Quantum Entanglement: Fundamentals and Relations with Consciousness/Mind

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Abstract
The “spooky action at a distance” and consequently quantum entanglement of physical systems/particles has been debated ever since the famous paper of Einstein, Podolsky and Rosen-EPR Argument. In spite of being considered highly controversial, there has been a considerable amount of work so far addressing the issue and many scientifically successful experiments were recently carried out proving that the entanglement is a physical reality. The influence of quantum mechanics and entanglement has been far from the imagination of most scientists, eventually reaching to the ultimate concept of consciousness/mind. The entanglement theory, its connection to the consciousness/mind and finally the entanglement theory of consciousness/mind therefore demands pure scientific interest. In order to maintain the requirements, the present work firstly summarizes the quantum entanglement theory together with its relations with Bell’s inequality and finally describes an “entanglement model of consciousness/mind”.

Key Words: quantum physics, entanglement, EPR Argument, Bell’s inequality, CHSH inequality, consciousness, mind.

Basics of Quantum Physics

1. Introduction
Quantum physics, probably the most influential scientific discipline of all, have been continuously feeding contemporary technologies and also other disciplines, such as biology, neuroscience, psychology, economics and even philosophy/theology (Penrose, 1989; 1994; Stapp, 1993; Eisert and Wiseman, 2007; Abbot et al., 2008). In this manner, quantum physics have a very distinct position within the overall scientific resources. The indispensable principles of quantum physics such as quantum superposition (De Martini, 2007; Romero-Isart et al., 2009), EPR argument paradox/entanglement (Aspect et al., 1982; Horodecki et al., 2009), decoherence/wave function collapse (Zurek, 1981; 1982; Paz and Zurek, 1999), quantum Zeno effect (Schwartz et al., 2005; Koshino and Shimizu, 2005; Stapp, 2009), in spite of being considered notorious, have already had foremost impact on the scientific world view. These phenomena are typically far removed from our intuitive limits and everyday experience. It is of interest of quantum physicists worldwide to explore various ways of bringing quantum phenomena closer to the macroscopic levels, to everyday life.
Quantum Entanglement is almost certainly the most prominent of all in terms of practical concerns and stands right at the centre of the quantum phenomena (Schrödinger, 1935) and inspires fundamental questions about the principles of the nature. Moreover, quantum entanglement is also the basis for a number of emerging technologies of quantum information processing such as quantum cryptography (Bennett and Brassard, 1984; Ekert, 1991), quantum teleportation (Bennett et al., 1993; Bouwmeester et al., 1997; Ursin et al., 2004; Riebe et al., 2004; Barret et al., 2004) and quantum computation (Deutsch and Ekert, 1998).

The scientific efforts on quantum phenomena are recently pushed even further to specifically explore ultimately challenging concepts such as consciousness/mind and relating human psychology (Stapp, 1991; 1993; 1995; Beck and Eccles, 1992; Penrose, 1994; Eccles, 1994; Jibu and Yasue, 1995; Hameroff and Penrose, 1996; Tegmark, 2000; Beck, 2008; Abbot et al., 2008; Conte et al., 2008, 2009a, 2009b; Khrennikov, 2005; 2006a; 2006b; Vitiello, 2003). The topic undoubtedly emerges interdisciplinary cooperation and has already attracted some considerable interest of physicists, biologists and neuroscientists. Quantum mechanics and specifically the entanglement phenomenon seem to have a strong scientific potential to prove that the consciousness/mind works in accordance with the quantum physical laws. The consciousness/mind and indeed brain have for centuries been the scouring topic of both scientific and philosophical issues and finally there seems to be a sparkling at the end of the tunnel. Therefore, fundamentals of certain subjects such as superposition principle, entanglement, decoherence, Bose-Einstein condensation, quantum Zeno effect and their scientific relations with consciousness/mind demand to be crystal clarified.

The present work hence aims to summarize the fundamental theory of quantum entanglement and also very closely relating topic of Bell’s inequality, specifically employed to test the validity of “local realism” and therefore “quantum entanglement”. Finally, an “entanglement model of consciousness/mind” and its relations with Bell’s inequalities are reported.

2. Fundamentals of Quantum Entanglement
Quantum mechanical particles/systems, having physical sizes approximately less than $10^{-7}$m, are accompanied by waves and spatial confinement of the particles leads the Schrödinger’s equation to produce a set of solutions, $\psi_n(x,t)$, (Eisberg and Resnick, 1974; Erol, 2010). These wave functions which are complete in the sense that any specific energy of $E_n$ with a corresponding wave function $\psi_n(x,t)$ must also satisfy the original form of the Schrödinger Wave Equation (SWE) that is Eigen value-Eigen function equation,

$$\hat{H}\psi_n(x,t) = E_n\psi_n(x,t)$$

(1)

where $\hat{H}$ denotes the Hamiltonian operator, $n$ indicates the relevant quantum state and is known as principle quantum number. According to the SWE, any quantum mechanical particle with a mass of $m$, total energy of $E$ and potential energy of $V(x)$ would physically have possible total energies of $E_n$, specifically, $E_1$, $E_2$, $E_3$, ..., $E_n$. The corresponding wave functions accompanying the particle/system can be expressed as $\psi_1(x,t), \psi_2(x,t), \psi_3(x,t), \ldots, \psi_n(x,t)$. Each of the wave function $\psi_n(x,t)$ is a particular solution of the Schrödinger’s equation for the same potential energy of $V(x)$ and is said to be a member of mathematically well defined “Hilbert Space” that is expressed by,

$$H = \{\psi_1(x,t), \psi_2(x,t), \psi_3(x,t), \ldots, \psi_n(x,t), \ldots\}$$

(2)

Since the SWE and the wave functions are linear then we expect that any linear combination of these functions will also be a solution of the SWE. This can clearly be extended to show that any arbitrary linear combination of all wave functions which are solutions to the SWE represent the overall wave function and can be given as,

$$\psi(x,t) = c_1\psi_1(x,t) + c_2\psi_2(x,t) + c_3\psi_3(x,t) + \ldots + c_n\psi_n(x,t) + \ldots = \sum_{n=1}^{\infty} c_n\psi_n(x,t)$$

(3)
This general principle is called “superposition principle”. It is mathematically well known fact that the wave functions of the Hilbert space must satisfy the orthonormality principle that is formulated by Kroenecker Function, 

$$\delta_{mn} = \int_{-\infty}^{\infty} \psi_n^*(x,t)\psi_m(x,t)dx = \begin{cases} \ m = n \Rightarrow 1 \\ m \neq n \Rightarrow 0 \end{cases}$$ (4)

The expression of $\psi(x,t)$ gives the most general form of the solutions to the SWE for a potential energy of $V(x)$. Its generality can be appreciated by noting that it is a function which is composed of a very large number of different functions combined in proportions governed by the adjustable constants of $c_n$. The quantum particle, associated with the wave, must be at a specific time $t$ and location $x$ whereas the waves are spread in space and can only have an amplitude, frequency and phase at that point and time. Talking about the physical meaning of the wave function, Born’s interpretation also known as Copenhagen interpretation, states that the measurable physical quantity in this case is “probability density-P(x,t)” and is defined as the probability per unit length of x axis, of finding the particle near the coordinate $x$ and time $t$. The finding probability must also be real and non-negative, whereas the wave function $\psi(x,t)$ is complex and obviously not possible to equate $P(x,t)$. However $\psi^*(x,t)\psi(x,t)$ is always real and non-negative and can be correlated to probability density by $P(x,t)=\psi^*(x,t)\psi(x,t)$. The wave function and so the probability density $P(x,t)$ must be continuous in space then the probability $P(x,t)dx$ is defined as the finding probability of the particle within $dx$, in other words, between $x$ and $x+dx$. The overall probability can be found by integrating the expression for the all space that is

$$P = \int_{-\infty}^{\infty} \psi^*(x,t)\psi(x,t)dx = 1$$ (5)

We now want to determine the probability of $P_n$ that is finding the quantum particle at a quantum state of $n$ and energy of $E_n$. In order to do so, we substitute the expression (3) into the equation of (5) and also employ the Kroenecker function then we obtain,

$$P = \sum_{n=1}^{\infty} |c_n|^2 = \sum_{n=1}^{\infty} P_n = 1$$ (6)

The equation shows that the constant $c_n$ has a very important meaning of probability. Specifically, $P_n=|c_n|^2$ means total finding probability of the particle/system at the quantum state of $n$ and energy of $E_n$. The definition of the constant $c_n$ can straightforwardly be formulated as

$$P_n = |c_n|^2 = \left(\int_{-\infty}^{\infty} \psi_n^*(x,t)\psi(x,t)dx\right)^2$$ (7)

which indicates the importance of the interaction between any specific state of $n$, $\psi_n(x,t)$, and the general wave function, $\psi(x,t)$, containing all the possible states.

The entanglement theory starts with consideration of two distinct quantum systems/particles, namely A and B, both obeying the basic principles described above and can be defined as linear combinations of all the possible quantum states (Horodecki et al., 2009). The superposition principle can be employed to express the wave functions mathematically; the overall individual Schrödinger wave functions are simply given by

$$\psi_A = \sum_{n=1}^{\infty} c_n \psi_n \ , \ \psi_B = \sum_{n=1}^{\infty} c_m \psi_m$$ (8)

If the quantum particles/systems A and B are somehow combined quantum mechanically and structure a “quantum composite system”, no matter how far they are spatially separated, then the “quantum composite system” must have the wave function of $\psi_{AB}$. The quantum mechanical laws state that the overall wave function of the composite system is formulated by the tensor product of the individual wave functions that is,

$$\psi_{AB} = \psi_A \otimes \psi_B$$ (9)

The equation (7) smartly defines the joint probability, also known as the correlation...
coefficient, that is “finding probability” of the composite system at a quantum state of \((n, m)\), which means the system A is at the quantum state of \(n\) and the system B at the quantum state of \(m\). The joint probability can quantum mechanically be expressed as

\[
P_{nm} = \int \left| c_n c_m (\psi_n \psi_m) \right|^2 dx
\]

\[
= |c_n|^2 |c_m|^2 \int \left| \psi_n \right|^2 \left| \psi_m \right|^2 dx = P_n P_m E_{nm}
\]

where \(P_n\) and \(P_m\) denotes individual probabilities of the system A and B at the quantum states of \(n\) and \(m\) respectively. The “entanglement parameter” defined by an integral equation of

\[
E_{nm} = \int \left| \psi_n \right|^2 \left| \psi_m \right|^2 dx
\]

and purely determines the entanglement /interaction/coupling level of the distant individual quantum particles/systems. This parameter also expresses the deviation of the “joint probability” of the quantum composite systems/particles from the textbook classical joint probability value of \(P_{nm} = P_n P_m\).

The classical probability does not hold unless one adds a quantum mechanical interference term. If the composite system behaves like a “classical composite system” that means no entanglement/coupling/interaction occurs between the sub-systems then the states are called “pure/separable” states then the entanglement parameter equals to unity. Pure/separable states also mean that the quantum entanglement parameter is given by the product of the individual “finding probabilities” of the sub-systems, that is

\[
P_{nm} = |c_n|^2 |c_m|^2 \left| \psi_n \right|^2 \left| \psi_m \right|^2 = 1
\]

The approach defined above offers a very basic but meaningful tool to test and understand any quantum particles or systems are entangled or not when two or more of them are combined together to structure a composite system. Basically, if the states of the composite system are pure/separable then the entanglement parameter, \(E_{nm} = 1\) and joint probability is given by \(P_{nm}^{separable} = P_n P_m\). If the states of the composite system are, on the other hand, entangled/non-separable/interacting then the entanglement parameter, \(E_{nm} \neq 1\) and the joint probability is simply given by

\[
P_{nm}^{entangled} = P_n P_m E_{nm}
\]

**Infinite Square Well Potential Paradigm**

In order to demonstrate basic definitions, given above, we consider the very well known “infinite square well potential” which assumes a quantum particle/electron is confined in an infinite potential well with a width of \(L\). Two individual quantum wells (bipartite), A and B, are separated spatially no matter what the distance is and only two quantum states, namely first and second, are considered for simplicity. The solution of the SWE gives the overall wave functions as (Eisberg and Resnick, 1974)

\[
\psi_A = c_1 \frac{2}{\sqrt{L}} \cos \left( \frac{\pi x}{L} \right) + c_2 \frac{2}{\sqrt{3L}} \sin \left( \frac{2\pi x}{L} \right),
\]

\[
\psi_B = c_3 \frac{2}{\sqrt{L}} \cos \left( \frac{\pi x}{L} \right) + c_4 \frac{2}{\sqrt{5L}} \sin \left( \frac{2\pi x}{L} \right)
\]

where the complex constants are randomly chosen as,

\[
c_1 = \frac{1}{\sqrt{3}}, \quad c_2 = \frac{1}{\sqrt{3}}, \quad c_3 = \frac{1}{\sqrt{5}}, \quad c_4 = \frac{2}{\sqrt{5}}
\]

Individual measurements give the probability of having the quantum state of \(n=1\) for A and \(m=1\) for the system B, as

\[
P_1^A = |c_1|^2 = \frac{1}{3}, \quad P_1^B = |c_1|^2 = \frac{1}{5}
\]

Consider the two systems are separated by very large distances and consistently structure a “quantum composite system” then the joint probability of having the quantum state of \(n=1\) for A and of \(m=1\) for the system B, is formulated as
The quantum entanglement parameter can be extracted straightforwardly as

$$P_{\text{11}} = \int \left| c_{\psi_1}^* c_{\psi_2} \left( \psi_1^* \psi_2^* \right)^* \psi_1 \psi_2 \right|^2 \, dx,$$

$$= \frac{1}{15} \frac{4}{L} \int \cos^2 \left( \frac{\pi x}{L} \right) \cos^2 \left( \frac{\pi x}{L} \right) \, dx \left| \right|^2 \int \cos \left( \frac{\pi x}{L} \right) dx \cos \left( \frac{\pi x}{L} \right) \, dx = \int \cos^2 \left( \frac{\pi x}{L} \right) \, dx \int \cos^2 \left( \frac{\pi x}{L} \right) \, dx$$

The quantum entanglement parameter would possibly be written and the entanglement integral and the quantum entanglement parameter would respectively be found as

$$I_{11} = \frac{L^2}{\pi^2} \frac{\pi^2}{4} = \frac{L^2}{4} \, E_{11} = \left| \frac{4L^2}{4} \right| = 1$$

The joint overall probability would be estimated as

$$P_{\text{separable}} = P_m P_n \frac{1}{15} = 0.0666$$

which is equal to the classical statistical joint probability. This would mean no “spooky action at a distance” occurs between the separable particles/systems.

### 3. Bell’s Inequality Theorem

Bell’s Inequality (Bell, 1964) and consequently Bell test experiments (Aspect et al., 1982; Weihs et al., 1998; Tittel et al., 1998; Gisin et al., 2000; Rowe et al., 2001; Matsukevich et al., 2008) provide to investigate simply the validity of the “local realism” under any “local hidden variable theory” and consequently validity of the quantum “entanglement effect”. Bell’s Theorem states that the universe is not locally deterministic and a “Bell inequality” must be obeyed under “local realism” however is to be violated under the influence of “quantum mechanics”. In the formulization of local realism used by Bell, the predictions of theory result from the application of classical probability theory to an underlying parameter space. By a simple argument based on classical probability, he then showed that correlations between measurements are bounded in a way that is violated by quantum mechanics. Bell’s theorem seemed to put an end to local realist hopes for quantum mechanics. Bell test
experiments to date overwhelmingly show that Bell inequalities are violated. These results provide empirical evidence against local realism and in favor of quantum mechanics. According to quantum theory, quantum correlations violating Bell inequalities simply happen, somehow from outside space-time, in the sense that there is no space-time explanation for their occurrence: there is no event here that somehow influences another distant event there. The term "Bell inequality" can mean any one of a number of inequalities, in real experiments, the "CHSH (Clauser, Horne, Shimony and Holt)" (Clauser et al., 1969) two channel inequality or "CH74" (Clauser and Horne, 1974) single channel inequality.

The standard probability theory assumes that "joint probability", also named as "correlation coefficient" of two individual systems/particles of A and B, is given by product of the independent probabilities of the two sides that is \( P_{ab} = P(a)P(b) \). In the case of classical probability theory, the joint probability is assumed to be determined by the "hidden variable" of \( \lambda \). The hidden variable \( \lambda \) is assumed to be drawn from a fixed distribution of possible states of the source, the probability of the source being in the state of \( \lambda \) for any particular trial being given by the density probability function of \( \rho(\lambda) \). Then "the expectation value of the joint probability" or "correlation coefficient" is given by

\[
\text{C}(a,b) - \text{C}(a',b') = \int \left[ P(a,\lambda)P(b,\lambda) - P(a,\lambda)P(b',\lambda) \right] \rho(\lambda) d\lambda \tag{13}
\]

This equation can also be written as

\[
\text{C}(a,b) - \text{C}(a',b') = \int \left[ P(a,\lambda)P(b,\lambda) + P(a',\lambda)P(b',\lambda) \right] \rho(\lambda) d\lambda
\]

Using \(|P(a,\lambda)| \leq 1\) and \(|P(b,\lambda)| \leq 1\) and also using the inequalities of

\[
[1 \pm P(a',\lambda)P(b',\lambda)] \rho(\lambda) \geq 0
\]

and \([1 \pm P(a',\lambda)P(b,\lambda)] \rho(\lambda) \geq 0\)

one can easily get the expression of

\[
|C(a,b) - C(a',b')| \leq \int [1 \pm P(a',\lambda)P(b',\lambda)] \rho(\lambda) d\lambda - \int [1 \pm P(a',\lambda)P(b,\lambda)] \rho(\lambda) d\lambda
\]

or, using the integral of \( \int \rho(\lambda) d\lambda = 1 \),

the expression can be written as

\[
|C(a,b) - C(a',b')| \leq 2 \pm [C(a',b') + C(a',b)]
\]

or in the more familiar form the final CHSH inequality expression is given by

\[
|S| = |C(a,b) - C(a',b') + C(a',b) + C(a',b')| \leq 2 \tag{14}
\]

The CHSH inequality is expected to be satisfied if the "local realism" is in power. If the CHSH inequality factor, \( S \), is numerically greater than 2, it has infringed the CHSH inequality and the experiment is declared to have supported the predictions of the quantum mechanics and hence "entanglement effect". The reader should note that the upper limit for the inequality is moved to \( 2\sqrt{2} \) and a reasonable amount of work has been studying the Tsirelson’s bound since then (Cirel'son, 1980). Considering the quantum system of A at a special setting of a with an orientation angle of \( \alpha \) and the quantum system of B at a special setting of b with an orientation angle \( \beta \), the CHSH theory formulates the correlation coefficient of C as,

\[
C(\alpha, \beta) = P_{III}(\alpha, \beta) + P_{IV}(\alpha, \beta) - P_{III}(\alpha, \beta) - P_{IV}(\alpha, \beta) \tag{15}
\]
where H and T denotes the possible two outcomes of the measurements of $\alpha$ and $\beta$. The expression can alternatively be formulated as,

$$C(\alpha, \beta) = \frac{1}{8} \left[ \cos^2(\alpha - \lambda) \cos^2(\beta - \lambda) \rho(\lambda) d\lambda \right]$$

which is a very useful expression when comparing the experimental outcomes of any specific measurement.

4. Quantum Mechanics and Consciousness/Mind

Information processing in the brain as a matter is mediated by the dynamics of large highly interconnected neuronal populations. The processes of the neurons have scientifically been resolved to some extent, however especially collective behavior of the neuron groups seems to be far from being resolved by simply employing biological, chemical, neuroscientific or classical physics laws. It is a solid fact, on the other hand, that inside any neuron, there are atoms, molecules and sub atomic particles which obey the principles of quantum mechanics and fulfill the space. It is therefore crystal clear that at atomic/molecular and subatomic levels quantum mechanical laws are decisive in the brain processes.

Consciousness/mind, on the other hand, has historically and philosophically been considered as a non-physical concept and rather considered to be a meta-physical entity. Therefore “binding problem” has been one of the most challenging topics of the scientific and philosophical contents. Recent research especially on “functional brain imaging techniques” and “anesthesia applications” clearly indicates the link between “consciousness/mind” and “brain” (Hameroff and Penrose, 1996). The basic but foremost conclusion here is that “the consciousness/mind is essentially energy hence certainly has a physical meaning”. This is apparent if one considers simply the Electroencephalography (EEG) measurements of consciousness/mind. Considering the minimum (delta waves, $f_{\text{min}}=1$Hz) and maximum (gamma waves, $f_{\text{max}}=100$Hz) frequencies of the brain waves correspond to the energy levels of; $E_{\text{min}}=hf=4,14.10^{-15}$eV and $E_{\text{max}}=hf=4,14.10^{-13}$eV. These energies can be considered the upper and lower limits of the energy quantization which can be considered as the consciousness/mind energy quanta, previously named as Thoughton/Ton by (Erol, 2010) and Psychon by Eccles (Beck and Eccles; 1992), of the mind/consciousness processes. If we compare these energy levels with the thermal energy levels at room temperatures, that is $E=kt=1,38.10^{-23}$ J/K $300K=0,025$eV and corresponding thermal fluctuation frequency is $f=E/h=6,25.10^{12}$Hz, it is clear that the thermal fluctuations and mind/consciousness energies never couple each other supporting the quantum consciousness/mind.

This scientific fact actually realized more than three quarters of a century ago and persistently the importance of the subject is underlined by a number of exceptional scientists (Lotka, 1925; Bohr, 1928; von Neumann, 1932; Whitehead, 1933; Bohm, 1952; Eccles, 1973; Walker, 1970; Bass, 1975). One of the founders of the quantum physics, Bohr himself, underlined the possible link between quantum mechanics and mind/consciousness and stated that “thoughts may involve energies at quantum levels” (Bohr, 1928). In spite of understandable resistance of the physics community, it’s now excitingly promising that the scientific research has recently been increased dramatically, especially following the apparent interest of the quantum physicists to the subject and recent research giving very strong positive signals about the sparkle future of the subject (Beck, 2008; Conte, 2008; Stapp, 1991; Hameroff and Penrose, 1996; Vitiello, 2003; Jibu and Yasue, 1995; Stapp, 1993, 1995; Khrennikov, 2005, 2006a, 2006b; Tegmark, 2000; Penrose, 1994; Conte et al. 2008, 2009a, 2009b; Eccles, 1994; Beck and Eccles, 1992; Abbott et al., 2008).

Amongst many quantum physical principles, the superposition principle (Clauser, 1997; Reiger et al., 2006; Romero-Izart et al., 2009; de Martini et al., 2005, 2007; Chan et al., 2003), entanglement/EPR paradox (Watterich, 2008), decoherence /
wave function collapse, quantum Zeno effect, Bose-Einstein condensation (Moore et al., 1999) and macroscopic quantum effects (Schrödinger, 1935; Marshall et al., 2003; Inouye et al., 1999; de Sarlo et al., 2005) are the topics immediately highlighting on being decisive of consciousness/mind processes.

The simple expression of the question (7), to our view, plays a central role in analyzing consciousness/mind operations because as it is clear from the equation any specific decision or behavior of the consciousness/mind, \( \psi_n(x,t) \), is very strongly influenced by the general state of the consciousness/mind, \( \psi(x,t) \), determined primarily by the instant information input via internal and external signals, previous experience/memory and environmental effects. Under the illuminations of the works and the thoughts above, we propose following hypothesizes to support the “quantum mind/consciousness” and the following theory of entanglement model of consciousness/mind is developed in accordance with the hypotheses (Erol, 2010).

**Hypothesis 1**

Consciousness/mind is a pure physical concept and energy, establishes at a time level of about 0.1s, space level of about \( 10^{-15} \) m and energy level of about \( 10^{-15} \) eV. Therefore it is well in the quantum regime and must be treated accordingly.

**Hypothesis 2**

Brain and mind/consciousness are identical and no separable (same) concepts at that energy and space levels and there is no “binding problem” as such.

5. Entanglement Model of Consciousness/Mind

The highly complicated mechanisms of consciousness/brain, such as decision making, believing, thinking, comparing, feeling etc., are to be governed by the quantum mechanical laws at atomic and sub-atomic scales. It is assumed that the consciousness/mind in fact is a “quantum field” with field particle/quanta of Ton. It can easily be speculated that the field is possibly the tachyonic field (Feinberg, 1967; Hari, 2008). Therefore the quanta of consciousness/mind, Ton, with a potential energy of \( V(x) \), mass of \( m \) and total energy of \( E \), must be governed by the Schrödinger Wave Equation. The solutions form a set of wave functions given by the equations (3) and (4) with a specific finding probability given by the equation of (7). The consciousness/mind model is, for simplicity, developed for guessing possible outcomes of a “tossing a coin” experiment, however can straightforwardly be generalized. Consider a person or consciousness/mind facing a “tossing a coin” trial; he/she has exclusively two choices, either head (H) or tail (T). The consciousness/mind, according to the equation (3) and before making the relating decision, is accompanied by a general wave function of,

\[
\psi = c_H \psi_H + c_T \psi_T
\]

The decision making process causes the wave function reduction/collapse to a specific state of either \( H \) or \( T \), through the decoherence mechanism of quantum mechanics (Zurek, 1981, 1982). Decoherence is the spontaneous interference of a quantum system with its environment leads to wave function collapse/reduction and explains how classical world may emerge from the quantum world and specifically underlines the other important issue of “measurement problem”. Wave function collapse is considered to be the choice of “observer” not the choice of “nature” and has a primary importance to understand the consciousness but beyond the scope of this paper and will be considered in the future studies. The general wave function is a member of orthonormal vector space that is two dimensional Hilbert space and instant state of the overall wave function depends up on the angle \( \alpha \). The angle \( \alpha \) can be assumed as “the point of view” and is primarily a function of time, internal dynamics, external dynamics, and memory. Two dimensional wave function/vector space or Hilbert space can simply be visualized as follows
The overall wave function accompanying the quanta/Ton of the consciousness/mind is basically given by

$$\psi = \psi_H + \psi_T$$  \hfill (18)

with the corresponding equations

$$\psi_H = \psi \cos \alpha,$$
$$\psi_T = \psi \sin \alpha,$$
and
$$\alpha = \tan^{-1}\left(\frac{\psi_T}{\psi_H}\right)$$

Considerations of the equation (17) and \(\sin^2 \alpha + \cos^2 \alpha = 1\) leads us to extract the constants of the superposition principle which are \(c_H = \cos \alpha\) and \(c_T = \sin \alpha\). The basic vector model of consciousness/mind suggested above can be extended to derive the equations for the quantum entanglement. In order to extract the basic equation for the entangled minds, consider two separate quantum systems / minds / persons of A and B with “the point of view” angles of \(\alpha\) and \(\beta\), respectively. The angles here are named as the point of view angles but essentially they are the angles of the overall instant wave functions with respect to the horizontal direction. Then the overall wave functions for the mind/person A and mind/person B are given by

$$\psi_A = \cos \alpha \psi_H^A + \sin \alpha \psi_T^A$$
$$\psi_B = \cos \beta \psi_H^B + \sin \beta \psi_T^B$$  \hfill (19)

The composite system, which consists of the two consciousnesses/minds and if the two construct a quantum system, has a wave function given by the tensor product of the two wave functions that is

$$\psi_{AB} = \psi_A(x,t) \otimes \psi_B(x,t) =$$
$$c_H^A c_H^B \psi_H^A \psi_H^B + c_T^A c_T^B \psi_T^A \psi_T^B$$
$$+ c_H^A c_T^B \psi_H^A \psi_T^B + c_T^A c_H^B \psi_T^A \psi_H^B$$  \hfill (20)

The joint probability or correlation coefficient of having H for the person A and again H for the person B is given by

$$P_{HH} = \left| \int c_H^A c_H^B \left| \psi_H^A \left| \psi_H^B \right|^2 \right| \ dx \right|^2$$
$$= |c_H^A|^2 |c_H^B|^2 \int \left| \psi_H^A \left| \psi_H^B \right|^2 \right| \ dx \right|^2$$  \hfill (21)

The “entanglement parameter-E_{HH}” can simply be derived by using the standard theory of entanglement presented previously. In the relating literature, several papers focus to test the CHSH inequality by similarly considering spin polarizations of the individual quantum particles. The angles in those papers are defined as the angles of the polarization and have very similar physical meaning. The quantum entanglement parameters for the very similar cases are calculated elsewhere by using “law of Malus” (Malus, 1809) as

$$E_{HH} = E_{TT} = \left| \int \left| \psi_T^A \left| \psi_T^B \right|^2 \right| \ dx \right|^2 = 2\cos^2(\beta - \alpha) \hfill (22)$$
$$E_{HT} = E_{TH} = \left| \int \left| \psi_H^A \left| \psi_T^B \right|^2 \right| \ dx \right|^2 = 2\sin^2(\beta - \alpha) \hfill (23)$$

Hence, joint probabilities can easily be formulated for the head-head, tail-tail, head-tail and finally tail-head combinations respectively as (Dehlinger and Mitchell, 2002)

$$P_{HH} = |c_H^A c_H^B|^2 \cdot 2\cos^2(\beta - \alpha) = \cos^2 \alpha \cos^2 \beta \cdot 2\cos^2(\beta - \alpha)$$
$$P_{TT} = |c_T^A c_T^B|^2 \cdot 2\cos^2(\beta - \alpha) = \sin^2 \alpha \sin^2 \beta \cdot 2\cos^2(\beta - \alpha)$$
$$P_{HT} = |c_H^A c_T^B|^2 \cdot 2\sin^2(\beta - \alpha) = \cos^2 \alpha \sin^2 \beta \cdot 2\sin^2(\beta - \alpha)$$
$$P_{TH} = |c_T^A c_H^B|^2 \cdot 2\sin^2(\beta - \alpha) = \sin^2 \alpha \cos^2 \beta \cdot 2\sin^2(\beta - \alpha)$$
The correlation coefficient, formulated by the LHVT, given below

\[ C(\alpha, \beta) = P_{tt} (\alpha, \beta) + P_{tt} (\alpha, \beta) - P_{tt} (\alpha, \beta) \]

is employed to calculate the actual correlation coefficient for the experimental setting angles of \( \alpha \) and \( \beta \) and found to be

\[ C_{\text{qen}}^{\text{entangled}} (\alpha, \beta) = 2\cos^2(\beta - \alpha)\left[ \cos^2 \alpha \cos^2 \beta + \sin^2 \alpha \sin^2 \beta \right] - 2\sin^2(\beta - \alpha)\left[ \cos^2 \alpha \sin^2 \beta + \sin^2 \alpha \cos^2 \beta \right] \] (24)

considering the separable states which leads to unity of the entanglement parameter gives the following equation

\[ C_{\text{qen}}^{\text{separable}} (\alpha, \beta) = \sin^2 \alpha (\sin^2 \beta - \cos^2 \beta) + \cos^2 \alpha (\cos^3 \beta - \sin^3 \beta) \] (25)

which is equivalent to the classical circumstances as expected. In general, when \( \psi_H = \psi_r \) and \( \alpha = \beta = \tan^{-1}\left( \frac{\psi_r}{\psi_H} \right) = 45 \)
then

\[ E_{\text{qen}}^{\text{entangled}} (\alpha, \beta) = \cos 2|\beta - \alpha| \]

can easily be found and is with excellent agreement with the previous findings (Dehlinger and Mitchell, 2002).

6. Conclusions

Fundamentals of the quantum entanglement is summarized and a quantum entanglement parameter, given by the equation (11) is defined. The entanglement parameter clearly indicates possible coupling/interaction of two distant quantum states. The coupling of the two distant quantum systems has been tested by means of the well-known Bell’s inequalities, specifically by using CHSH inequality relation that is the equation (14). The CHSH inequality is derived and the relation with the entanglement is expressed both theoretically and experimentally. The subtle topic of scientific relation between quantum mechanics and consciousness / mind is explained with the support of the relating literature. Finally, a basic model of the consciousness/mind is explained and a quantum entanglement model of the consciousness/mind is developed. The equations of (14), (15) and (24) can be employed to experimentally test the validity of the quantum entanglement of the minds/consciousnesses which is planned to be the specific aim of a forthcoming paper.
References


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