Chaos, Fractal in Biology, Biothermodynamics and Matrix Representation on Hypercycle

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ABSTRACT
First, we discuss biofield and some nonlinear theories in biology. Next, chaos in biology and its application to cancer are researched. Third, fractal and complex dimension in biology are searched. Fourth, nonlinear biothermodynamics and in which possible entropy decrease are investigated. Fifth, we propose the matrix representations of hypercycle theory. Hypercycle can be defined by a degree of connectivity. Its fuzzy element corresponds to that each element is fractal. NeuroQuantology should be nonlinearity and quantization, and may relate to quantum chaos, quantized matrix, etc.

Key Words: biology, nonlinearity, chaos, fractal, biothermodynamics, entropy, hypercycle, matrix

1. Introduction
Various nonlinear theories are very important aspects in modern science. They include fractal (Mandelbrot, 1977; 1983; Falconer, 1990), chaos and soliton, etc.

Assume that some corresponding relations exist between biology and physics, such the biosystem and corresponding mechanics are related each other. Bone and its system correspond to the elastic mechanics (Cowin, 1981) and the structural mechanics. Blood and the circulation systems correspond to hydrodynamics (Burton, 1965; Rushmer 1970; Caro et al., 1978). Hydrodynamics is generally nonlinear, and is contacted with the limit cycle and various periods or quasi-periods, whose branch-chaos obtain fractal, and derive the Julia set.

integration of classical, quantum and subquantum concepts for emergence hypothesis, and discussed a theory of everything as introduction of consciousness in Schrödinger equation, standard model, and loop quantum gravity and string theory and unification of experiences with fundamental forces.

Based on the inseparability and correlativity of the biological systems, we proposed the nonlinear whole biology and four basic hypotheses (Chang, 2001, 2012b). It may unify reductionism and holism, structuralism and functionalism. Further, the loop quantum theory is applied to biology, and proposed the model of protein folding and lungs, and obtain four approximate conclusions (Chang, 2012b). Globus (2013) discussed Bohr, Heidegger, the unspeakable and disclosure as an exercise in quantum neurophilosophy. For different sources on the duality of soul-body, mind-body, mind-brain or consciousness-brain, Tarlac (2013) used the expression consciousness-brain, and discussed terms of the soul, or something that existed other than the material body, and developments on mind-body duality, mind-brain and consciousness-brain. This is symbolized in the psi-phi problem. Li (2013) investigated a timeless and spaceless quantum theory of consciousness. In this paper we discuss some nonlinear theories in biology and biothermodynamics and the matrix representations of hypercycle.

2. Biofield and Nonlinear Biology
Rich (1978) discussed the biofield. Rubik (2002) investigated the biophysical basis on the biofield hypothesis and its role in medicine. Rein (2004) discussed the bioinformation within the biofield on beyond bioelectromag-netics. Prigogine proposed the theory of dissipative structure, in which a well-known formula is:

\[ dS = dS + dS. \] (1)

According to (1), its rate assumes that

\[ \frac{dS}{dt} = \sum_j J_j X_i, \] (2)

where \( J_j \) are the flow, and \( X_i \) are the corresponding internal forces. For the statistical independence, they are very small. Such the theory of dissipative structure may derive the extensive force and mechanics.

Generally, in biosystems there are movement and flow, they form various biological fields, which may be the scalar, vector, spinor, tensor fields, etc. These fields must be nonequilibrium and nonuniform, and they form gradient for the scalar field, and divergence and rotation for the vector field. In the n-dimensional space, the gradient is:

\[ \text{Grad} = \sum_i \frac{\partial u_j}{\partial x_i}, \] (3)

The n-dimensional divergence is:

\[ \text{Div} = \sum_i \frac{\partial P_j}{\partial x_i}, \] (4)

The n-dimensional rotation is more complex, and is probably:

\[ \text{Rot} = \sum_{ij} \left( \frac{\partial P_j}{\partial x_i} - \frac{\partial P_i}{\partial x_j} \right). \] (5)

Their flow or movement may be described by the diffusion equation and the equations of hydrodynamics, etc.

Biosystem is based on the simpler physics, and produces the structural complexity or the compositional complexity, and forms finally the functional complexity (Rescher, 1998). But, these complex systems as the set of the same individual should obey the statistical rule. At present a main direction of scientific development is a self-organization on various systems and natural dynamics on higher complex systems from disorder to order, from stochasticity to regularity, from chaos to structure.

Generally, biomechanics should be nonlinear. We discussed the fractal, chaos and soliton in nonlinear biology and neurobiology, in which soliton may keep the integrality and veracity of information in neural transfer. The nonlinear mechanism of memory is researched. Based on the extensive quantum theory in which the formulations are the same with the quantum mechanics and only quantum constant h is different, we proposed the extensive quantum biology (Chang, 2012a). The post-Newtonian mechanics is a type of nonlinear mechanics, and its development is general relativity. We researched the possible applications of the Yang-Mills gauge theory in biology (Chang, 2012b).

Human biological system in order to transmit information does not only use biological channels but also electromagnetic,
acoustic and soliton waves; electric, electromagnetic and torsion fields as well as bioplasma. The bioelectronic model assumes that protein structures, DNA, RNA, and melanin have the properties of an electronic material in which local and nonlocal quantum processes can occur. Adamski (2013) used a hypothetico-deductive method in order to explain the nature of consciousness and formulates the thesis that the nature of consciousness is located in quantum processes.

Usually, the fractal, chaos and soliton will appear in the nonlinear biology, and there are soliton-chaos double solutions (Chang, 2013a). Many biological systems possess various oscillations, in which period or chaos exist, for example, electroencephalogram (EEG) shows chaos, in particular, under thought. Therefore, thinking originates from chaos. A strong disorderly thought field is the super-normal functional state, in which the thoughts surge, and super-normal creative power gives fully play (Chang, 1988b; 2003).

3. Chaos in Biology and Application to Cancer

In nonlinear mechanics there are strange attractors and chaos. The nonlinear biomechanics and the general nonlinear biology are the basis of chaos in biological systems. The nonlinear equations in biology have Lorenz equations, Duffing equation, Chay model and Van der Pol equation, etc.

In biology and physiology there are widely various chaos (Chang, 2012a) and the chaotic dynamics. They include the excited or stimulated state of neuron and whose Hodgkin-Huxley equation (Aihara et al., 1984), and the heart beats rhythmically and whose Bonhoeffer-Van der Pol equation (Fitzhugh, 1969), and the exudation of hormone, and the dynamical disease and epidemic, and the complex structure of living systems (Cramer, 1988), etc. Hartline equation in the inhibitory neural network may add the nonlinear terms, and then chaos will appear.


Kotini et al. (2007) proposed that if there is any nonlinearity in multiple sclerosis patients, the use of nonlinear analysis in comparison with controls may find out the mechanisms underlying their brain waves, and chaotic activity of multiple sclerosis patients is lower than in the normal brain. Tsuda et al. (2007) discussed chaos reality in the brain, in which dynamic behaviors on cortical transitory are observed during task-related actions of animals, and provide a dynamical interpretation of such transitory behaviors in terms of chaotic itinerancy. Bruzzo et al. (2007) established a model to delineate the emergence of “self” in the brain making recourse to the theory of chaos. Self-similarity is discussed as a latent menace of a pathological confusion between “self” and “others”. It relates to the functional connectivity of neural nets (structure), chaotic dynamics, and neural Darwinism. Cignetti et al. (2009) discussed effects of fatigue on inter-cycle variability in cross-country skiing.

The elastic mechanics may derive the oscillation. Van der Pol equation

\[ x'' + \alpha (x^2 - 1)x' + \beta x = \delta \cos(\omega t + \phi) \]  

describes approximately the pulsation of heart. The nonlinear oscillation equation may derive soliton. The non-harmonic oscillator with force is:

\[ x'' + kx' - \beta x + \alpha x^3 = b \cos(\omega t) \]  

It has the chaos solution. If \( x''=0 \) and \( b=0 \), it will be an ordinary differential form of Heisenberg unified equation (Heisenberg, 1966). If \( x''\neq0 \) and \( k\neq0 \), it will be an ordinary differential form of the square of the nonlinear Dirac equation:

\[ \frac{\partial^2 \psi}{\partial x^2} + b - a \psi^2 \psi = 0, \]  

and

\[ (\partial_x^2 + 2b - \frac{\partial}{\partial x}) + b^2 - 2ab \psi^2 \psi \]

\[-2a \psi' \frac{\partial}{\partial x} \psi' + a^2 \psi^3 = 0. \] 

Self-organization is an important character of biology. It is defined as the
phenomenon by whose internal structure independent of external causes. Self-organization of cells into complex interacting systems can be described using a branch of mathematics called nonlinear dynamics, which includes the study of chaos. Coffey (1998) used nonlinear dynamics and chaos as the new biology for medicine, and explained complex biological systems and self-organization on the events leading to disorders as varied as epilepsy, heart disease and cancer.

A cancer cell is a strange attractor, which aggregates much cell. Such the formation of cancer may apply the Diffusion-Limited Aggregation (DLA) model (Witten et al., 1981) and its fractal dimension is:

\[ D_f = \frac{d^2}{d+1}; \quad (10) \]

or apply the Kinetic Cluster Aggregation (KCA) model, and its fractal dimension is:

\[ D_f = \frac{5}{6}d. \quad (11) \]

It is interesting that for the 3-dimensional space (d=3) both models obtain the same \( D_f = 2.5 \).

Now control chaos is a developing direction in chaotic. It includes using small perturbations to control chaos (Ott et al., 1990; Shinnbrot et al., 1993; Hubingeret al., 1993), adaptive control for nonlinear systems (Sinha et al., 1990; Sinha, 1991), and continuous self-controlling feedback (Pyragas, 1992; Quet et al., 1993). Further, control chaos was applied to cardiac chaos (Garfinkelet al., 1992; Christiniet al., 1996), chaos in network (Molgedey, 1992) and chaos in the brain (Schiff et al., 1994). Therefore, if chaos may be applied to describe cancer, such various methods on control chaos correspond probably to treatment of cancer.

Moreover, the acupuncture points of traditional Chinese medicine have the fractal structure for our body, and \( D = 0.631 \). It is the fractal pathology.

4. Fractal and Complex dimension in Biology

Our body is simply covered with fractals. They are everywhere from the circulatory system to the lymph system, the lungs, the muscle tissue, the calyx filters in the kidney, the small intestine to the folding patterns on the surface of the brain (Sardaret et al., 1998). Goldberger et al. (1985) proposed the fractal hypothesis on a mechanism of cardiac electrical stability, and some observations on the question as ventricular fibrillation ‘chaos’ (Goldberger, 1986). The fractal in biology may be dependent on gene, DNA and their structural formations. They are continuously embedded by some self-similarity, and form various organism. A self-similarity may construct the same new cells, synapses and so on. The fractal dimensions \( D = 2.3 \) for the blood-vessel system, \( D = 2.17 \) for the lungs, \( D = 2.76 \) for the surface of the brain (Mandelbrot 1983). They and the breathing system, the neural systems, etc., possess the fractal characters. We investigated the neural synergetics, Lorenz model of brain, soliton-chaos double solutions and physical neurobiology (Chang, 2013a). The fractal dimension of Lorenz model is \( D = 2.06 \).

In the neurobiology Hodgkin-Huxley equations, FitzHugh-Nagumo equations and much interactions are all nonlinear. FitzHugh-Nagumo dynamical equations of a single neuron are:

\[ c \frac{dV(t)}{dt} = V(t) - V^3(t) - y(t), \quad (12) \]

\[ \frac{dy(t)}{dt} = \gamma V(t) - y(t) + b + \sqrt{2D} \xi(t). \quad (13) \]

Here \( b \) and \( c \) are constants, and \( \xi(t) \) represents Gauss white noise. Eq.(12) as a nonlinear equation may derive chaos. We apply the qualitative analysis theory of the nonlinear equations, the characteristic matrix of Eqs.(12) and (13) is:

\[ \begin{pmatrix} 1 - 3V^2 & -1 \\ \gamma & -1 \end{pmatrix}. \quad (14) \]

The solutions of \( 3V^2 - 1 + \gamma = 0 \) are \( (\pm \sqrt{3/\gamma}, 0) \) and \( (0, 1) \), and the results of the qualitative analysis are independent of \( b, c \) and a random term \( \xi(t) \). This shows the scaling invariance and self-similarity, such it has a fractal character.

Further, fractal is developed to multifractals. Grebogi et al. (1982) discussed the occurrence of sudden qualitative changes of chaotic dynamics. For this case, the chaotic region can suddenly widen or disappear. Halsey, et al. (1986) proposed fractal measures and their singularities, which are characterized by two indices: \( \alpha \), which determines the strength of their singularities; and \( f \), which describes how densely they are distributed. The spectrum of singularities is described by the possible range of \( \alpha \) values and the function

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The new formalism allows an introduction of smooth functions to characterize the measures. Kohmoto (1988) introduced an analogy to the formalism of statistical mechanics, an entropy function and a free energy for multifractals. For the invariant set of a dynamical system, these functions are furthermore related to the measure-theoretic (Kolmogolov-Sinai) entropy, the topological entropy, and the Lyapunov exponent.

The generalized multifractal dimension is

\[ D_q = \frac{r(q)}{q-1} = \lim_{\epsilon \to 0} \frac{\ln \sum [p_i + (q-1)p_i \ln p_i]}{(q-1)\ln \epsilon} \]  

which changes with different q. When q=0, \( D_0 = -r(0) = D^* \) is usual Hausdorff dimension. When q=1,

\[ D_1 = \sum p_i \ln p_i (\epsilon \to 0) \]  

is information dimension. \( \sum p_i \ln p_i \) correspond to the information entropy of system. Falconer (1994) discussed the multifractal spectrum of statistically self-similar measures.

We extended the fractal dimension D into the complex dimension, in aspects of both mathematics and physics (Chang, 1988a). The representation of complex dimension is:

\[ D_c = D + iT \]  

When the complex dimension is combined with relativity, whose dimensions are three real spaces and one imaginary time, it expresses a change of the fractal dimension with time or energy, etc., and exists in the fractal description of meteorology, seismology, medicine and the structure of particle (Chang, 1989).

In the continuous change process of parameter, a whole structure of strange attractor may change suddenly. For example, the fractal dimension \( D = 0.538 \) of the logistic mapping for \( \lambda_c \) point, and \( D = 0.34 \) for \( 3^\ast \) period. The strange attractors possess fractal dimension, whose change corresponds just to the complex dimension. The fractal dimension of protein forms probably the spectral dimension. It is namely the complex dimension to a certain degree.

Further, we researched the fractal dimensional and complex dimensional mathematics and the physics. From this is discussed the fractal relativity, which connects with self-similarity of the Universe and an extensive quantum theory. Combining the quaternion, etc., the high dimensional time \( i\epsilon t \to ic\epsilon t + jfc\epsilon t + kfc\epsilon t \) is introduced. The fractal dimensional time is obtained, and space and time possess completely symmetry. The higher dimensional, fractal, complex and super-complex space-time theory covering all might be constructed preliminarily (Chang, 2010).

The fractal studies a set. The multifractal studies a measure. The complex dimension should be a set of complex number, or a changeable set.

5. Nonlinear Biothermodynamics

The multifractal is related to thermodynamics and statistics. The nonlinear thermodynamics is in fact dynamics (Volkenstein, 1982). In the statistics a basic principle is statistical independence (Landau et al., 1980). It shows that various interactions among these subsystems should not be considered. But, if various internal complex mechanism and interactions cannot be neglected, a state with smaller entropy (for example, self-organized structure) can appear under some conditions. In these cases, the statistics and the second law of thermodynamics should be different (Chang, 1994; 1997; 2005; 2012c; 2013b, 2013c). Because internal interactions bring about inapplicability of the statistical independence, entropy decrease in an isolated system is possible. In particular, it has a possibility for attractive process, internal energy, system entropy and so on. Therefore, we proposed that a necessary condition of entropy decrease in isolated system is existence of internal interactions. The internal interactions often are related with nonlinearity (Chang, 1997).

Any biological system has the complexity. We proposed a universal formula for any isolated system (Chang, 2005):

\[ dS = dS^a + dS^i \]  

where \( dS^a \) is an additive part of entropy and is always positive, and \( dS^i \) is an interacting part of entropy and can be positive or negative. Eq.(18) is similar to the formula (1). But, two formulae are applicable for internal or external interactions, respectively.

Further, based on the Eq.(18), a sufficient and necessary condition of entropy decrease in
isolated system may be expressed quantitatively (Chang, 2012c):
\[ \alpha > ds^i > -ds^a, \]
i.e., \[ |ds^i| > ds^a \] (for negative \( ds^i \)). (19)

In usual cases, the condition corresponds to that in isolated systems there are some stronger internal attractive interactions. Negative temperature is based on the Kelvin scale, we find that it will derive necessarily entropy decrease. The known negative temperature is a fallacy in thermodynamics, and is contradiction with usual meaning of temperature and with some basic concepts of physics and mathematics (Chang, 2012c).

Assume the rate \( ds^i \) is:
\[ \frac{ds^i}{dt} = \sum \frac{J_kX_i}{\tau}, \] (20)
where \( J_k \) and \( X_i \) are the flow and internal forces. If the flow
\[ J_k = \sum I_{a_k}X_i, \] (21)
so
\[ \frac{ds^i}{dt} = \sum \frac{(\sum I_{a_k}X_i)X_i}{\tau}. \] (22)

For a simplest case, \( J_k = I_0X_i \), so
\[ \frac{ds^i}{dt} = \sum \frac{I_0X_i^2}{\tau}. \] (23)

The rate of the interacting entropy is a nonlinear function of internal forces \( X_i \).

In a real number field, there is \( \frac{ds^i}{dt} < 0 \) for \( I_0 < 0 \). It shows that some internal interactions may derive entropy decrease. For \( X_i = 0 \) or \( J_k = 0, ds^i = 0 \), so \( ds = ds^a > 0 \). For a general case, \( ds^i \) may be positive or negative. If the total entropy decrease,
\[ ds = ds^a + ds^i - ds^f < 0, \] (24)
the necessary condition will be
\[ ds^i > ds^a + ds^f > ds^a. \] (25)

This includes that soliton is an order; fractal and chaos are related to periods, whose entropy cannot increase always.

We proposed entropy decrease due to fluctuation magnified and internal interactions in some isolated systems, and calculated quantitatively the possibility of entropy decrease (Chang, 2005). Some possible tests for entropy decrease are researched in isolated systems in physics, chemistry, astronomy (Chang, 2013b,c,e). In biology the neuroscience, the permeable membrane, the molecular motor, etc., are all some internal interactions. We discussed quantitatively the moderate input negative entropy flow, and introduced the thermodynamics of physiology and psychology, and Qigong and various practices are often related to these order states with entropy decrease (Chang, 2013b). Generally, it should be confirmed by many stable states in Nature.

The complex biological systems provide some modes on entropy decrease in an isolated system. This is known that any organism all is a typical self-organized system, and must be an order process of entropy decrease. As long as this process is isolated at least in a certain time, it all is a violation for the second law of the classical thermodynamics (Chang, 2013b).

The base of bioenergetics is the cell membrane, in which the ion channel exists. The biologic membranes may choose a direction self-motion. The cell membrane and ferment show a control for direction. A permeable membrane is namely the Maxwell demon, which may be entropy decrease. Enzyme as catalysis relates closely to electronic move. While electron always moves, which correspond to quantum mechanics and quantum biology.

Semiconductor may control a voltaic direction, which is analogy with the biologic membranes. Therefore, the semiconductor theory may be applied to the biologic membranes, which should have the corresponding heat effect, light effect, electric and magnetic effect, and the similar Hall effect.

The emergence and formation of living individual are more due to internal interactions. The growth of a cuvette baby in test tube should be a classical isolated system at least in a certain time. Generally, many biological and chemical changes in test tube are all the isolated systems. The homeostatic organisms are the internal interaction with auto-adjustment, which keep small entropy, so entropy cannot increase continuously.

Molecular self-assembly is an autonomous process that forms molecules or polymer under non-external influence is nanostructure technology. Crane (1950) proposed two basic principles of molecular self-assembly. Further,
Adleman (1994), Winfree et al. (1998) and Ignatova et al. (2008) discussed the self-assembly of DNA structures by the molecular and DNA computation.

Generally, biological evolutions are increases of complexity and bioinformation. They correspond to entropy decrease. In an evolitional process with long time, life forms a nonlinear complex and complete system with multi-levels: gene, cell, tissue, organ, system, individual, population, community, ecosystem, bio-sphere. If the biosystem is isolated at a certain time, the second law of thermodynamics will be violated.

We proposed an entropy index of health on human body, and supposed that entropy dS can extend to the complex number (Chang, 2013b), which corresponds to the complex dimension.

Biology provides a wide region for research of entropy decrease in various isolated systems, which has possibly different levels in biological systems, for example, membrane, enzyme, adenosine triphosphates (ATP) and molecular motor, etc.

The second law of thermodynamics is essentially science; it should not become a religion.

6. Graph Theory and Matrix Representations of Hypercycle

It is known that there are various life periods and cycles in biology. In 1971, Eigen proposed the hypercycle theory. This is a scientific theory on the relationship between protein and nuclei acid, and on the origin of life, and discussed self-organization of matter and the evolution of biological macromolecules (Eigen, 1973). It is in order to model prebiotic evolution governed by the Darwinian principles of competition between species and mutations, and leads to a new level of evolution. Here cooperative behaviors are reflected by intrinsically nonlinear reaction mechanisms. Further, the hypercycle is a principle of natural self-organization, and it as a beautiful form is a tool (Eigen et al., 1979) not only in biomolecules, and may be widely applied to many fields. In the hypercycle cooperative behavior is reflected by intrinsically nonlinear reaction mechanisms, and the dynamics is described by a system of coupled nonlinear differential equations, for example, a second-order hypercycle model (Figure-1) and an extension of hypercycle evolution principle (Figure-2) (Eigen et al., 1979). In the hypercycle each cycle as a whole has self-enhancing growth properties, and different sets of the nonlinear equations carry information and function (Eigen, 1973). Kaneko (2003) discussed recursiveness, switching and fluctuations in a replicating catalytic network, and relation with hypercycle. Solenov et al. (2006) analyzed the nonunitary quantum walks on hypercycles. Braxenthaler et al. (1997) proposed that the dynamic evolution of proteins exhibits chaos property. This is related to the hypercycle.

![Figure 1. Second-order hypercycle model.](image1)

![Figure 2. Extension of hypercycle evolution principle.](image2)
7. Discussion and Conclusion

Biology possesses some characters of whole, self-organization and jump-evolution, etc. An important character of the nonlinear interactions is the formation of self-organization, which should decrease entropy. From chaos to order (Prigogine et al., 1984), the attractor is formed, and it is also a process of formation on hypothesis and theory.

Different sensation systems are usually independent each other. Based on the open out the potential of blind children and other research, we proposed a hypothesis: The neural excitable cell is continuously induced and excited, then grow out new synapse and dendrite, and the feeling system, hearing system, smell system, etc., may joint to visual system, and form a new neural network, and achieve finally a transformation among vision and other sensations. Further, we proposed some possible tests, for example, for trained mammal, etc., and research possible theories. This is a testable application of the nonlinear whole neurobiology, and may build a bridge between modern science and traditional culture, religion (Chang, 2013e).

In a word, NeuroQuantology should be nonlinearity and quantization. Therefore, it is related to quantum chaos (Stockmann 1999), quantized soliton and matrix, and network, etc. Various mathematical and physical methods apply continuously different aspects of biology, this will accelerate deep development of modern biology.

Chaos possesses a chaotic region with $2^n$ period, and the quasi-period. This corresponds to the fuzzy mathematics (Zadeh, 1968; Thomason, 1977). Different chaotic regions in biological systems form the chaotic hypercycle theory. Therefore, the chaotic hypercycle should be the fuzzy matrix, whose elements are decimal, and the operations are intersection and union. It is a development of hypercycle, in which each element all is fractal structure. Even element in hypercycle may be complex number, which is the complex matrix, and corresponds to changeable element and the complex dimension.
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