Threshold Selection Based on Fuzzy Tsallis Entropy and Particle Swarm Optimization

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Abstract
Tsallis entropy is a generalization of Shannon entropy and can describe physical system with long range interactions, long time memories and fractal-type structures. In this paper, a novel threshold selection technique for image segmentation is proposed by combining Tsallis entropy and fuzzy c-partition. The image to be segmented is firstly transformed into fuzzy domain using membership function. Then, the fuzzy Tsallis entropies for object and background are defined. The threshold is selected by finding a proper parameter combination of membership function such that the total fuzzy Tsallis entropy is maximized. To reduce the computational complexity, particle swarm optimization (PSO) is used to search the optimal parameter combination. The main advantage of the proposed method is that it considers not only the information of object and background but also interactions between them in threshold selection procedure. Experimental results show that the proposed method can give better segmentation performance than methods based on traditional Shannon entropy.

Key Words: Tsallis entropy, particle swarm optimization

1. Introduction
Image analysis usually refers to processing of images with the goal of finding objects presented in the image. Image segmentation is a crucial step in image analysis. Thresholding is a widely employed technique for image segmentation. The approaches are based on the assumption that object and background can be distinguished by their intensity values. Many techniques for automatically selecting threshold had been developed over the past years (Sahoo et al., 1988). Among these methods, Shannon entropy based methods, such as Kapur’s maximum entropy method (Wong et al., 1985), minimum cross-entropy method (Li and Lee, Yin, 1993, 2007), Reny’s entropy method (Sahoo, 1997), were intensively studied and proven to be effective. To take ambiguity or uncertainty into consideration in image segmentation, fuzzy entropy methods, which integrate fuzzy set theory and Shannon entropy, also attracted many researchers’ attention. In (Cheng et al., Cheng et al., 1998, 1999), Cheng et al. introduced the concept of fuzzy c-partition and defined fuzzy entropy for each fuzzy partition. First, fuzzy sets for object and background were defined using membership function with parameters. An optimal threshold was determined by finding a proper parameter combination such that the total fuzzy entropy of each fuzzy partition was maximized. Then, Cheng et al. (Cheng et al., 2000) extended his method to two-dimensional case. Zhao et al (Zhao et al., 2001) proposed an other entropy function using fuzzy c-partition and probability partition. Tao et al. (Tao et al., 2003) designed a three-level...
thresholding method for image segmentation on the basis of Zhao’s method. Tao et al. (Tao et al., 2007) investigated the performance of the fuzzy entropy approach when it was applied to the segmentation of infrared objects.

The basic characteristic of Shannon entropy is extensive or additivity, which implies that if a physical system is decomposed into two statistical independent subsystems \( A \) and \( B \), the Shannon entropy of the system is the sum of the Shannon entropy of subsystems \( A \) and \( B \). Although Shannon entropy is simple in mathematical operation, it ignores the interactions between subsystems. In specialty, when Shannon entropy is applied to select threshold for image segmentation, the interactions between object and background is ignored, which may lead to inaccurate segmentation results sometimes. Recently, Tsallis entropy (Tsallis, Furuichi, 1988, 2005) was proposed as a generalization of Shannon entropy. Compared to Shannon entropy, Tsallis entropy is non-extensive or non-additivity, which is suitable to describe systems with long range interactions, long time memories and fractal-type structures. Owing to this property of Tsallis entropy, Albuquerque et al. (Albuquerque et al., 2004) applied maximum Tsallis entropy to select threshold for image segmentation and got ideal segmentation results. Sahoo et al. (Sahoo and Arora, 2006) extended it to two-dimensional case.

In this paper, a novel technique, called fuzzy Tsallis entropy, is proposed to select threshold for image segmentation. This method is described in section 2. The basic of PSO is briefly reviewed in section 3. Experimental results and conclusion remarks are given in section 4 and section 5 respectively.

2. Fuzzy Tsallis entropy thresholding method

2.1 Tsallis entropy

Considering a probability distribution \( \{p_1, p_2, \ldots, p_k\} \), where \( 0 \leq p_i \leq 1 \), \( \sum_{i=1}^{k} p_i = 1 \) and \( k \) is the total number of states. The traditional Shannon entropy is defined as

\[
S = -\sum_{i=1}^{k} p_i \ln(p_i)
\]  

This formalism has been shown to be validity in Boltzmann-Gibbs-Shannon (BGS) statistics. BGS statistics seem to describe nature when the effective microscopic interactions and the microscopic memory are short ranged. Generally, systems that obey BGS statistics are called extensive systems. If we consider a physical system that can be decomposed into two statistical independent subsystems \( A \) and \( B \), the probability of the composite system is \( p^{A+B} = p^A \cdot p^B \). It has been verified that the Shannon entropy has the extensive property (additivity):

\[
S(A + B) = S(A) + S(B)
\]

However, for a certain class of physical systems, which entail long-range interactions, long time memory and fractal-type structures, some kind of extension appears to be necessary. Inspired by multi-fractals concepts, Tsallis (Tsallis, 1988) has proposed a generalization of the BGS statistics, i.e., Tsallis statistics. The Tsallis statistics is currently considered to be useful in describing the thermo-statistical properties of nonextensive systems and it is based on a generalized entropic form. Tsallis entropy is defined as

\[
S_q = \frac{1 - \sum_{i=1}^{k} (p_i)^q}{q - 1}
\]  

where the real number \( q \) is an entropic index that characterizes the degree of nonextensivity. This expression meets the BGS entropy in the limit \( q \rightarrow 1 \). The Tsallis entropy is nonextensive in such a way that for a system that can be decomposed into two statistical independent
systems, the entropy of the system is defined by the following pseudo additivity entropic rule

\[ S_q(A + B) = S_q(A) + S_q(B) \]

where \( S_q \) is the Tsallis entropy and \( q \) is a real parameter. \( S_q(A) \) is the Tsallis entropy of the pixel whose gray level is \( k \) in the partition of \( D \), and \( D \) is composed of pixels with high gray levels.

\[ S_q(B) = (1 - q) \cdot S_q(A) \cdot S_q(B) \]

2.2 Fuzzy Tsallis entropy thresholding method for image segmentation

Let \( D = \{(i, j): i = 0,1, \ldots, M - 1; j = 0,1, \ldots, N - 1\} \) and \( G = \{0,1, \ldots, l - 1\} \) where \( M \), \( N \) and \( l \) are three positive integers. Let \( I(x, y) \) be the gray level value of the image at the pixel \( (x, y) \) and \( D_k = \{(x, y): I(x, y) = k, (x, y) \in D\}, k = 0,1, \ldots, l - 1. \]

Denote \( T \) as the threshold, which segments an image into the background and object. In a gray levels image, the domain \( D \) of the original image is classified into two parts, \( E_d \) and \( E_b \). \( E_d \) is composed of pixels with low gray levels and \( E_b \) is composed of pixels with high gray levels.

\[ \prod_2 = \{E_d, E_b\} \] is an unknown probability partition of \( D \), whose probability distribution is described as

\[ p_d = P(E_d), p_b = P(E_b) \]

For each \( k = 0,1, \ldots, 255 \), let

\[ D_{kd} = \{(x, y): I(x, y) \leq T, (x, y) \in D_k\} \]
\[ D_{kb} = \{(x, y): I(x, y) > T, (x, y) \in D_k\} \]

then the following equations hold:

\[ p_{kd} = P(D_{kd}) = p_k \cdot p_{dk} \]
\[ p_{kb} = P(D_{kb}) = p_k \cdot p_{bk} \]

It is clear that \( p_{dk} \) and \( p_{bk} \) are the conditional probability of a pixel that is classified into the class ‘d’(dark) and ‘b’ (bright), respectively, under the condition that the pixel belongs to \( D \), with \( p_{dk} + p_{bk} = 1(k = 0,1, \ldots, 255) \). In this paper, the conditional probability \( p_{dk} \) and \( p_{bk} \) are set to be equal to the grade of a pixel whose gray level value is \( k \) belonging to the class ‘d’(dark) and ‘b’ (bright), i.e., \( \mu_d(k) \) and \( \mu_b(k) \) respectively. Then the following equations hold:

\[ p_d = \sum_{k=0}^{255} p_k \cdot p_{dk} = \sum_{k=0}^{255} p_k \cdot \mu_d(k) \]
\[ p_b = \sum_{k=0}^{255} p_k \cdot p_{bk} = \sum_{k=0}^{255} p_k \cdot \mu_b(k) \]

The grade \( \mu_d(k) \) are obtained using \( Z(k,a,c) \)-function as the membership function and \( \mu_b(k) \) using \( S(k,a,b,c) \)-function. The two membership functions are shown in the following:

\[ \mu_d(k) = \begin{cases} 1, & k \leq a \\ 1 - \frac{(k-a)^2}{(c-a)\cdot(b-a)}, & a < k \leq b \\ \frac{(k-c)^2}{(c-a)\cdot(c-b)}, & b < k \leq c \\ 0, & k > c \end{cases} \]

\[ \mu_b(k) = \begin{cases} 0, & k \leq a \\ \frac{(k-a)^2}{(c-a)\cdot(b-a)}, & a < k \leq b \\ 1 - \frac{(k-c)^2}{(c-a)\cdot(c-b)}, & b < k \leq c \\ 1, & k > c \end{cases} \]

where parameters \( a \), \( b \), \( c \) satisfy \( 0 \leq a < b < c \leq 255 \). The graph of \( Z(k,a,c) \)-function and \( S(k,a,b,c) \)-function is shown in Figure 1.
For each class, the fuzzy Tsallis entropy function is given as following,

\[
H_d = \frac{1 - \sum_{k=0}^{255} p_k \mu_d(k)^q}{p_d^{q-1}} \quad (10)
\]

\[
H_b = \frac{1 - \sum_{k=0}^{255} p_k \mu_b(k)^q}{p_b^{q-1}}
\]

Then the total fuzzy Tsallis entropy function is given as

\[
H(a, b, c) = H_d + H_b + (1-q) \ast H_d \ast H_b \quad (11)
\]

The total fuzzy Tsallis entropy varies along with three variables \(a, b, c\). We can find a combination of \(a, b, c\) such that the total fuzzy entropy \(H(a, b, c)\) achieves the maximum value. Then the most appropriate threshold to segment the image into two classes can be computed as:

\[
\mu_d(T) = \mu_b(T) \quad (12)
\]

As is shown in Figure.1, threshold \(T\) is the point of intersection of the \(\mu_d(k)\) and \(\mu_b(k)\) curve.

According to formulas (8) and (9), the solution can be given as following:

\[
T = \begin{cases} 
 a + \sqrt{(c-a) \ast (b-a)^2}, & (a+c)/2 \leq b \leq c \\
 c - \sqrt{(c-a) \ast (c-b)^2}, & a \leq b \leq (a+c)/2 
\end{cases}
\]

\[
X_i(t+1) = X_i(t) + \frac{V_i(t+1) + c_1 r_1 (P_g(t) - X_i(t)) + c_2 r_2 (P_g(t) - X_i(t))}{w}
\]

\[
X_i(t+1) = X_i(t) + V_i(t+1), i = 1, 2, \ldots, n \quad (14)
\]

where \(n\) denotes the number of particles in the swarm, \(V_i(t)\) and \(X_i(t)\) represent the velocity and position of particle \(i\) in the solution space at \(t\)-th iteration step respectively; \(r_1\) and \(r_2\) are two random numbers uniformly distributed in the range \([0,1]\); \(c_1\) and \(c_2\) are acceleration constant, usually \(c_1 = c_2 = 2.0\); \(w\) is the inertia weight. Generally, the value of each component in \(V_i\) can be clamped to the range \([V_{min}, V_{max}]\) to control excessive roaming of particles outside the search space. Each particle flies toward a new position according to equation (??) and equation (14). In this way, all particles of the swarm find their new position and apply these new position to update their individual best position \(p_i(t)\) and global best position \(P_g(t)\) of the swarm. This process is repeated until a user-defined stopping criterion, usually is maximum iteration number \(t_{max}\), is reached.

It is shown that inertia weight \(w\) has great effect on the searching performance of PSO. The inertia weight provides a balance between global and local search. Shi (Shi and Eberhart, 1998) suggested a linearly decreasing manner to adjust the value of inertia weight during searching processing, i.e.,

\[
w = w_{max} - \frac{w_{max} - w_{min}}{t_{max}} \ast t \quad (15)
\]
where $w_{\text{max}}$ and $w_{\text{min}}$ denote the maximum and minimum of inertia weight. For more details about PSO, one can refer to (Kennedy et al., 2001).

4. Experimental results

In this section, a set of casting images derived from X-ray inspection system, which is a well-accepted technique for identification and evaluation of internal defects in casting, are used to evaluate the performance of the proposed method. The quality of casting image is poor such that classical threshold techniques can not give satisfactory segmentation results. To demonstrate the superiority of fuzzy Tsallis entropy method, the segmentation results obtained by fuzzy Tsallis entropy method are compared with Tao's method (Tao et al., 2007), Cheng's method (Cheng et al., 1998), Kapur's method (Wong et al., 1985) and Otsu method (Otsu, 1979).

In experiments, the parameters of PSO are set as follows: $c_1 = c_2 = 2$, $w_{\text{min}} = 0.4$, $w_{\text{max}} = 0.9$. The maximum iteration number is 100 and number of particles is 50. In PSO, each particle is a 3-dimension vector $(a, b, c)$ representing the parameter combination of $Z$-function and $S$-function. All the algorithms are implemented on Pentium 4, 2.0GHz PC using Matlab6.5 programming language.

The segmentation results for three type defects image, i.e., air holes, nonmetallic inclusions and shrinkage cavities, are shown in Figure 2--4. The thresholds, parameter combination of membership function obtained by PSO using each method are listed in Table 0.

The segmentation results using the proposed fuzzy Tsallis entropy are shown in Figure 2(b)--4(b) when nonextensive coefficient $q$ equals to 0.1, 1.8 and 0.1 respectively. Figure 2(c),(d),(e),(f)--4(c),(d),(e),(f) are the results using Tao's method (Tao et al., 2007), Cheng's method (Cheng et al., 1998), Kapur's method (Wong et al., 1985) and Otsu method (Otsu, 1979).

Figure 2 is a defected casting image of air holes, we find the fuzzy partition with parameters $q = 0.1$ as shown in Figure 2(g). It can be seen form Figure 2(b) that the main components of the image are well segmented, and the extra impurity are eliminated. Although other methods can acquire the main features of the image, overmuch impurities are included in the picture, especially the Kapur's entropy method.

Figure 3 is a defected casting image of shrinkage cavities. In Figure 3(b), the main features of the image are well deserved, particular the detail parts of the defects' edges; Tao's fuzzy entropy method losses much detail information of the defects, as shown in Figure 3(c); Other approaches can acquire the defects information completely, while they all introduce a lot of external noise.

In Figure 4, we adopt each method to segment the foreign collision defects casting image. Each method can distinct the object from the background well, but the proposed method is more integrate and clear than Cheng's fuzzy entropy method and Otsu's method, and less noise than Kapur's method and Tao's method.

For the above experiments, it is evident that the defects are well segmented using the proposed fuzzy Tsallis entropy method with no or less extra noise while other methods lead to much extra noise. However, how to choose the proper value of the parameter of $q$ is still a open problem. In this paper, we select positive values for $q$ and investigate the threshold level by visual inspection. In order to illuminate the influences of the parameter $q$, the variation curve of threshold $T$ for different $q$ is shown in Figure 5.
Figure 3. Segmentation results of nonmetallic inclusions image (a) Original image (b) The proposed method (c) Tao's method (d) Cheng's method (e) Kapur's method (f) Otsu's method (g) membership function (h) convergent curve of PSO.

Figure 5(a) is the thresholding variation curve in different $q$ of air holes image and Figure 5(b) is shrinkage cavities image. Threshold holds on 168 when $q$ keeping in the range of (0.1–0.6) in Figure 5(a) while sharp declines when $q \geq 0.7$ results in increasing noise dots and blurring the objects' border; There are two parts of stable value in Figure 5(b), but the first part is taken in the proposed method considering the emergence of pseudo-target. However, we can choose the thresholding by tests and take the best one you thought.

Figure 5. Thresholding variation curve in different $q$ (a) Air holes image (b) Nonmetallic inclusions image
In experiments, PSO is used to find parameter combination of membership function. The convergent curve of PSO for each image are shown in Figure 2(h)-4(h). It can be seen that in 10-20 steps, the PSO is convergent which implies that the time used to segmentation is very short. The proposed method can be applied to real time environment.

5. Conclusion
In this paper, a novel thresholding method is proposed by combining non-extensive of Tsallis entropy and fuzzy c-partition. The proposed method considers interactions between object and background furthermore in the procedure of selecting threshold, which can get better segmentation result than methods that ignore the interactions. On the other hand, PSO is used and can find the parameter combination in short time. Experimental results show that the proposed method is effective.
References


