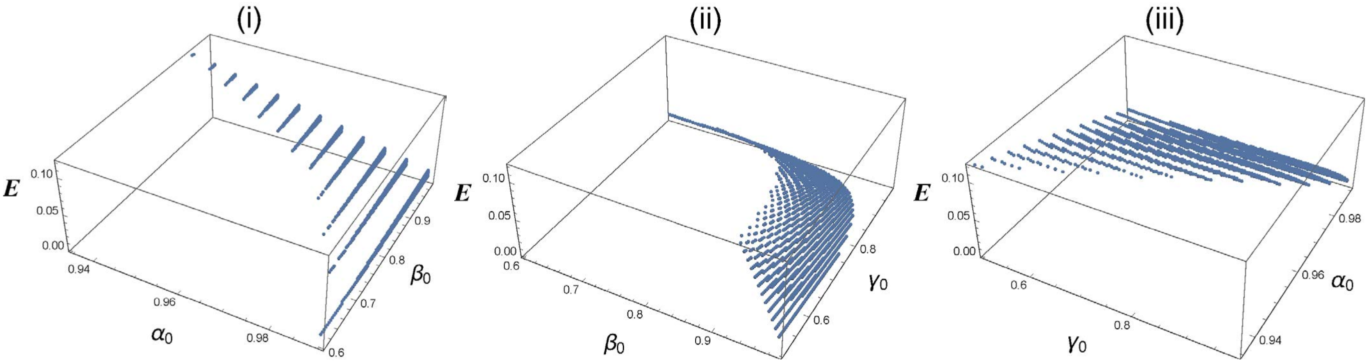

Figure 4. Average maximal entanglement for coefficients that satisfy (5,6,7,9), where the outcome approximates the weakest link. In particular, (i) shows the relation between the coefficients α_0 and β_0, (ii) between β_0 and γ_0, and (iii) between γ_0 and α_0.

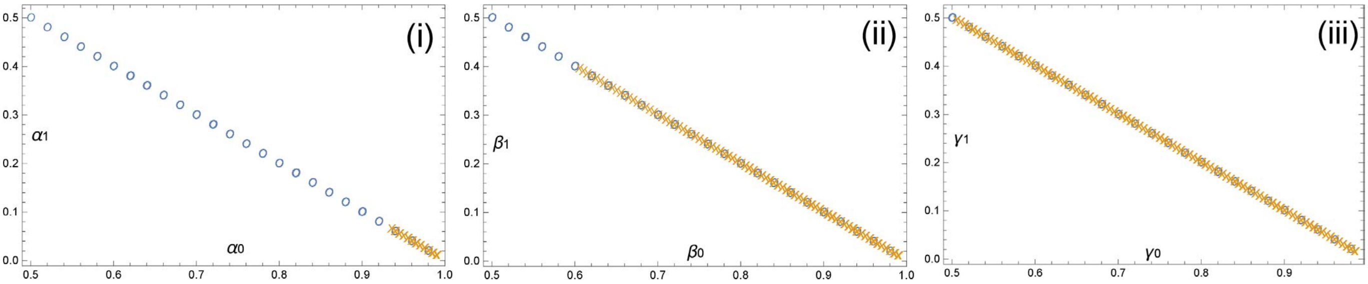

Figure 5. Comparison of the distribution between the coefficients satisfying (5-8) (with *O*'s) and the non-maximal states with (5,6,7,9) (with *X*'s).

between the observer and the physical system. Moreover, this aspect of quantum theory, which is often considered philosophical, has recently yielded extremely practical and powerful applications, such as breaking current cryptographic protocols (Shor, 1994) or establishing an unbreakable crypto-system (Bennett *et al*., 1984). In particular, entanglement has been shown to be an important element in quantum information science, such as teleportation (Bennett *et al*., 1993), key distribution (Ekert, 1991), and computing (see (Nielsen *et al*., 2000) for a review). In this paper, we have numerically examined certain aspects of quantum correlation that are critical in many realizations of next-generation quantum technology, such as long-distance entanglement (Tittel *et al*., 1998).

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