Neural Networks that Emulate Qubits

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Abstract
Recurrent neurons can store simultaneous true and false with probabilistic behaviors usually reserved for quantum states. It is speculated below that neural networks can simulate certain quantum computations and, although less potent than quantized particles, they nevertheless significantly exceed the capabilities of classical deterministic circuits.

Key Words: qubits, probabilistic, neurons, parallelism, toggling

I. Introduction
One powerful argument against the quantum mind proposition is that quantum states are based on controlled atomic particles (Quantum Mind, 2011). These are thought to be unstable and thus would decohere well before brain signaling begins. However, below it is speculated that ordinary neurons can achieve in part what qubits do. Neurons so configured do not have as many states as a system of qubits. But they are shown below to be more capable than ordinary bits.

Classically, explicit long term memory has been envisioned as associative arrays of recurrent neurons, each element being essentially a ring oscillator, which would be microscopic, active only when called and inscrutable (Burger, 2009). Part and parcel of this simple minded approach is that when a neuron is active with a high frequency of pulses, a deterministic true is defined; otherwise when frequency approaches zero (rest), a deterministic false is defined.

The approach below is to employ frequency and phase in a recurrent neuron to define a sphere, the top of which represents true with certainty, and the bottom of which represents false with certainty. Other points on the sphere represent less than full probability for true and false. To be more exact, a recursive neuron becomes a neural multivibrator. By controlling multivibrator frequency and phase there results a novel method of controlled toggling. Such controlled toggles may bring about the amazing feats of gifted savants (Burger, 2011).

The first section below, Working with Neurons, suggests that neurons may have certain qubit-like properties even though they are based on electrical as opposed to atomic and subatomic forces. These limited qubit-like properties lead to advantages over classical neural networks. The second section Neural Networks with Quantum-like Advantages includes 1) Packing Data into a State Vector, 2) Transmitting Binary Functions via a State Vector.

Working with Neurons
Neurons are nonlinear. Once triggered, they produce bursts of pulses with similar amplitudes. Each pulse tends to have a brief width but the separation can vary; the
frequency of pulses within a burst may range from a few hundred hertz down to below one hertz, depending on parameters. The full purpose of these multifaceted signals has yet to be uncovered.

Figure 1 speculates how neurons might evolve to constitute multivibrators. Frequency and therefore duty cycle may be changed by adjusting the feedback Delays (F0 and F1). Moreover, the relative phases of the output signals may be adjusted using Delays 0 and 1. All multivibrator waveforms are assumed to be harmonically related and synchronized. For example, if the low frequency is 1 Hz, then frequency can step up to perhaps 400 Hz in 1 Hz steps. Figure 2 Illustrates a waveform with 50% duty cycle and so represents true with 50% probability and false with 50% probability. To observe true or false the waveform is sampled as in Figure 3.

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**Figure 1.** Multivibrators for true (top) and false (bottom).

**Figure 2.** Probability generator for zero and one.
Figure 3. Random pulse to sample combined zero and one; z is either true (an action potential) or false (at rest with no pulses).

Transforming from fundamental waveforms as in Figure 1, where the expected output is one or zero with certainty, to a combined waveform as in Figure 2, where the observed output could go either true or false, involves a judicious adjustment of delay for the low frequency (zero) multivibrator. The lower frequency may be specified to be $f_o > 0$ and the higher frequency may be specified to be $f_i$. An intermediate frequency may be specified to be $f_x$. The width of a pulse may be specified to be $\tau_x$. The probability of seeing a true as an output (signal $z$) can be formulated to be:

$$P_{True} \approx \tau_x f_x$$

Therefore the probability of seeing a false is:

$$P_{False} = 1 - \tau_x f_x$$ (1)

The greatest chance of seeing a false occurs for $f_x = f_o$ (assuming a sampling window that cannot see a single pulse). The greatest chance of seeing a true occurs for $f_x = f_i$ (assuming a sampling window that sees two or more pulses).

II. Delay

Biological signal delay can be regulated via the density of conductive pores (or ion channels) in an unmyelinated neural conductor and also by local ionic concentrations. This will control local charging currents and therefore the speed of signaling. To do so there must be a nearly uniform covering of synapses and possibly an ionic environment that limits currents in ion channels. To increase delay and lower frequency, the surface is subjected to a shower of inhibitor neurotransmitters; to decrease delay, and raise frequency, inhibitor neurotransmitters are withdrawn.

III. Analogy to Qubits

A single neural multivibrator is roughly analogous to a quantum particle or “qubit” $|\psi>$ based on two states 0, 1, which in vector form are $|0> = [1 0]'$, and $|1> = [0 1]'$ (the prime denotes a transpose and is a way to express a vertical vector on a horizontal line). A qubit is usually presented with its special $|\psi>$ symbols, and in quantum mechanics it is a mathematically linear combination of its two states:

$$|\psi> = \alpha|0> + \beta|1>$$ (2)

In general $\alpha$ and $\beta$ are complex numbers. To properly conserve probability, a mathematical constraint is that:

$$\alpha^2 + |\beta|^2 = 1$$ (3)

A single qubit is sometimes visualized as locating a point on a sphere as in Figure 5 (Nielsen and Chuang, 2000; Pittenger, 1999). Note that a qubit is thought of a rotating through an angle $\theta$ within the x-z plane. Relative phase shift involves another angle $\phi$ about the z axis. The result is that any qubit vector can have a positive or a negative value, or any angle in between. In particular, negative signs are permitted, for instance, either $+|\psi>$ or $-|\psi>$ is possible. Note that $-|1>$ is totally unrelated to $|0>$.

A sphere of probability is easy to visualize, of course, it portrays only the relative phase of $\alpha$ relative to $\beta$, and it is good for only a single qubit. In a multivibrator, given an arbitrary mix of frequency and phase, an operating point can be visualized as a vector $a = [a_1, a_2]'$ which could be anywhere on the sphere. Figure 5 shows approximately, a 50-50 mix of the two independent states, zero and one. This mix is denoted as $a = \eta [1 1]'$, where $\eta = 1/\sqrt{2}$ to satisfy equation (2) as developed above for a quantum qubit, that is, $(1/\sqrt{2})^2 + (1/\sqrt{2})^2 = 1$.

Suppose now that two qubits are prepared at 50 % duty cycle each. Then $|a> = \eta [1 1]'$ and $|b> = \eta [1 1]'$. Upon readout, there is a 25 % chance of any given combination 00, 01, 10, 11. Thus there are four possible states. These states can be expressed using a “direct” product. That is,
\[ |\psi> = [a, b, a', b'] = [a_1, b_1, a_2, b_2, a_3, b_3] \]
\[ \eta = \begin{bmatrix} a_1 \\ b_1 \\ a_2 \\ b_2 \\ a_3 \\ b_3 \end{bmatrix}. \]  

(4)

So in this case:

\[ |\psi> = \eta^2 [1 1 1 1]' \]

A significant difference is that “direct product” is not appropriate for multivibrators. Basically, n multivibrators will give up to 2n independent states, and not the full 2^n as expected of n qubits (Figure 6 illustrates the states of three multivibrators with independent states \(a, b, a', b', a_2, b_2, a_3, b_3\), shown for convenience with equal probabilities of ones and zeros). This amounts to one of many possible codes of data but not the full 2^n codes held by a quantum state vector. The above multivibrator system, as given, does not allocate probability between multivibrators. This means, for example, that the probability of seeing something from state \(a\) cannot be reduced to zero. It also means that the state \([0 0]'\) is not included in this formulation. Although unnecessary here, there are plausible neural circuits that arbitrarily distribute the probability for each neural multivibrator.

Other differences are that probabilities take on discrete values since the available frequencies are assumed to be harmonically related. For example, if \(f_0 = 1\) Hz, then \(f_x\) is an integer; probability takes on discrete values (according to Equations 1). An insignificant difference is that the author places \(0\) as being a lower frequency and therefore, on the bottom of a sphere (usually a Bloch sphere has \(|0>\) at the top).

IV.
V. The Phase of the One
It has not been explained yet how the phase of a one is controlled. This may be done by using an auxiliary multivibrator whose frequency is fixed at \( f_1 \). If the media were linear, the signals might simply add producing two tones at once, as is common mechanically. Unfortunately, neurons are far from linear. In the case of neural signals, they may combine logically, for instance, in an AND gate with a symbol as in Figure 7, although other logical combiners are also possible.

![Figure 7. AND function](image)

The AND gate is readily available in a neuron and it will preserve frequency, phase and duty cycle information. A waveform (not drawn to scale) is portrayed in Figure 8. This waveform results by increasing the frequency of multivibrator 0 and keeping the frequency \( f_1 \) of an auxiliary multivibrator fixed. The width of each pulse is arranged to vary from \( \tau_0 \) at the lower frequency to \( \tau_1 \) at the higher frequency. This is done because pulse width might in practice be affected by frequency, and to ensure that the phase information in the higher frequency is not lost as the lower frequency is driven to approach the higher frequency.

![Figure 8. Showing No Phase Shift for a One Relative to a Zero](image)

**Neural Networks With Quantum-like Advantages**
Neural multivibrators like those outlined above hold more information than toggle circuits or flip flops.

**Packing Data Into a State Vector** – State vectors saved in the form of efficient multivibrators could be copious and useful as memory elements. For instance, binary-like information may be created with combinations of \([1\ 0]\)', \([0\ 1]\)' and \([1\ 1]\)' using \([1\ 0]\)', \([0\ 1]\)' the following probability vectors may be created: \([1\ 0]\) (1 (0) 1)', \((1\ 0)\ (0\ 1)\)', \((0\ 1)\ (1\ 0)\)', \((0\ 1)\ (0\ 1)\)', these being nothing but ordinary binary coding 0, 0, 1, 0, 1, 1. In addition, using \([1\ 0]\)', \([1\ 1]\)' the following advanced codes may be created: \([1\ 0]\) (1 1)', \([1\ 0]\) (1 0)' using \([0\ 1]\)', \([1\ 1]\)' the following codes may be created: \([0\ 1]\) (1 1)', \([0\ 1]\) (0 1)'; and finally there is: \(\eta^2\ (1\ 1)\)' These advanced codes are in addition to ordinary binary codes for \( n = 2 \).

Generally the number of additional data items grows exponentially with the number \( n \), the number of independent variables using multivibrators. By considering independent elements as being \([1\ 0]\)', \([0\ 1]\)' and \([1\ 1]\)' there could be \(2^n \) codes. This calculation by the author is the binary count \(2^n\) of the basic variables 0, 1; plus the binary count with \([1\ 1]\)' in place of one variable; plus the binary count with \(\eta\ [1\ 1]' \) in place of two variables; and so on to \(\eta^n\ [1\ 1]' \) in place of all variables.

The net count using multivibrators is far more than for binary coding with a mere \(2^n\) codes using \( n \) bits. However, since each advanced code is probabilistic, waveforms would have to be sampled several times in order to read out faithfully the original data. This may be accomplished in this classical system by permitting several random sampling pulses and letting the data accumulate in a register for this purpose.

**Transmitting Binary Functions via a State Vector** – It is possible to apply a version of Deutsch's algorithm to multivibrators. This algorithm is perhaps not very practical but it has served to prove that quantum computation is in a sense more potent than classical binary computing. In its minimal form, multivibrator \( a \) is initialized to be \([1\ 0]\)' and then is transformed to: \( a = [a_1, a_2] = [1\ 1]' \). Mathematically this can be expressed linearly as
\[ a = \eta (a_1 + a_2) \] (5)

\[ a_1 = \eta = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad a_2 = \eta = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \]

A function \( f(a) \) is now applied that can go true under certain conditions. For instance, it could go true whenever \( a_2 \) occurs. The result of applying the function can be that a negative sign is placed on \( a_2 \). Then \[ a = \eta \begin{bmatrix} 1 \\ -1 \end{bmatrix} \]. The waveforms for this are suggested in Figure 9.

Probability processing is available to distinguish \( \eta \begin{bmatrix} 1 \\ 1 \end{bmatrix} \) from \( \eta \begin{bmatrix} 1 \\ -1 \end{bmatrix} \). Let \([p \ q]' = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \). Then:

\[ a_1 = (1 - \eta^2)p + \eta^2q = 1 \]

\[ a_2 = (1 - \eta^2)p - \eta^2q = 1 - 2\eta^2 = 1 - 1 = 0 \] (6)

Note that \( \eta^2 = \tau_x f_x = 0.5 \). So \( \eta \begin{bmatrix} 1 \\ 1 \end{bmatrix} \) transforms to \( \begin{bmatrix} 1 \\ 0 \end{bmatrix} \) or \( 0 \) which can be determined with certainty using one observation. In the case of \( \eta \begin{bmatrix} 1 \\ -1 \end{bmatrix} \), let \([p \ q]' = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \). Then:

\[ a_1 = (1 - \eta^2)p + (\eta^2)(q) = 1 - 2\eta^2 = 0 \]

\[ a_2 = (1 - \eta^2)p - (\eta^2)(q) = 1 \] (7)

So \( \eta \begin{bmatrix} 1 \\ -1 \end{bmatrix} \) transforms to \( \eta \begin{bmatrix} 0 \\ 1 \end{bmatrix} \) or \( 1 \) which can be determined with certainty using one observation.

It may be determined that \( a \) carries more information than expected. If after processing and measurement, \( a = \pm \begin{bmatrix} 1 \\ 0 \end{bmatrix} \) it may be concluded with certainty, upon study, that the function is constant: either \( f(a_i) = 0 \) or \( f(a_i) = 1 \) for \( i = 1, 2 \); otherwise, if \( f(a_i) = a_i \) or \( f(a_i) = \text{NOT}(a_i) \), the complement of \( a \) one measures with certainty \( a = \pm \begin{bmatrix} 0 \\ 1 \end{bmatrix} \). Such global properties are determined by forwarding and observing \( a \) only once to see if it is zero or one. Considering the sign and the phase, the exact function, one of the four possible with a single binary variable, may be determined. This information normally requires both the unknown function and evaluating of it at least two times using ordinary bits. For larger \( n \) this small advantage becomes more significant.

Generalizing, a state for \( n \) multivibrators may be defined to be \([a_1 \ a_2 \ b_1 \ b_2 \ldots]'\) and prepared to be \([1 \ 1 \ 1 \ 1 \ldots]'\). Application of a certain class of functions (symmetric and antisymmetric functions) can result in minus signs in the state vector. For example a function of two binary variables with the truth table \( 0 \ 1 \ 1 \ 0 \) is symmetric; it can be made to create a state vector \([1 \ -1 \ -1 \ -1]'\). This transforms to \([0 \ 1 \ 0 \ 1]'\) and is recognized when observed to be \(1 \ 1 \).

The procedure for relating a truth table to the states of multivibrator qubits is illustrated for the case \( n = 2 \) in Table 1. To solve for \([a_1 \ a_2 \ b_1 \ b_2]'\) the author uses the structure in the fourth column. Begin by assuming that \( a_1 = 1 \). Then it follows that \( a_1 = 1, a_2 = -1, b_1 = 1, b_2 = -1 \) in order to provide the pattern in the third column. The end result must be that the list entries corresponding to the \( 1 \)'s in the truth table are negated. This is what must be accomplished in order to apply a function physically.

<table>
<thead>
<tr>
<th>Truth Table</th>
<th>Prepared List</th>
<th>Operated on List</th>
<th>Assumed List Structure</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 1 1 0</td>
<td>1 1</td>
<td>1 1</td>
<td>a_1 b_1</td>
</tr>
<tr>
<td>1 1 -1 -1</td>
<td>1 -1</td>
<td>-1 -1</td>
<td>a_1 b_2</td>
</tr>
<tr>
<td>0 1 1 0</td>
<td>1 1</td>
<td>1 1</td>
<td>a_1 b_2</td>
</tr>
</tbody>
</table>

2 A symmetric or antisymmetric function of dimension \( k \) is symmetric or antisymmetric about the center of its truth table and is either symmetric or antisymmetric in each binary subdivision of \( 2^{k-1} \) entries of the table \( i=1,2,\ldots,2^{k-1} \). This class of binary functions has an even number of true entries in its truth table for \( k>1 \).
VI. Conclusions

This work is definitely not about quantum mechanics although it increases understanding of this abstract topic. Above it is shown that recurrent neurons can have limited quantum-like properties. In particular, these circuits may represent probabilistic proportions of zero and one concurrently, using a three dimensional probability-phase space similar to the space of a qubit.

A neural multivibrator in the above system provides a high probability of a logical zero when operating at lower frequencies and a high probability of a logical one at higher frequencies. Frequencies are controlled by varying the delay in the feedback path to achieve both one and zero with given probabilities. Phase shifts are controlled by varying the delay in the output path. Consequently there is a lot of information in a multivibrator, far more than in a classical flip flop or charge storage device. As an element of biological long term memory, a multivibrator could represent the presence of an attribute of a mental image and also encode its strength (with frequency) and color (with phase).

Obviously books could be written about what we do not know about the intelligent functioning of a brain. Although it is uncertain how multivibrator qubits into brain theory, it is now known that a set of n neural multivibrators may hold far more than \(2^n\) words, which is a great deal of data. This extra capacity may come into play when memorizing visual scenes or musical pieces, for instance. Also it was found that within the class of symmetric and antisymmetric functions, an observation of n multivibrators can discern and identify \(2^{n+1}\) functions. This property could be related to human abilities. Memorized tables, for instance, with many details up to \(2^{n+1}\) might be compressed into simple codes representing n attributes.

Last, but not least, there is now more clarity on the possibility of controlled toggle circuits within the brain. A Multivibrators constructed of neurons with probabilistic readout could serve as a controlled toggles. This is advances the possibility of reversible parallel computing within the brain as discussed elsewhere.

References


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