LQR Suppression of Hopf Bifurcation in Hodgkin-Huxley Neurons

Yexin Lin, Haiyuan Liu* and Li Hu

ABSTRACT

The work of Hodgkin and Huxley on nerve conduction has long been recognized as an outstanding scientific achievement. The Hodgkin-Huxley equations are modeled by a number of parameters and perform diverse behaviors depending on the various parameters. The purpose of this research is to solve the bifurcation control problem in the Hodgkin-Huxley nerve fibers. In this study, we focus on the Hopf bifurcation in the HH model, which arises by the external current injection. We present a feedback controlled approach to control the bifurcation phenomenon in the Hodgkin-Huxley nerve fibers via washout filter which can stop the repetitive firing in a particular region of the nervous system validly. A washout filter is augmented to the HH dynamics and the output of the filter is fed to an external controller generator through a linear gain. The linear projective control theory is applied to compute the control gain. The stable controller is performed by two similar control rules that are designed by the integration of the washout filters and static projective control theory. The first case of the control rule filters all the dynamic variables including the membrane potential and the channel activation, while the second case of the control rule filters merely the membrane potential. The MATCONT software package is used for analysis of the bifurcation points in conjunction with MATLAB.

Key Words: Hodgkin-Huxley model, Hopf bifurcation, Washout filter, Bifurcation control

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Introduction

The Hodgkin-Huxley nonlinear model (Hodgkin et al., 1952) which describes the electrical excitations of the squid giant axon is one of the greatest challenges in the electrophysiology and neural computing. The first complete mathematical equation set of neuronal membrane dynamics were proposed by British biologists Hodgkin and Huxley in 1952. The membrane potential is modeled by circuit theory and kinetic method, and a fourth order highly nonlinear differential equation is presented. Many scholars put forward their own improvement and innovation on the basis of the HH model in the following decades. The voltage-clamped experiments on rat muscles were carried out by Duval (Duval et al., 1978), Adrian (Adrian et al., 1977) and Pappone (Pappone, 1980). The FitzHugh-Nagumo model which was suggested by Richard FitzHugh in 1961 is a simplified version of the Hodgkin-Huxley model (Fitzhugh, 1962). After Hodgkin and Huxley, various kinds of ionic channels have been discovered and discussed, and their open-close characteristics have been identified for diverse neuronal membranes (Hille, 2001). These electrical excitations have been modeled under the HH formalism (Chay et al., 1988). Although the mentioned HH-type models may perform much more complex dynamics than the original HH model, they share common nonlinear characteristics and dynamics in many aspects.

Bifurcation phenomenon refers to qualitative changes in the solution structure of kinetic system when the system parameters vary slightly. Bifurcation
is problematic and complicated in biological neural networks. The etiology of some neurological diseases including Parkinson disease, pathological heart rhythms is closely interrelated to the bifurcation phenomenon. Controlling bifurcations in the neuron dynamical system can give an assistance to the treatment of those diseases (An, 2006). Bifurcation control aims to design a controller that can eliminate the repetitive firing to avoid undesirable instability around bifurcation points and obtain the expected dynamical behaviors by modifying the system bifurcation properties. System bifurcations can be controlled by different methods including dynamic feedback based on washout filter (Zhou et al., 2003), linear or nonlinear state feedback (Abed et al., 1986; Abed et al., 1994), on time-delayed feedback control (Chen et al., 1999), applying the quadratic invariants in the normal form (Doruk et al., 2013), and control by automatic temperature (Doruk et al., 2018).

In this paper, we focus mainly on eliminating the Hopf bifurcation caused by the external current injection in the HH model. In this process, we applied the washout filter as a controller and combined the washout filter controller with the linear quadratic theory (LQR) to obtain the control gain. We proposed and designed two control methods based on the washout filter. The first is to filter all the dynamic system variables while the second is to filter merely the membrane potential. Both methods applied to the HH close-loop model are computed by MATCONT software package (Dhooge et al., 2003).

The paper proceeds as follows: we introduce the HH equations in detail and describe the washout filter in Section 2. The bifurcation theory and the results of simulation analysis in the HH open-loop system are shown in Section 3. We deduce the linear control gain based on LQR and apply the washout filter to the HH close-loop model with LQR in the Section 4. The responses of the designed HH close-loop system are shown in the Section 5. Conclusion and prospect are presented in the Section 6.

The HH equations

The Hodgkin-Huxley model is a fourth order highly nonlinear differential equation derived for representing the electrical excitations of the squid giant axon.

The HH model can be described as

\[
\begin{align*}
\frac{dV}{dt} &= \frac{1}{C_m} \left[ I_{\text{ext}} - g_N a m^3 h (V - V_{Na}) - g_K n^4 (V - V_K) - g_L (V - V_L) \right] \\
\frac{dn}{dt} &= \alpha_n (1 - n) - \beta_n n \\
\frac{dm}{dt} &= \alpha_m (1 - m) - \beta_m m \\
\frac{dh}{dt} &= \alpha_h (1 - h) - \beta_h h
\end{align*}
\]

(1)

\[
\begin{align*}
\alpha_n &= \frac{0.1 - 0.01V}{\exp(1 - 0.1V) - 1}, \beta_n = 0.125 \exp \left( -\frac{V}{80} \right) \\
\alpha_m &= \frac{2.5 - 0.01V}{\exp(2.5 - 0.1V) - 1}, \beta_m = 4 \exp \left( -\frac{V}{18} \right) \\
\alpha_h &= 0.07 \exp \left( -\frac{V}{20} \right), \beta_h = \frac{1}{\exp(3 - 0.1V) + 1}
\end{align*}
\]

(2)

In the HH model, \( V \) is the displacement of the cell membrane potential from its resting value in mV; dimensionless state variables \( n, m \) and \( h \), which take continuous values between zero and one, represent the proportion of activating molecules of the potassium channel, the proportion of activating molecules of the sodium channel and the proportion of inactivating molecules of the sodium channel. The \( I_{\text{ext}} \) denotes the external current injection, in units of \( \mu \text{A/cm}^2 \). \( g_{Na} \), \( g_K \) and \( g_L \) represent the sodium channel conductance, the potassium channel conductance and the current leakage channel conductance in mS/cm². \( V_{Na} \), \( V_K \) and \( V_L \) represent the sodium channel resting potential, the potassium channel resting potential and the level of potential where the leakage current reduces to zero in mV. \( C_M \) is the membrane capacitance in \( \mu \text{F/cm}^2 \).

The following shows the values taken for the simulation purpose

\[
\begin{align*}
g_{Na} &= 120 \text{mS/cm}^2 \\
g_K &= 36 \text{mS/cm}^2 \\
g_L &= 0.3 \text{mS/cm}^2 \\
V_{Na} &= 115 \text{mV} \\
V_K &= -12 \text{mV} \\
V_L &= 10.613 \text{mV} \\
C_M &= 0.91 \mu \text{F/cm}^2
\end{align*}
\]

(3)

When the HH model is simulated with the above parameter values under no external current injection, i.e. \( I_{\text{ext}} = 0 \). The steady state values of HH model can be obtained as

\[
X_e = [V_e, n_e, m_e, h_e] = [0.0036206688, 0.3177323999, 0.0529550868, 0.5959941247]
\]

(4)
The washout filter theory

A washout filter (also sometimes called a washout circuit) is a high-pass filter that washes out (rejects) steady state inputs, while passing transient inputs (Hassoun et al. 2004). The biggest advantage of using washout filter is that all the equilibrium points of the open-loop system can be preserved, in other words, the locations of those equilibrium points are not drifted. Although washout filters have been successfully used in many control applications, there is no systematic way to choose the constants of the washout filter. The washout filter in feedback control does not drift the position of the equilibrium points of the open-loop system. We apply the washout filter to the Hodgkin-Huxley close-loop system to control the bifurcation phenomenon. Nevertheless, there are still some limitations in using stable washout filter in feedback control.

Hopf Bifurcation of the HH Model

Bifurcation phenomenon refers to a state change in the number of candidate operating conditions of a nonlinear dynamic system when a system parameter is slightly varied. The candidate operating condition of system includes equilibriums, periodic solutions, limit points and other invariant subset of its limit set. Different bifurcations occur with the change of different bifurcation parameters. Among the diverse bifurcations encountered in the HH model, we focus on the Hopf bifurcations. At the Hopf bifurcation point, a periodic solution of small amplitude emerges from the equilibrium state when the bifurcation parameter value is varied across the Hopf bifurcation point.

Hopf bifurcation. A continuous fundamental control-phase path of an equilibrium point loses its stability as it intersects a secondary path of a periodic solution. The location of a Hopf bifurcation on the equilibrium point is characterized by a complex conjugate pair of linear eigenvalues of the Jacobian matrix whose real part passes through zero. When the secondary path is stable, it is the supercritical Hopf bifurcation. Otherwise, when the secondary path is unstable, it is subcritical Hopf bifurcation (Tompson et al., 1993).

The Jacobian matrix of HH model is given

\[
A = \begin{bmatrix}
\frac{\partial \dot{V}}{\partial V} & \frac{\partial \dot{V}}{\partial n} & \frac{\partial \dot{V}}{\partial m} & \frac{\partial \dot{V}}{\partial h} \\
\frac{\partial \dot{n}}{\partial V} & \frac{\partial \dot{n}}{\partial n} & 0 & 0 \\
\frac{\partial \dot{m}}{\partial V} & 0 & \frac{\partial \dot{m}}{\partial m} & 0 \\
\frac{\partial \dot{h}}{\partial V} & 0 & 0 & \frac{\partial \dot{h}}{\partial h}
\end{bmatrix}
\]  

Here we analyze the Hopf bifurcation in the HH model when the external current injection \(I_{ext}\) varies. Applying bifurcation theorem and using MATCONT as a tool, we can simulate and analyze the Hopf bifurcation in the HH model. Before the simulation work is carried out, we first need to calculate a set of the equilibrium point of the open-loop HH equations without the external stimulus to start the MATCONT software packages. The value of equilibrium point without the external current injection is obtained in (4). Starting the MATCONT uses the above the steady state values. The following results are found by tracing over the external current injection while holding the other parameters at nominal values. We can get two Hopf bifurcation points of HH model when the external current injection \(I_{ext}\) changes. The two Hopf bifurcation points are exhibited in the Table.1.

Table 1. Bifurcation analysis results derived by the MATCONT software.

<table>
<thead>
<tr>
<th>Case</th>
<th>Parameter</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(I_{ext}) = 9.515396</td>
<td>(I_{ext}) = 154.223130</td>
<td></td>
</tr>
<tr>
<td>Eigenvalues</td>
<td>(\lambda_1 = -4.88804)</td>
<td>(\lambda_1 = -10.104019)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(\lambda_2 = j0.603383)</td>
<td>(\lambda_2 = j1.074441)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(\lambda_3 = -j0.603383)</td>
<td>(\lambda_3 = -j1.074441)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(\lambda_4 = -0.137961)</td>
<td>(\lambda_4 = -0.310617)</td>
<td></td>
</tr>
<tr>
<td>Equilibrium Points</td>
<td>(V = 5.247813)</td>
<td>(V = 21.925581)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(n = 0.4002203)</td>
<td>(n = 0.643052)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(m = 0.096220725)</td>
<td>(m = 0.419248)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(h = 0.40959479)</td>
<td>(h = 0.070483)</td>
<td></td>
</tr>
</tbody>
</table>
In order to show the bifurcation characteristics of the model, it will be convenient to show the bifurcation diagrams obtained by the MATCONT software package for the varying values of the external current injection. Therefore we choose the external current injection as the bifurcation parameter, the simulation result is shown in Fig. 1.

The responses of equation (1) with different values of the external current injection are shown in the Fig. 2-4. The case 1 and 2 are shown in Fig. 2 and Fig. 3. Fig. 4 shows the response of open-loop system simulation with the $I_{\text{ext}} = 100$.

**The Controlled HH Model**

*The close-loop HH model based on a washout filter controller*

In this paper, the parameters of washout filter can be selected arbitrarily provided that the filter has a stable behavior. The gain is tuned by static projective control approach (Doruk et al., 2013). That is a linear output feedback method which approximates the close-loop spectrum of a full state feedback control through orthogonal projection. Thus one first is in need of a full state feedback linear control law designed for the considered problem. In order to have

![Figure 1. The bifurcation diagram for external current injection $I_{\text{ext}}$ variation](image1.png)

![Figure 2. Results of the open-loop system simulation for case $I_{\text{ext}} = 9.515396$](image2.png)
a simple and quick algorithm one can use the LQR for obtaining the full state feedback control prototype before the projection operation.

The washout filter controller can either filter all of the states of the HH model or merely filter the membrane potential. Both cases are examined and simulated in this paper. The first case: the first controller filters all the dynamic variables of the HH nerve fibers which requires an estimator to compute the values of the activations from the membrane potential measurements. This method brings complex computation. The second case: controller merely filter the membrane potential and only a single gain needs to be computed. Both methods are investigated in order to compare the performance of the washout filters in the two cases. The design procedure involves the discussion of the washout filter dynamics to the HH model. Then the modified model is linearized around the bifurcation equilibrium points and finally a linear control method is applied to obtain a state feedback controller. The orthogonal projection operation is applied on the full state feedback control law and a feedback only from the washout filter output is constructed.

For the first case: controller filters all the dynamic variables.
All the states of the HH model are filtered so the components of the parameters ($A_n$ and $\hat{A}_n$) of the washout filter in (5) can be designed as

$$A_n = \begin{bmatrix} a_{n1} & 0 & 0 & 0 \\ 0 & a_{n2} & 0 & 0 \\ 0 & 0 & a_{n3} & 0 \\ 0 & 0 & 0 & a_{n4} \end{bmatrix}, \quad B_n = \begin{bmatrix} \beta_{n1} & 0 & 0 & 0 \\ 0 & \beta_{n2} & 0 & 0 \\ 0 & 0 & \beta_{n3} & 0 \\ 0 & 0 & 0 & \beta_{n4} \end{bmatrix}$$ (9, 10)

And the controller is

Where the matrix $K \in \mathbb{R}^4$ is the control gain and the matrix $Y$ is the output of the washout filter. The close-loop HH model is obtained as

$$\frac{dV}{dt} = \frac{1}{C_m} \left[ I_{ext} - g_{Na} m^3 h (V-V_{Na}) - g_K n^4 (V-V_K) - g_L (V-V_L) + u \right]$$

$$\frac{dn}{dt} = a_{n1} (1-n) - \beta_{n1} n$$

$$\frac{dm}{dt} = a_{n2} (1-m) - \beta_{n2} m$$

$$\frac{dh}{dt} = a_{n3} (1-h) - \beta_{n3} h$$

$$\frac{dh}{dt} = a_{n4} (1-h) - \beta_{n4} h$$

$$z_1 = \alpha_{n1} z_1 + \beta_{n1} V$$

$$z_2 = \alpha_{n2} z_2 + \beta_{n2} n$$

$$z_3 = \alpha_{n3} z_3 + \beta_{n3} m$$

$$z_4 = \alpha_{n4} z_4 + \beta_{n4} h$$

$$y_1 = \alpha_{u1} y_1 + \beta_{u1} z_1$$

$$y_2 = \alpha_{u2} y_2 + \beta_{u2} n$$

$$y_3 = \alpha_{u3} y_3 + \beta_{u3} m$$

$$y_4 = \alpha_{u4} y_4 + \beta_{u4} h$$

In the development of the control algorithm the linearized version of equation (1) is

$$\frac{d\hat{y}}{dt} = \begin{bmatrix} \frac{\partial f}{\partial x} \hat{y} + \frac{\partial f}{\partial u} u + \frac{\partial f}{\partial x_{delay}} x_{delay} \\ \frac{\partial h}{\partial x} \hat{y} + \frac{\partial h}{\partial u} u + \frac{\partial h}{\partial x_{delay}} x_{delay} \\ \frac{\partial k}{\partial x} \hat{y} + \frac{\partial k}{\partial u} u + \frac{\partial k}{\partial x_{delay}} x_{delay} \\ \frac{\partial l}{\partial x} \hat{y} + \frac{\partial l}{\partial u} u + \frac{\partial l}{\partial x_{delay}} x_{delay} \end{bmatrix}$$

And the output of the filter is designed by

$$Y = \begin{bmatrix} \alpha_{u1} & 0 & 0 & 0 \\ 0 & \alpha_{u2} & 0 & 0 \\ 0 & 0 & \alpha_{u3} & 0 \\ 0 & 0 & 0 & \alpha_{u4} \end{bmatrix} z_1 \begin{bmatrix} \beta_{u1} & 0 & 0 & 0 \\ 0 & \beta_{u2} & 0 & 0 \\ 0 & 0 & \beta_{u3} & 0 \\ 0 & 0 & 0 & \beta_{u4} \end{bmatrix}$$ (14)

We need to calculate the control gain matrix $K = [k_1, k_2, k_3, k_4]$.

For the second case: controller merely filter the membrane potential.

The state space of the washout filter is designed as

$$\dot{z} = \alpha_n z + \beta_n V$$

$$y = \alpha_n z + \beta_n V$$ (15)

And the controller is

$$u = -ky$$ (16)

The close-loop HH model is in a similar form with the first case. In the development of the control algorithm the linearized version of (13) is

$$\begin{bmatrix} \frac{\partial f}{\partial x} \hat{y} + \frac{\partial f}{\partial u} u + \frac{\partial f}{\partial x_{delay}} x_{delay} \\ \frac{\partial h}{\partial x} \hat{y} + \frac{\partial h}{\partial u} u + \frac{\partial h}{\partial x_{delay}} x_{delay} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \frac{\partial f}{\partial x} \hat{y} + \frac{\partial f}{\partial u} u + \frac{\partial f}{\partial x_{delay}} x_{delay} \\ \frac{\partial h}{\partial x} \hat{y} + \frac{\partial h}{\partial u} u + \frac{\partial h}{\partial x_{delay}} x_{delay} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

With the washout filter controller output as

$$y = \alpha_n z + \beta_n V$$ (18)

We need to calculate the control gain $k$.

We will introduce the LQR theory to compute the gain $K$ of the designed washout filter controller in next section.

**Linear quadratic output feedback using projective control theory to calculate the control gain**

In control theory, the linear-quadratic-Gaussian (LQG) control problem is one of the most fundamental optimal control problems. It concerns linear systems driven by additive white Gaussian noise. The problem is to determine an output feedback law that is optimal in the sense of minimizing the expected value of a quadratic cost criterion. The projective control approach is a linear control method which derives an output feedback controller from a full state feedback system design. The orthogonal projection is applied to the close-loop controlled system in the theory. Consider a linear system in the following form

$$x = Ax + Bu$$

$$y = Cx$$

Where $x \in \mathbb{R}^n, u \in \mathbb{R}^m, A \in \mathbb{R}^{n \times n}, B \in \mathbb{R}^{n \times m}, C \in \mathbb{R}^{q \times n}$.
The full state feedback controller is designed as 
\[ u = -K_vx \] 
where \( K_v \) is a gain matrix. The control gain matrix can be found from the linear quadratic theory which minimizes the following quadratic performance function. We can obtain control gain matrix by minimizing the infinite horizon quadratic gain index shown below

\[ J = \int_0^\infty (x^T Q x + u^T R u) dt \quad (20) \]

And the close-loop dynamics can be described as

\[ \dot{x} = Ax + Bu = Ax - BKx = (A - BK)x \quad (21) \]

The characteristic equation of the close-loop dynamics can be obtained as

\[ (A - BK)V = V\Lambda \quad (22) \]

Where \( \Lambda \) is a diagonal matrix whose elements are the eigenvalues of the close-loop dynamics and \( V \) is the corresponding eigenvectors. Consider the output feedback form 
\[ y = Cx \]
we can obtain the new form of the controller 
\[ u = -KY \quad (28) \]

The output variables are

\[ y = y_1, y_2, y_3, y_4 \]

The obtained gain matrix is

\[ K = [k_1, k_2, k_3, k_4] \quad (31) \]

For the second case (controller merely filters the membrane potential), the washout filter parameters can be taken as \( \alpha_w = -0.01, \beta_w = 1 \). In this condition, the washout filter keep stable. The state variables are

\[ x = [V, n, m, h, z]^T \quad (32) \]

The controller is designed as 
\[ u = -ky \quad (33) \]

The output variables are

\[ y = \beta_w y \]

The obtained gain is

\[ K = K \quad (36) \]

Where the quadratic cost weights \( Q \) and \( R \) in (20) can be designed as diagonal matrices for the sake of simplicity.

\[ Q = qI, R = 1 \quad (37) \]
Where \( I \) is a unit matrix.

The external current injection \( I_{\text{ext}} = 9.515396 \)

For the first case (controller filters all the dynamic variables):

With the application of LQR to the HH-WF simulation, \( q_f = 100 \), the full state feedback gain matrix can be computed as

\[
K = \begin{bmatrix}
9.9621 & -155.043 & 117.8773 & 10.1162 & 9.8838 & 0.1496 & 0.1037 & 0.3159 \\
-0.0033 & 0.0010 & 0.0027 & -0.1813 & -0.0001 & 0.0000 & 0.0000 & 0.0000 \\
-0.0042 & -0.1527 & -0.0108 & -0.0002 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\
0.0004 & -0.0014 & -0.0056 & -0.0010 & -0.1206 & -0.0000 & 0.0000 & 0.0000 \\
-0.0864 & 0.3846 & 0.7037 & 0.0774 & 0.0139 & 0.0119 & 0.0333 & -0.0102 \\
0.0007 & -0.0033 & -0.0027 & 0.9803 & 0.0007 & 0.9950 & -0.7239 & 0.1081 \\
0.0004 & 0.0495 & 0.0147 & 0.0009 & 0.0002 & -0.0589 & -0.0384 & 0.9924 \\
-0.0000 & 0.0005 & 0.0036 & 0.0055 & 0.9926 & 0.0803 & 0.6848 & 0.0583
\end{bmatrix}
\]

(38)

The close-loop spectrum for the new case is

\[
\mathbf{E} = \begin{bmatrix}
11.5450 & 3.0926 & 1.0193 & 0.1949 & 0.1315 & 0.0100 & 0.0000 & -100 \\
-0.0100 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & -100 \\
-0.0000 & 0.0003 & 0.0027 & 0.9803 & 0.0007 & 0.9950 & -0.7239 & 0.1081 \\
-0.0000 & 0.0042 & 0.1527 & -0.0108 & -0.0002 & 0.0000 & 0.0000 & 0.0000 \\
-0.0864 & 0.3846 & 0.7037 & 0.0774 & 0.0139 & 0.0119 & 0.0333 & -0.0102 \\
0.0007 & -0.0033 & -0.0027 & 0.9803 & 0.0007 & 0.9950 & -0.7239 & 0.1081 \\
0.0004 & 0.0495 & 0.0147 & 0.0009 & 0.0002 & -0.0589 & -0.0384 & 0.9924 \\
-0.0000 & 0.0005 & 0.0036 & 0.0055 & 0.9926 & 0.0803 & 0.6848 & 0.0583
\end{bmatrix}
\]

(39)

The corresponding eigenvectors are

\[
\mathbf{V} = \begin{bmatrix}
0.9963 & 0.9389 & 0.7103 & -0.0143 & -0.0017 & 0.0000 & 0.0000 & 0.0000 \\
-0.0003 & 0.0010 & 0.0027 & -0.1813 & -0.0001 & 0.0000 & 0.0000 & 0.0000 \\
-0.0042 & -0.1527 & -0.0108 & -0.0002 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\
0.0004 & -0.0014 & -0.0056 & -0.0010 & -0.1206 & -0.0000 & 0.0000 & 0.0000 \\
-0.0864 & 0.3846 & 0.7037 & 0.0774 & 0.0139 & 0.0119 & 0.0333 & -0.0102 \\
0.0007 & -0.0033 & -0.0027 & 0.9803 & 0.0007 & 0.9950 & -0.7239 & 0.1081 \\
0.0004 & 0.0495 & 0.0147 & 0.0009 & 0.0002 & -0.0589 & -0.0384 & 0.9924 \\
-0.0000 & 0.0005 & 0.0036 & 0.0055 & 0.9926 & 0.0803 & 0.6848 & 0.0583
\end{bmatrix}
\]

(40)

With the projection application, the filter output gain is computed

\[
K_0 = \begin{bmatrix}
9.8427 & -141.2563 & 116.1719 & -1878.8
\end{bmatrix}
\]

(41)

The Fig. 5 shows results of the close-loop system with the washout-projection controlled.

For the second case (controller filter merely the membrane potential):

With the application of LQR to the HH-WF simulation, \( q_f = 10 \), the full state feedback gain matrix can be computed as

\[
K = \begin{bmatrix}
1.5646 & 687.6336 & 154.4712 & 442.2513 & 3.0434 & 0.0354 & 0.0797 & -0.0231
\end{bmatrix}
\]

(42)

With the projection application, the filter output gain is computed

\[
K_0 = \begin{bmatrix}
1.4411 & -9.1086 & 135.0861 & 128.8953
\end{bmatrix}
\]

(43)

The Fig. 6 shows the response of the close-loop system with the membrane potential controlled.

For the first case (controller filters all the dynamic variables):

With the application of LQR to the HH-WF simulation, \( q_f = 100 \), the full state feedback gain matrix can be computed as

\[
K = \begin{bmatrix}
6.5638 & -1056.6 & 280.4767 & 661.4 & 9.8593
\end{bmatrix}
\]

(44)

With the projection application, the filter output gain is computed

\[
K_0 = \begin{bmatrix}
5.0664
\end{bmatrix}
\]

(45)

The Fig. 7 shows results of the close-loop system with the membrane potential controlled.

For the second case (controller filter merely the membrane potential):

With the application of LQR to the HH-WF simulation, the full state feedback gain matrix can be computed as

\[
K = \begin{bmatrix}
3.3874 & -147.8976 & 83.7953 & 11.0184 & 3.1209
\end{bmatrix}
\]

(46)

With the projection application, the filter output gain is computed

\[
K_0 = \begin{bmatrix}
3.2097
\end{bmatrix}
\]

(47)
Figure 6. Results of the close-loop system with the membrane potential controlled $I_{ext} = 9.515396$

Figure 7. Results of the close-loop system with the washout-projection controlled $I_{ext} = 154.22313$

Figure 8. Results of the close-loop system with the membrane potential controlled $I_{ext} = 154.22313$. 
The Fig. 8 shows the response of the close-loop system with the membrane potential controlled.

**Conclusion and Prospect**

In this paper, a feasible and effective method is presented to develop an algorithm for feedback controlled electrical nervous system stimulation, with which the Hopf bifurcation that arises from the external current injection in the HH model can be eliminated stably. We have presented a method which gives a control rule to control the repetitive firing of the membrane potential in nerve fibers. The source of bifurcations (bifurcation parameter) in our model is the external current injection which is also the case in various sources found in literature. In order to achieve the aim, the repetitive firing of membrane potential in the nervous system is associated with the bifurcation control theory so that the unstable oscillations in the nonlinear HH dynamic model can be controlled. The stable controller is performed by two similar control rules that are designed by the integration of the washout filters and static projective control theory. The first case of the control rule filters all the dynamic variables including the membrane potential and the channel activation, while the second case of the control rule filters merely the membrane potential. The latter case yields a simpler and practically more applicable algorithm but the control performance is lower. In both cases, the linear quadratic output feedback method is applied to calculate the control gain. In short, one can give a conclusion that the feedback controller algorithm provides a successful treatment process providing that the parameter conditions which lead to a specific type of bifurcation are obtained. In recent researches, the parameter conditions are generally unknown due to the complex and unsure measurement and estimation of conditions of instability in nervous systems through membrane voltage feedback is needed. This may be a subject in the future research.

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