Mind Efforts, Quantum Zeno Effect and Environmental Decoherence

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ABSTRACT

In this article we present the mathematical formalism of Quantum Zeno Effect (QZE) and explain how the QZE arises from suppression of coherent evolution due to external strong probing action. Then we prove that the model advocated by Henry Stapp, in which mind efforts are able to exert QZE upon brain states, is not robust against environmental decoherence if the mind is supposed not to violate the Born rule and to act only locally at the brain. The only way for Stapp to patch his proposal would require postulation of mind efforts acting globally on the brain and the entangled environment, which would be regression to a theory predicting paranormal Psi effects. Our arguments, taken together with the lack of scientifically valid confirmation of paranormal Psi effects such as telekinesis, imply that Stapp’s model does not have the potential to assist neuroscientists in resolving the mind-brain puzzle.

Key Words: Quantum Zeno Effect, mind, brain, Stapp’s interactive dualism, decoherence

1. Introduction

The mind-brain problem is undoubtedly one of the most fascinating in natural sciences and there has been hope that one can get deeper insight in it by bringing together neuroscience and quantum mechanics (QM) (cf. Tarlacı 2010a,b). The fruitfulness of the latter approach however critically depends on the possibility of the brain to sustain quantum coherence for biologically relevant timescales (cf. Tegmark, 2000) and the quantum mind advocates have been actively searching for physical mechanisms able to prolong desired quantum coherent states so that they may have functional consequences in the operating neuronal networks.

Surprisingly, the quantum physicist Henry Stapp claimed that he has found a quantum mechanism that allows for the mind to govern the evolution of the brain despite the environmental decoherence (Stapp, 2000; 2001; 2007). It was suggested that mind efforts could prolong certain brain states via Quantum Zeno Effect (QZE) so that the statistical averaging over macroscopic time intervals of brain's actions does not "wash out of all effects that depart from what would arise from a classic statistical analysis that incorporates the uncertainty principle as simply lack of knowledge" (Schwartz et al., 2005; p.1320). Formally, in order for the mind action upon brain to exert a form of QZE, Stapp introduced two novel postulates (axioms) according to which the metaphysical (phenomenal) mind is given the freedom to locally operate at the brain and is able to select both the timing and the measurement basis in which the brain state is collapsed: “the freedom to choose which questions are put to nature, and when they are asked, allows mind to influence the behavior of the brain” (Stapp,
Because the metaphorical mind represents the subjective aspects of experience (qualia) and possesses no extra degrees of freedom different from those already possessed by the physical brain, the mind does not have its own wavefunction or density matrix and does not obey the Schrödinger equation. Stapp’s model is not based and cannot be based only on the axioms of QM introduced by von Neumann because in von Neumann’s formulation there are no such things as minds, spirits, ghosts or souls, which do not possess their own wavefunctions or density matrices but can interact with other physical objects. Exactly because the mind in Stapp’s model does not have its own wavefunction or density matrix, it should not be able to interact with anything physical according to von Neumann’s formulation. However, Stapp postulates that the mind actions could be modeled with the use of quantum operators operating upon the brain states (brain density matrix), which necessarily requires the inclusion of the two novel axioms allowing metaphysical objects (such as minds) to interact with physical objects by choosing which questions to ask, and when to ask them. Importantly, Stapp did not postulate that the mind is able to choose the outcome of the wavefunction collapse, which would be a radical violation of the Born rule in the standard QM formalism: “the brain does practically everything, but mind, by means of the limited effect of consenting to the rapid re-posing of the question already constructed and briefly presented by brain, can influence brain activity by causing this activity to stay focused on the presented course of action” (Stapp, 2001; p.1489). The latter clarification is necessary, because if mind efforts are postulated to violate the Born rule then they could override easily the effects due to environmental decoherence providing us with unlimited power to perform Psi effects at our will.

In the following exposition we present the basic mathematical principles that account for the QZE in a simple two-level system and without loss of generality we explain the mechanism that makes QZE possible (Section 2). Then we introduce a laboratory setup to illustrate the discussion and to show that generally in case of environmental decoherence the mind efforts cannot exert QZE upon brain without acting globally on the brain and the entangled with the brain environment (Section 3). From this follows that either the mind efforts acting locally at the brain are ineffective and not robust against environmental decoherence contrary to Stapp’s claims, or that mind efforts act globally at the brain and the entangled environment, which implies telekinetic effects of the mind upon entangled with the brain physical apparatuses. Finally, we prove a quantum mechanical theorem based on concrete calculation of the von Neumann entropy $S(\hat{\rho})$ of the brain density matrix $\hat{\rho}$, which shows that the QZE exerted by the mind locally onto the brain in general cannot slow down or reverse the effects of environmental decoherence (Section 4). Our argument, taken together with the lack of scientifically valid confirmation of paranormal Psi effects such as telekinesis, implies that Stapp’s model does not have the potential needed to resolve the mind-brain puzzle.

2. Mathematics of the QZE
The QZE was derived originally for unstable (decaying) systems where sufficiently quick repeated measurements delay the decay of the system. In the continuous measurement limit the system never decays (Misra and Sudarshan, 1977). Experimentally the QZE was first observed by Itano et al. (1990), who suggested that the QZE occurs because every measurement causes the wavefunction to collapse to a pure eigenstate of the measurement basis. Yet, this viewpoint was harshly criticized by several authors (Ballentine, 1991; Block and Berman, 1991; Home and Whitaker, 1992; 1997), which correctly pointed out that the quantum coherent evolution (according to the Schrödinger equation) is disturbed by frequent detector entanglements that lead to qubit mixed states, therefore the QZE results from suppression of the coherent evolution of the qubit, and does not require at all wavefunction collapse. Insofar as we are not concerned with the reversibility of the QZE setup in the following discussion we will leave it to the reader to choose an interpretation for the evolution of the measured system: either it can be viewed as evolving into proper mixture of states as a result of wavefunction collapses or just evolving into improper mixture of states as a result of external entanglements. Here we remind that according to the terminology...
introduced by d’Espagnat (1999), a *proper mixture* is an ensemble for which can be given an ignorance interpretation. Such a system is in one of several possible pure states for which we could only ascribe different non-zero statistical probabilities. That is why such a state is also referred to as a *statistical mixture*. In contrast, an *improper mixture* is represented by a density matrix for which cannot be given an ignorance interpretation. For improper mixture the individual systems are quantum entangled and cannot be thought to be in some definite state of which we are ignorant; rather, the reduced density operator is the only description that they can have (d’Espagnat, 1999; Timpson and Brown, 2005). Stapp agrees that the realities created by two different preparation procedures that give the same density matrix of the universe are one and the same reality. Thus if certain proper or improper brain mixtures are described by identical brain density matrices, they will be considered one and the same reality and the mechanism through which the mixtures are obtained (external entanglements or wavefunction collapses) will be irrelevant for our discussion.

The following mathematical sketch of QZE is rigorous and independent on the interpretation of QM chosen. Let us have a two-level system (qubit) undergoing decay and let the quantum state at \( t = 0 \) be \( |\psi_o\rangle \). At time \( t \) the state evolves according to the Schrödinger equation to \( \exp(-i\hat{H}t/\hbar)|\psi_o\rangle \). The Hamiltonian \( \hat{H} \), which is an operator corresponding to the total energy of the system, is assumed to have no explicit time dependence (Peres, 1980; Facchi and Pascazio, 1998). The survival (non-decay) probability is given by:

\[
(1) \quad p = |\langle \psi_o | \exp(-i\hat{H}t/\hbar) |\psi_o \rangle|^2
\]

Expanding as a power series:

\[
(2) \quad \exp(-i\hat{H}t/\hbar) = \sum_{n=0}^{\infty} \frac{(-i\hat{H}t/\hbar)^n}{n!}
\]

and for sufficiently small \( t \) we can approximate:

\[
(3) \quad \exp(-i\hat{H}t/\hbar) \approx 1 - \frac{i\hat{H}t}{\hbar} - \frac{1}{2} \frac{\hat{H}^2t^2}{\hbar^2}
\]

therefore for the survival probability we obtain:

\[
(4) \quad p = 1 - \left( \frac{t}{\tau_z} \right)^2
\]

where the Zeno time is given by:

\[
(5) \quad \tau_z = \frac{\Delta \hat{H}}{\hbar}
\]

where

\[
(6) \quad \Delta \hat{H} = \sqrt{\langle \psi_o | \hat{H}^2 |\psi_o \rangle - \langle \psi_o | \hat{H} |\psi_o \rangle^2}
\]

Now suppose we make \( n \) quick measurements at times \( t = \frac{T}{n}, \frac{2T}{n}, \frac{3T}{n}, ..., T \). The survival probability is:

\[
(7) \quad p = \left[ 1 - \left( \frac{T}{n\tau_z} \right)^2 \right]^n
\]

and one easily verifies that in the continuous limit if \( n \to \infty \) we obtain \( p \to 1 \). Mathematically the QZE relies on the fact that for \( n > 1 \):

\[
(8) \quad 1 - \left( \frac{T}{n\tau_z} \right)^2 > 1 - \left( \frac{T}{\tau_z} \right)^2
\]

Thus essential feature of the QZE is that if the qubit is sufficiently frequently measured there is a very high probability for finding the qubit in undecayed state (Figure 1), or in other words, the evolution of the qubit is delayed.

In the following discussion, we will suppose without loss of generality that the brain could be modeled as an evolving two-level quantum system, which has two basis states \( |\text{Yes}\rangle \) and \( |\text{No}\rangle \) that correspond to Yes and No answers to a binary question, and that the brain Zeno time allows in principle for QZE to be realized (see also Stapp, 2001). Then we will explain why the QZE could be effective only if the two-level brain being measured in a certain basis is not being measured by a third party also. The latter result implies that a putative QZE depending upon mind efforts in Stapp’s model could not be effective in case of environmental decoherence of the brain.
3. QZE and Environmental Decoherence

As we have briefly outlined in the introduction, Stapp’s model relies on two postulates: the phenomenal mind, which has no extra physical degrees of freedom is able (1) to measure the physical brain in a freely chosen basis and (2) to choose the timing of this measurement, which is supposed to work through wavefunction collapse and creation of proper mixture of brain states rather than mind-brain entanglement and creation of improper mixture of brain states. Before we address the environmental decoherence effects upon brain states let us introduce an easy to grasp laboratory setup of QZE with polarized photon, a version of which has been actually performed by Kwiat et al. (1995, 1996). First, we need a source of coherent light (laser). Then we pass the beam through vertical polarization filter, after which we place \( n \) equally spaced polarization rotators that rotate the polarization in clockwise direction by an angle \( \vartheta = \pi / 2n \). After passing through the \( n \) polarization rotators the initially vertically polarized photon with state:

\[
|\psi_i\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}
\]

evolves into horizontally polarized state:

\[
|\psi_f\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}
\]

The evolution of the quantum state of the photon from vertical linear polarization into horizontal linear polarization in the two-dimensional complex Hilbert space \( \mathcal{H} \) plays the role of “decay” (Figure 2A). The Hamiltonian of the system \( \hat{H} = \kappa \hat{\sigma}_z \), where \( \hat{\sigma}_z \) is the first Pauli spin matrix, is formally equivalent to the one governing the evolution of two-level atom undergoing coherent Rabi oscillations (cf. Gagen, 1993).

Next, we insert perfect polarizing beamsplitters after each polarization rotator, which are oriented so that the plane of incidence coincides with the vertical plane and the angle of incidence of the laser beam is equal to the Brewster’s angle of the material the beamsplitter is made of. Thus each beamsplitter entangles the photon polarization with the photon path (cf. Walborn et al., 2002; 2003) and in effect non-destructively measures photon polarization so that the vertically polarized component will be transmitted through the beamsplitter, while
the horizontally polarized component will be reflected. The vertically polarized transmitted ray through all beam splitters is detected by detector $D_1$, while the reflected horizontally polarized rays are detected by detectors $D_2$ through $D_{n-1}$ (Figure 2b). Since the detectors are assumed to be located in distinguishable spatial locations they automatically get entangled with the photon polarizations as they absorb the photon traveling along one of the distinguishable paths. Notice that because the beamsplitter orientation defines the plane of incidence and effectively decides the measurement basis of photon polarization, hereafter we will refer to each beamsplitter as the photon polarization and the corresponding detectors.

After the first polarizing beamsplitter the vertically polarized photon state $|\psi_1\rangle$ will be transmitted towards detector $D_1$, while the horizontally polarized photon state $|\psi_2\rangle$ will be reflected towards detector $D_2$. Thus after the first polarizing beamsplitter the composite detector-photon system will be in a quantum entangled state:

$$|\Phi_1\rangle = \cos \theta |\psi_1\rangle |D_1\rangle + i \sin \theta |\psi_2\rangle |D_2\rangle$$

and the photon state will be an improper mixture described by reduced density matrix:

$$\hat{\rho}_1 = \begin{pmatrix} \cos^2 \theta & 0 \\ 0 & \sin^2 \theta \end{pmatrix}$$

In order to make a connection with Stapp’s description of the QZE, which uses density matrices and projection operators, we should say that the action of the first polarizing beamsplitter is formally equivalent to a measurement of the photon in the $|\psi_1\rangle, |\psi_2\rangle$ basis. After the measurement, the photon will be either in the state $|\psi_1\rangle$, or in the state $|\psi_2\rangle$. The choice of a measurement basis is equivalent to a choice of a family of orthogonal projection operators that sum up to the unit operator $\hat{I}$. The projection operator asking the question “Is the photon in the state $|\psi_1\rangle$?” is given by:

$$\hat{P}_{1} = |\psi_1\rangle \langle \psi_1| = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

The projection operator asking the question “Is the photon not in the state $|\psi_1\rangle$?” is given by:

$$\hat{I} - \hat{P}_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

For the two-dimensional Hilbert space of the photon, the projection operator $\hat{I} - \hat{P}_1$ is the same as the projection operator $\hat{P}_2 = |\psi_2\rangle \langle \psi_2|$, asking the question “Is the photon in the state $|\psi_2\rangle$?” The two operators $\hat{P}_1$ and $\hat{I} - \hat{P}_1$ are orthogonal because $\hat{P}_1 (\hat{I} - \hat{P}_1) = 0$. These two orthogonal operators also sum up to the unit operator $\hat{P}_1 + (\hat{I} - \hat{P}_1) = \hat{I}$ and thus define a choice of a measurement basis.

Before the first polarization beamsplitter the density matrix of the photon is:

$$\hat{\rho} = \begin{pmatrix} \cos^2 \theta & -i \cos \theta \sin \theta \\ i \cos \theta \sin \theta & \sin^2 \theta \end{pmatrix}$$

After the first polarization beamsplitter the density matrix of the photon evolves as:

$$\hat{\rho} \rightarrow \hat{\rho}_1 = \hat{P}_1 \hat{\rho} \hat{P}_1 + (\hat{I} - \hat{P}_1) \hat{\rho} (\hat{I} - \hat{P}_1)$$

Substituting the matrices for $\hat{\rho}_1$, $\hat{P}_1$ and $\hat{I} - \hat{P}_1$, and performing the matrix multiplication gives:

$$\hat{\rho}_1 = \begin{pmatrix} \cos^2 \theta & 0 \\ 0 & \sin^2 \theta \end{pmatrix}$$

which was already obtained considering the photon-detector entanglements.

Because an advocate of the “consciousness causes collapse” interpretation of QM could theoretically replace the detectors with the eyes of human observers so that the photon evolves into proper mixture, it is obvious that the conclusions from the proposed laboratory setup will not depend on the interpretation of QM used. Indeed it is a basic quantum mechanical theorem that proper or improper mixtures described by the same density matrix are experimentally indistinguishable.

With every next beam splitting the photon will evolve into an improper mixture of being horizontally polarized and thus reflected and detected by corresponding detector, or being reset to its original state of
vertical polarization and transmitted towards $D_i$. After passing through the $n^{th}$ polarizing beamsplitter the photon state will be in a mixture in which the passage through all detectors without decaying will have probability of $\cos^{2n} \theta$. And this is exactly the QZE - the photon decay probability from one has been decreased to $1-\cos^{2n} \theta$ due to insertion of the polarizing beamsplitters, which measure the photon polarization and entangle it with the corresponding detectors. In the final state $|\Phi_n\rangle$ resulting from $n$ successive entanglements:

\begin{equation}
|\Phi_n\rangle = \cos^n \theta |\psi_1\rangle |D_1\rangle + \sin \theta |\psi_2\rangle |D_2\rangle + icos \theta \sin \theta |\psi_3\rangle |D_3\rangle + \ldots
+ icos^{n-1} \theta \sin \theta |\psi_n\rangle |D_n\rangle,
\end{equation}

one can sort out the component $|\psi_1\rangle |D_1\rangle$ that is realized with probability $\cos^n \theta$ and is responsible for the QZE. Of course there are also $n$ orthogonal possibilities $|\psi_2\rangle |D_2\rangle$, $|\psi_3\rangle |D_3\rangle$, ..., $|\psi_n\rangle |D_n\rangle$, which are realized respectively with probabilities $\sin \theta$, $\cos \theta \sin^2 \theta$, ..., $\cos^{n-1} \theta \sin^n \theta$. The reduced density matrix of the photon becomes:

\begin{equation}
\hat{\rho}_n = \begin{pmatrix}
\cos^{2n} \theta & 0 \\
0 & 1-\cos^{2n} \theta
\end{pmatrix}
\end{equation}

In the continuous limit when $n \to \infty$, the latter $n$ orthogonal states are realized with probability zero, while the state responsible for the QZE is realized with probability of one (cf. Joos, 1984). Therefore despite the fact that entanglements produce mixed state of the photon, in the continuous limit $n \to \infty$ the probabilities of reflected orthogonal states vanish, and the final result is a coherent photon state $|\psi_1\rangle$ at detector $D_1$, in which the quantum amplitude of one is not caused by abrupt localization of the photon wavefunction due to the von Neumann’s collapse postulate, but is a mathematical consequence of the continuous observation of the photon state. In the continuous limit $n \to \infty$ the reduced density matrix of the photon is a pure state:

\begin{equation}
\hat{\rho}_\infty = \begin{pmatrix}
1 & 0 \\
0 & 0
\end{pmatrix}
\end{equation}

Up to this point we have constructed a laboratory QZE that results solely from measurements due to entanglements and we have shown that mathematically there is no need for one to make recourse to measurements due to von Neumann’s collapses. If the measurement of the evolving qubit is continuous then a coherent state can be produced by the entanglements. Mathematically the QZE is result of suppressing the quantum coherent (also known as Schrödinger or unitary) evolution of the target system, because in general the evolution from a state $|a\rangle$ to state $|b\rangle$ requires passage via intermediate state $|a\rangle+|b\rangle$, which entails interference (coherence). Unitary (Schrödinger) evolution from $|a\rangle$ to $|b\rangle$ requires all the phase relations contained in the intermediate state $|a\rangle+|b\rangle$ and since the phase relations are destroyed by measurements, it is not surprising that motion becomes impossible in quantum theory if coherence is completely absent (cf. Joos, 2006). Thus QZE in the laboratory setup is a result from diagonalization of the reduced density matrix of the photon and it is irrelevant whether such a diagonalized matrix is proper or improper mixture.

Mathematically, the QZE can be also reformulated in terms of density matrices and operators. Unitary evolution according to the Schrödinger equation is given by:

\begin{equation}
\hat{\rho}(t) = \exp(\frac{-iHt}{\hbar} )\hat{\rho}_0 \exp(\frac{iHt}{\hbar} )
\end{equation}

where $\hat{\rho}_0$ is the initial density matrix of the quantum system at $t=0$, and the density matrix of the system at time $t$ is:

\begin{equation}
\hat{\rho}(t) = \exp(-i\hat{H} t/\hbar) \hat{\rho}_0 \exp(i\hat{H} t/\hbar)
\end{equation}

If the quantum system is not subject to external measurements, it will evolve unitarily and would be able to “decay”. However if the system is subject to an external measurement (described by a family of orthogonal projection operators that sum up to unity), the density matrix of the system would evolve non-unitarily as:

\begin{equation}
\hat{\rho} \to \hat{\rho}' = \hat{P} \hat{\rho} \hat{P} + (I-\hat{P}) \hat{\rho}(I-\hat{P})
\end{equation}

If these non-unitary evolution steps $\hat{\rho} \to \hat{\rho}'$ are sufficiently frequent, the system will not be able to evolve at all i.e. QZE will occur. The non-unitary evolution can be achieved either through external detector entanglements or through von Neumann.
wavefunction collapses. In both cases the mathematics is the same, therefore the putative mechanism for achieving QZE is irrelevant if one wants to study the effects of environmental decoherence. Although Henry Stapp repeatedly stressed that the QZE achieved through von Neumann wavefunction collapses is “different” from the QZE realized experimentally (Itano et al., 1990; Kwiat et al., 1995; 1996), in view of the preceding discussion it is clear that whatever that “difference” is, it does not and cannot affect the mathematics that is used to describe the QZE.

Nevertheless, in order not to misrepresent Stapp’s position we will now require the reader to imagine that all the intermediate measurements have been performed by a metaphysical (phenomenal) mind that acts upon and collapses the wavefunction of the target two-level system, which in the discussed setup is the photon and not the brain. Though from physical perspective the phenomenal mind does not have extra degrees of freedom to get entangled with the photon (or the brain respectively) we will assume that this is not a conceptual problem for Stapp’s model and the mind is guiding the brain evolution via process that is equivalent to controlling the beamsplitter orientation in the laboratory setup, which effectively decides the measurement basis.

Now what remains in order to incorporate the environmental decoherence in our setup is to introduce a set of measuring devises (in our case beamsplitters) controlled by a third party called environment, which will be able to choose the measurement basis independently of the mind choices. Thus, let us double the number of rotators and beamsplitters to \(2n\) and let all odd numbered beamsplitters be controlled by the environment and all even numbered beamsplitters be controlled by the mind. Since the environmental decoherence could happen in any basis, we decide to investigate the effect of the environment if the measurements (projections) by the odd numbered beamsplitters are performed in a complementary basis:
\[ |\psi_+\rangle = \frac{1}{\sqrt{2}}(|\psi_1\rangle + |\psi_2\rangle) \]
\[ |\psi_-\rangle = \frac{1}{\sqrt{2}}(|\psi_1\rangle - |\psi_2\rangle) \]

compared to the measurement basis chosen by the mind. This corresponds to all beamsplitters operated by the environment being oriented with their plane of incidence at \(-\pi/4\) so that they transmit the state \(|\psi_+\rangle\) and reflect the state \(|\psi_-\rangle\). The projection operators associated with this choice of a measurement basis are:

\[ \hat{P}_+ = |\psi_+\rangle \langle \psi_+ | = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \]
\[ \hat{P}_- = |\psi_-\rangle \langle \psi_- | = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \]

The operators \( \hat{P}_+ \) and \( \hat{P}_- \) are orthogonal and they sum up to unity: \( \hat{P}_+ + \hat{P}_- = \hat{I} \), as can be directly checked.

It is now easy to calculate that the relevant probabilities are:

\[ p_1 = \cos\left(\frac{\pi}{4} - \theta\right)^{2n} \cos\left(\frac{\pi}{4} + \theta\right)^{2n} \]
\[ p_2 = \sum_{k=1}^{n} \cos\left(\frac{\pi}{4} - \theta\right)^{2k-2} \cos\left(\frac{\pi}{4} + \theta\right)^{2k+2} \]
\[ p_3 = \sum_{k=1}^{n} \cos\left(\frac{\pi}{4} - \theta\right)^{2k} \cos\left(\frac{\pi}{4} + \theta\right)^{2k-2} \]

In the continuous limit \( n \to \infty \), we have \( p_1 = 0 \), which means that the photon does not arrive at all at \( D_1 \). Instead the photon ends with probability \( 1/3 \) in state \(|\psi_+\rangle\) and with probability \( 2/3 \) in state \(|\psi_-\rangle\). Of course one might modify the setup in alternative ways and suppose for example that the mind can also measure the reflected \(|\psi_+\rangle\) photon wavefunction branches, or alternatively the environment can measure the reflected \(|\psi_-\rangle\) branches. Straightforward calculations show that in either modification the QZE breaks down. This is due to the third party measurement (done by the environment) in a complementary measurement basis, which effectively puts the photon polarization back into superposition of the states \(|\psi_+\rangle\) and \(|\psi_-\rangle\). The result is not surprising and would follow if the mind itself started random switching of the measurement basis (that is random switching of the projection operators). The proof is transparent: if the mind itself does not use QZE strategy and starts random switching of the measurement basis the QZE will break down; therefore the QZE will break down also in the case when the mind is operating together with a non-cooperating environment (e.g. one that uses a complementary measurement basis or one that switches randomly the measurement basis).

4. QZE cannot Slow Decoherence

What remains to be addressed is the claimed proof by Stapp (2007), according to which the QZE exerted by mind efforts upon the brain would work despite the environmental decoherence, or to be more precise, the QZE exerted by the mind upon the brain in general can slow down the environmental decoherence of the brain. First, we note that Stapp investigates a particular case in which the brain is measured by the environment in the very same measurement basis in which the mind makes wavefunction collapses. But such an argument is fallacious, and practically says that if the environmental decoherence assists the mind QZE and happens to be in the same measurement basis with the one chosen by the mind, then the environmental decoherence will not destroy the attempted by the mind QZE upon brain states. Only in the special case when environmental decoherence happens to be in the desired by the mind measurement basis, mind efforts will be assisted by such decoherence. This is not surprising because the environmental decoherence of a macroscopic body can be formally understood as a form of QZE in which interference terms between different possible locations of the macroscopic object are suppressed e.g. Joos (1984) argued that macroscopic body decoheres in a position basis due to interaction with the photons forming the background thermal radiation. Obviously if the mind wanted to decohere the macroscopic brain in position basis, then the environmental thermal decoherence will assist the job. Moreover, if this were the only basis the mind can choose, then it is not clear what is the function of such metaphysical mind and what kind of freedom it has, since its role would be redundant with the one performed by the thermal radiation.
If the environment exerts the very same QZE intended by the mind it will assist the mind efforts. But this does not suffice for a general proof of robustness against environmental decoherence, because it is possible for the decoherence basis to be complementary to the one intended by the mind. Thus Stapp’s discussion either presupposes that the environment already uses a measurement basis intended by the mind or that the mind cannot choose other measurement basis but the one chosen by the environment.

In general, environmental decoherence can happen in any basis, although position and energy basis might be preferred (Zeh, 1996; 2000; 2005; Tegmark and Wheeler, 2001; Zurek, 2003; Hornberger, 2009). Nevertheless, for the construction of the one chosen by the environment it does not really matter if the environmental decoherence can happen in any basis or only in a certain preferred basis. What is important is that whatever the basis in which the environmental decoherence happens, the mind could not possibly exert an effective QZE in a complementary basis to the one chosen by the environment. This contradicts with Stapp’s main thesis that mind choices are simultaneously effective and absolutely free.

For those readers who are already familiar with the basics of quantum information theory we will prove a basic theorem, which directly invalidates Stapp’s claim that QZE can slow down environmental decoherence (Stapp, 2007). In the proof of the theorem we will use the concept of von Neumann entropy $S(\hat{\rho})$ of the brain density matrix $\hat{\rho}$, which provides a precise quantitative measure of the brain decoherence. The von Neumann entropy $S(\hat{\rho})$ of a quantum system described by a density matrix $\hat{\rho}$ is given by:

$$S(\hat{\rho}) = -\text{Tr} \hat{\rho} \ln \hat{\rho}$$

which is equivalent to:

$$S(\hat{\rho}) = -\sum \lambda_i \ln \lambda_i$$

where $\lambda_i$ are the eigenvalues of the density matrix $\hat{\rho}$. If the density matrix of the brain is a pure state, then the von Neumann entropy is zero. If the density matrix of the brain is a completely mixed state then the von Neumann entropy is maximal. In other words, the von Neumann entropy $S(\hat{\rho})$ quantifies the departure of the system from a pure state.

First important feature of the von Neumann entropy $S(\hat{\rho})$ is that it remains unchanged under unitary (quantum coherent) evolution of the density matrix according to the Schrödinger equation. In other words, the change of the von Neumann entropy in time for unitary evolution is zero:

$$\frac{dS(\hat{\rho})}{dt} = 0$$

If the system is in a pure state with zero von Neumann entropy and evolves coherently, then it will also have zero entropy after any period of time $t$.

Second important feature of the von Neumann entropy $S(\hat{\rho})$ is that if the quantum system of interest undergoes environmental decoherence, the change of the von Neumann entropy in time is positive:

$$\frac{dS(\hat{\rho})}{dt} \geq 0$$

Only for a completely mixed state the change of the von Neumann entropy in time would be zero, but this is because the state already has reached the maximal possible von Neumann entropy, which means that all possible quantum states are equally probable.

Since we are interested into realization of QZE, it is obvious that a completely mixed state having maximal von Neumann entropy cannot be a starting point for QZE. If all quantum states are equally probable, then with or without any intervention at a later period of time $t$ all quantum states will be also equally probable. From the mathematical sketch in Section 2, we have seen that in order for QZE to be realized, one should start either with a pure state or with a quantum system that is with very high probability in a certain initial quantum state $|\psi_0\rangle$. Then by performing suitable QZE measurements one can keep high the probability for the quantum system to be in the state $|\psi_0\rangle$ even at a later time $t$. Note that the QZE is not equivalent with the claim that the state of the system does not change in time. The state may change in time and the probability for the system to be in the state $|\psi_0\rangle$ may actually decrease in time. The important feature is that at the end the...
probability for the system to be in a certain state \(|\psi_0\rangle\) remains high (although it may not be 100%). This requirement is violated for a completely mixed state where the state of the quantum system may not change in time, but all quantum states are equally probable, and therefore no QZE occurs.

After ruling out a completely mixed state as being the initial brain state in Stapp’s QZE model, we can say that that environmental decoherence will always increase the von Neumann entropy of the brain density matrix in time:

\[
dS(\hat{\rho}) \bigg/ dt < 0
\]

Stapp’s claim that in general the mind action onto the brain density matrix can slow down the environmental decoherence implies that the mind action should be able to decrease the von Neumann entropy of the brain density matrix in time:

\[
dS(\hat{\rho}) \bigg/ dt > 0
\]

Indeed, if the mind QZE action cannot generally decrease the von Neumann entropy of the brain in time, then it would be either ineffective or may actually speed up environmental brain decoherence, which always increases the von Neumann entropy in time.

To show that Stapp’s claim for mind efforts slowing down the environmental decoherence is erroneous, we will prove as a theorem that the von Neumann entropy production by the mind efforts is always:

\[
dS(\hat{\rho}) \bigg/ dt \geq 0
\]

According to Stapp (2007) in QZE due to the external probing mind action the density matrix of the brain evolves as:

\[
\hat{\rho}_0 \rightarrow \hat{\rho}_t
\]

where \(\hat{\rho}_0\) is the initial brain density matrix just before the mind action, \(\hat{\rho}_t\) is the brain density matrix at time \(t\) given by:

\[
\hat{\rho}_t = \hat{P}\hat{\rho}_0\hat{P} + (I - \hat{P})\hat{\rho}_0(I - \hat{P})
\]

and \(\hat{P}\) is an arbitrary projection operator.

We will prove that if \(S(\hat{\rho}_0)\) is the von Neumann entropy of the initial brain density matrix \(\hat{\rho}_0\) and \(S(\hat{\rho}_t)\) is the von Neumann entropy of the density matrix \(\hat{\rho}_t\), the inequality \(S(\hat{\rho}_t) \geq S(\hat{\rho}_0)\) holds for every \(\hat{P}\). We will prove the latter statement through direct algebraic calculation of the relevant von Neumann entropies.

Keeping in mind that matrix multiplication is not commutative we can rewrite \(\hat{\rho}_t\) as:

\[
\hat{\rho}_t = \hat{P}\hat{\rho}_0\hat{P} + (I - \hat{P})\hat{\rho}_0(I - \hat{P})
\]

\[
\hat{\rho}_t = \hat{P}\hat{\rho}_0\hat{P} + (I - \hat{P})\hat{\rho}_0(I - \hat{P})
\]

\[
\hat{\rho}_t = \hat{P}\hat{\rho}_0\hat{P} + (1 - \hat{P})\hat{\rho}_0(1 - \hat{P})
\]

\[
\hat{\rho}_t = \hat{P}\hat{\rho}_0\hat{P} + (I - \hat{P})\hat{\rho}_0(I - \hat{P})
\]

Then we express the initial density matrix \(\hat{\rho}_0\) in the basis in which it is diagonal as:

\[
\hat{\rho}_0 = \begin{pmatrix} k & 0 \\ 0 & 1-k \end{pmatrix}
\]

where \(k\) and \(1-k\) are the eigenvalues of the density matrix \(\hat{\rho}_0\). We note that because the density matrix is a Hermitian operator, it always has an orthonormal eigenbasis in which the matrix is diagonal.

Next, we consider the most general case for which \(\hat{P}\) chosen by the mind projects onto an arbitrary vector \(|\psi\rangle\) and we express this vector in the basis in which \(\hat{\rho}_0\) is diagonal as:

\[
|\psi\rangle = \begin{pmatrix} a \\ b \end{pmatrix}
\]

where \(a\) and \(b\) are complex numbers. Then we use the fact that introducing an arbitrary pure phase \(\exp(\imath \varphi)\) does not change the physical interpretation of the vector and fix \(a\) to be real (see also Susskind 2006, 2008, 2012). From the definition of a real number we have:

\[
a' = a
\]
where \( a^* \) is the complex conjugate of \( a \).

Next, we use the fact that the state is normalized so that \( aa^* + bb^* = 1 \) (again see Susskind 2006, 2008, 2012). Simple algebraic work gives us:

\[
1 - a^2 = \frac{b^*}{b}
\]

Since it is required that \( b \neq 0 \) in order for the above fraction to be defined, the case in which \( b = 0 \) will be investigated separately.

Thus for the projection operator \( \hat{P} \) we obtain:

\[
\hat{P} = |\psi\rangle\langle\psi| = \begin{pmatrix} a \\ b \end{pmatrix} \begin{pmatrix} a^* & b^* \end{pmatrix}
\]

\[
\hat{P} = \begin{pmatrix} a^2 & a(1-a^2) \\ ab & b(1-a^2) \end{pmatrix}
\]

Next we substitute into eq. (43) the matrices \( \hat{P} \) given by eq. (49) and \( \hat{\rho}_i \) given by eq. (44) in order to obtain \( \hat{\rho}_i \) in matrix form:

\[
\hat{\rho}_i = \begin{pmatrix} A & B \\ C & D \end{pmatrix}
\]

where

\[
A = a^2(2-4k) + k + a^4(-2+4k)
\]

\[
B = -(a - 3a^3 + 2a^5)(-1+2k) / b
\]

\[
C = a(-1+2a^2)b(-1+2k)
\]

\[
D = 1 + a^2(-2-4k) - k + a^4(-2+4k)
\]

Although \( \hat{\rho}_i \) looks complicated, it has simple eigenvalues:

\[
\lambda_i = k + a^2(1-2k)
\]

\[
\lambda_z = 1 - k - a^2(1-2k)
\]

The von Neumann entropy of the matrix \( \hat{\rho}_i \) is given by:

\[
S(\hat{\rho}_i) = -\lambda_i \ln \lambda_i - \lambda_z \ln \lambda_z
\]

It is easy to see by inspection that always \( S(\hat{\rho}_i) > S(\hat{\rho}_o) \) because:

Case 1. \( b = 0 \), \( a = 1 \) or \( b = 1 \), \( a = 0 \). The projection operators \( \hat{P} \) and \( \hat{I} - \hat{P} \) project onto the basis in which the density matrix \( \hat{\rho}_o \) is diagonal. Therefore \( S(\hat{\rho}_i) > S(\hat{\rho}_o) \).

Case 2. \( k = 1 - k = 1/2 \). Then \( a^2(1-2k) = 0 \), from which follows that:

\[
\lambda_i = k = 1/2 \\
\lambda_z = 1 - k = 1/2
\]

and the von Neumann entropy is maximal for both density matrices: \( S(\hat{\rho}_i) = S(\hat{\rho}_o) \). (In addition, such scenario is not suitable for QZE because the initial state cannot be completely mixed).

Case 3. \( k \neq 1/2 \), \( a \neq 0, 1 \). First, let us note that the bigger the absolute difference between the two eigenvalues of the density matrix, the smaller the von Neumann entropy of the density matrix is. Therefore in order to determine which density matrix \( \hat{\rho}_i \) or \( \hat{\rho}_o \) has bigger von Neumann entropy it suffices to compare the absolute differences between the eigenvalues of \( \hat{\rho}_o \) and \( \hat{\rho}_i \), respectively.

The absolute difference between the eigenvalues of \( \hat{\rho}_o \) is:

\[
|\lambda_i - \lambda_z| = |1-2k - 2a^2(1-2k)| = |1-2k(1-2a^2)| = |1-2k|.|1-2a^2| > 0
\]

For \( a \neq 0, 1 \), we have \( 0 < |1-2a^2| < 1 \), and it is evident that the von Neumann entropy of the density matrix \( \hat{\rho}_i \) is always higher than that of the density matrix \( \hat{\rho}_o \): \( S(\hat{\rho}_i) > S(\hat{\rho}_o) \).

Taking together Cases 1-3 implies that the von Neumann entropy production by the mind efforts is always non-negative:

\[
\frac{dS(\hat{\rho})}{dt} \geq 0
\]

From the above theorem follows immediately that QZE cannot decrease the entropy of the brain density matrix \( \hat{\rho}_o \), even if it projects \( n \) times with an arbitrarily chosen operator \( \hat{P} \) or operators \( \hat{P}_1, \hat{P}_2, \ldots, \hat{P}_n \), because for each projection \( \hat{P}_i \) the inequality \( S(\hat{\rho}_i) \geq S(\hat{\rho}_o) \) holds. Projecting \( n \) times with any operators \( \hat{P}_i \) cannot decrease the von Neumann entropy of the brain.

Finally, we would like to add that even though the von Neumann entropy production by the mind efforts is always non-negative,
there are possible special scenarios for which the environmental decoherence could be slowed down, but these require special requirements on the coupling between the brain and its environment. Of course, if there are some special requirements in the way the brain decoherence occurs then QZE could slow down such a special form of environmental decoherence, but this is something very different from Stapp’s original claim for general validity of his model irrespective of the form in which the environmental decoherence occurs. We could further clarify that the mentioned special requirements on the brain-environment coupling concern the Hamiltonian describing the brain-environment interaction, and therefore there is no easy “patch” of Stapp’s model by simply switching the decoherence basis or performing another simple trick. Indeed, it is not a big surprise that Stapp failed in creating a working mind-brain model, because he did not use in the construction of his model even the essential minimum of quantum information theory. The possibility for an effective QZE (in case of non-negative von Neumann entropy production by mind efforts) for a special case of suitably chosen Hamiltonian describing the brain-environment interaction is a completely novel claim presented in this work and therefore should not be attributed to Stapp whose QZE construction violates the basics of quantum information theory.

5. Mind Physics and Psi Effects
In the preceding discussion we have presented the theory of the QZE arising from standard QM formalism. We have also assumed that the mind affects directly the brain (or the photon in the laboratory setup) by choosing the measurement basis. Moreover, since such an influence has been postulated by Stapp’s theory it is not considered here as a paranormal Psi effect. However, any influence that might lead to detectable effect upon the interacting with the brain environment (such as laboratory apparatus) is referred to as a paranormal Psi effect (telekinesis).

First, we briefly discuss the possible violation of the Born rule, in which one insists that the mind can chose the outcome of the wavefunction collapse. Due to interaction between the physical brain and observed external quantum system e.g. a photon passing through a beamsplitter, the brain and the reflected or transmitted photon will get entangled. This ensures consistency between the brain perceiving a flash due to the transmitted photon and the actual transmission of the photon. If the mind locally operating on the brain is able to choose the outcome of the collapse it can always choose to see the flash, and thus the photon will be always transmitted at the beamsplitter. Such an effect is not dependent on external decoherence, because regardless of the probability of the quantum event observed, the mind by selectively actualizing the event (probability becomes equal to one) will affect the externally entangled agent and it will not obey Born rule too. Indeed several advocates of the “mind causes collapse” proposal have tried to detect experimentally such mind paranormal Psi effects and came up with publications according to which the mind affects by thought the output from random number quantum generators (Jahn and Dunne, 1986; Goswami, 1989; 1990; 1997), conscious observer affects the decay of a qubit (Bierman, 2003) or can change the outcome of distant processes such as water crystal formation (Radin et al., 2006). All such positive (successful) Psi experiments were found to contain either statistical flaws (Alcock, 2003; Jeffers, 2003; Bösch et al., 2006) or when performed by independent researchers end up with negative results (Hall et al., 1977). Therefore we assume that paranormal Psi influence upon brain environment such as telekinesis would be a regression for Stapp’s model. That is why we have stressed earlier that Stapp does not use as a postulate possible violation of the Born rule.

Next, we show that if mind efforts were acting globally upon the composite system brain-environment they could be effective in exerting QZE upon brain states, but also would exert QZE upon the environment. Indeed, the only way for the mind to be generally effective in controlling the brain in case of environmental decoherence would be if the mind is allowed to perform global measurement of the composite system describing both the brain and its environment. Suppose that in addition to the brain, modeled as a two-level quantum system with corresponding two-dimensional complex Hilbert space, the spanned by the two orthonormal basis states |Yes⟩ and |No⟩, we have also a very simple two-level environment
with corresponding two-dimensional complex Hilbert space \( \mathcal{H}_2 \) spanned by an orthonormal basis \( |\xi_1\rangle, |\xi_2\rangle \). The composite brain-environment system should be described in four-dimensional complex Hilbert space \( \mathcal{H}_{\text{total}} = \mathcal{H}_2 \otimes \mathcal{H}_4 \) (cf. Griffiths, 2003; Susskind, 2006; 2008; 2012), which now has four basis vectors \( |\text{Yes}\rangle|\xi_1\rangle, |\text{Yes}\rangle|\xi_2\rangle, |\text{No}\rangle|\xi_1\rangle, |\text{No}\rangle|\xi_2\rangle \). If the composite brain-environment system is in a pure state with zero von Neumann entropy (or if it is in a state that is close to a pure state) one can show that the mind could achieve QZE upon both the brain and the environment if it were allowed to perform global measurements in the space \( \mathcal{H}_{\text{total}} \) and provided that this composite system is not decohered by the rest of the Universe. Indeed, it is a standard approach to exert QZE upon the local environment (e.g. electromagnetic cavity) in order to delay the decoherence of a target quantum system (e.g. trapped ions) provided that the actions upon this local environment are not overridden by decoherence due to interaction with the rest of the Universe (cf. Maniscalco et al., 2008). Such a global effect of mind efforts however would imply existence of paranormal Psi influence of the mind upon the environment, which we have already noted is an experimentally disproved option. It could be added, that if one thinks of QZE performed on the composite system containing the brain and its local environment then it would be required for this composite system to be isolated so that it does not suffer decoherence from the rest of the Universe. However, because the latter requirement cannot be physically justified, one would be further led to postulate that the mind actually operates upon the whole Universe, because at present it is considered that only the Universe as a whole does not suffer external decoherence. Such a possibility does not seem to be attractive scientific program to be further pursued, although it might be enthusiastically cheered by those who believe in paranormal Psi effects (cf. Bem and Honorton, 1994; Grinberg-Zylberbaum et al., 1994; Thaheld, 2010; de Castro, 2011; Erickson, 2011; Haas, 2011; Krippner and Fracasso, 2011; Persinger and St-Pierre, 2011; Temkin, 2011; Wolf, 2011).

Finally, there is a possibility to require some special conditions for the brain-environment coupling. Such an option, however, is hardly a justification for the general claim that QZE exerted by mind efforts is robust against environmental decoherence, and, if pursued further, it would face the main problem that all quantum mind theories are supposed to answer – how the brain avoids decoherence and why there should be any special conditions within or around the brain that suppress environmental decoherence rather than enhance it?

6. Discussion

To the best of our knowledge the central argument of the current work that mind efforts in Stapp’s model are not robust against environmental decoherence is novel, although concerns for possible paranormal mind effects in Stapp’s model have been voiced before (cf. Donald, 2003). We have rigorously shown that Stapp’s model cannot satisfy and have at the same time: (1) mind that does not violate the Born rule and does not actively choose the outcome of the collapse, (2) mind that acts only locally at the brain, (3) mind that exerts a form of QZE upon brain, which is robust against environmental decoherence, and (4) mind, which chooses the basis in which the QZE is exerted. One of these four requirements should be relaxed if Stapp is to be consistent, but none is satisfactory. Relaxing conditions (1) or (2) leads to prediction of paranormal Psi effects exerted by the mind upon the environment surrounding the physical brain, relaxing the condition (3) implies that mind efforts are generally ineffective in the case of environmental decoherence, and relaxing condition (4) implies that the mind has no free will whatsoever. In the latter case the mind action would be redundant with the action of the environment to decohere the brain (with the possible exception that the mind introduces irreversibility in the overall process making quantum erasure impossible). Moreover, in such a case the mind will observe the QZE caused by the environment rather than cause the QZE.

Stapp’s description of how the mind exerts QZE upon the brain manages to obscure the necessity to associate state vectors (and density matrices) both to the mind and the brain, and this problem remains “hidden” in the operator terminology attributed to von Neumann. In the current work we have purposefully stressed upon the fact that the
metaphysical mind has neither a state vector nor a density matrix within Stapp’s model. Although similarly to Stapp we have assumed that the actions of the phenomenal mind can be described by quantum operators that represent the measurement processes performed by the mind, we made it clear that this assumption requires introduction of new postulates (axioms). Thus, we consider that Stapp misleadingly claims that his model is equivalent to von Neumann’s formulation of QM. It is not!

The central result of the current work is that the original exposition by Stapp (2007), Sec. 11.2, Figures 11.3-11.7 describing how the mind efforts can slow down environmental decoherence is erroneous and contradicts a basic quantum mechanical theorem proven in Section 4 according to which the von Neumann entropy production by the mind efforts is always non-negative. This result, taken together with our objections against the feasibility of paranormal Psi effects, implies that Stapp’s model does not have the potential to assist neuroscientists in resolving the mind-brain puzzle.

Appendix

Tegmark’s Decoherence Formula

If the mind is supposed to be effective in controlling the brain states it follows that the mind dynamic timescale \( \tau_{\text{dyn}} \) should be comparable but not slower than the brain decoherence timescale \( \tau_{\text{dec}} \). From neuroscience it is well known that the input of sensory and the output of motor information from the brain cortex happen to be at millisecond timescale via electrical firing of axons. The decoherence timescale due to ion-ion collisions for a single firing neuron was estimated by Tegmark (2000) to be:

\[
\tau_{\text{dec}} \approx \frac{4\pi \varepsilon_0}{m} \sqrt{\frac{m(kT)^3}{Nq_e^4n}}
\]

where \( \varepsilon_0 = 8.85 \times 10^{-12} \) F m\(^{-1} \) is the vacuum permittivity, \( m = 3.85 \times 10^{-26} \) kg is the mass of the sodium ion, \( N = 1.15 \times 10^9 \) is the number of sodium ions that enter into the cortical pyramidal neuron per single action potential (Laughlin and Attwell, 2004), \( n = 1.75 \times 10^{26} \) m\(^{-3} \) is the total density of ions (both positive and negative) in the brain corresponding to physiological osmolarity of 290 mOsm, \( q_e = 1.6 \times 10^{-19} \) C is the elementary charge, \( k = 1.38 \times 10^{-23} \) J K\(^{-1} \) is the Boltzmann constant, and \( T = 310 \) K is the temperature. After numerical substitution with the biologically relevant values we obtain \( \tau_{\text{dec}} \approx 5.14 \times 10^{-24} \) s. Since in the mental representation of sensory information (or for generation of motor output) is involved not a single neuron but millions of neurons in a particular field of the cerebral cortex, from Tegmark’s estimate follows that the decoherence time of the brain cortical neuronal network due to ion-ion collisions \( \tau_{\text{dec}} \approx 5 \times 10^{-30} \) s is only few orders of magnitude above the Planck scale. This estimate essentially fixes the dynamical timescale of mind choices in Stapp’s model due to the requirement \( \tau_{\text{dyn}} \leq \tau_{\text{dec}} \).

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