A Linear Approximate Model of Creativity in Quantum and Chaos Theory

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ABSTRACT

The appearance of creativity is one of the most attractive issues in neuroscience and psychology. Creativity has been investigated qualitatively by using concepts of quantum and chaos theory since the end of the 20th century. Here, we show an explicit mathematical model which tries to explain the dynamics on the development of creativity. The present model is constructed by a linear approximation of chaos theory in nonlinear dynamical system. Also, it is the classical approximation of quantum mechanics, in which Newtonian mechanics is derived. Significant feature of the creativity is an existence of discontinuity which may relate to sudden appearance of idea. Such discontinuous nature relates not only to nonlinear dynamics but also to quantum theory. In this study, we first investigate the characteristics of creative attitudes by means of factor analysis and abstract two chief factors of the creative attitudes; that are "efforts and durability" and "independence and originality". Moreover, we find a significant positive relationship between the emotional experiences and the two creative attitudes. Scholastic ability judged by paper tests, on the other hand, also has a significant positive relationship with the factor of "effort and durability" but not with that of "independence and originality". Secondly, we build a mathematical model on the change of the two chief factors of creative attitudes using the linear approximation in nonlinear dynamical systems. We use the Dirac delta function to express the discontinuity. The calculated results are expressed in a relation between the novelty and the workload. The present linear approximation model should be the first one to build mathematically a predictive model of creativity.

Key Words: Creative Attitudes, Discontinuity, Linear Approximation of Chaos Theory, Classical Approximation of Quantum Mechanics, Emotional Experiences

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Introduction

Creativity is generally defined as the ability to produce work that is both original and useful. One of the most attractive issues in neuroscience and psychology is the appearance of creativity. The study of creativity has a long history (Albert and Runco, 1999; Glover *et al*., 1989; Runco, 1988; Sternberg and Lubart, 1999; Torrance, 1961) since Guilford challenged to pay attention as a very important one (Guilford, 1950). Developing in parallel with the cognitive approach, work in the social-personality research has focused on personality variables, motivational variables, and the sociocultural environment as sources of creativity. Despite the various studies done so far, the mechanism on the development of creativity is still unclear, as it is one of the most complicated human behaviors.

Creativity has been investigated by using physical concepts of quantum theory and nonlinear dynamics since the end of the 20th century (Goswami, 1996; 1999; Schuldberg, 1999). However, these studies just focused on qualitative correspondence with the phenomena in creativity, and no mathematical model has been proposed so far, despite the achievement

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of quantum approaches in consciousness and mind theory (Hameroff and Penrose, 2003; Tarlaci, 2010a; 2010b). Then, some explicit mathematical model which can explain the dynamics of the development of creativity is awaited (Abraham *et al*., 2012).

Here, we try to construct an explicit mathematical model which explains the dynamics on the development of creativity. To build a mathematical model as the first step, high approximation should be necessary in the complicated phenomena of creativity. Creativity is known to relate to the two sides: one is chaos theory in nonlinear dynamical system and the other is the quantum theory. As the first approximation, our model is constructed by a linear approximation of chaos theory in nonlinear dynamical system. On the other hand, it is the classical approximation of quantum mechanics, in which Newtonian mechanics is derived (Dirac, 1958; Feynman, 1948). The present model relates to the two sides as shown in Figure 1.

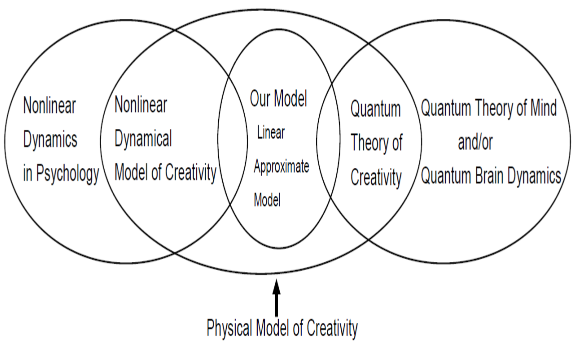

Figure 1. Relationship among the present model, chaos theory in nonlinear dynamical system and the quantum theory
The present model is constructed by a linear approximation of chaos theory in nonlinear dynamical system. Also, it is the classical approximation of quantum mechanics, in which Newtonian mechanics is derived.

For the specific modeling of the dynamics on creativity, we should find some dominant factors, controlling the complicated phenomena of creativity. Here, we focus on the creative attitudes, and abstract chief factors by factor analysis based on the statistical investigations, as described later.

One of the significant features of the creativity is an existence of discontinuity such as "aha" experiences which relates to a sudden appearance of idea (Bowden and Jung-Beeman, 2003). Such discontinuous nature may relate not only to quantum theory but also to the concepts of bifurcation and chaos in nonlinear dynamical system. In nonlinear dynamics, a system can suddenly change into qualitatively different one, something termed "bifurcation" (Abraham *et al*., 2012; Krippner *et al*., 2012; Schuldberg, 1999). For example, a system may change from periodic to chaotic behavior when the values of its parameters are changed. In our model, we use the Dirac delta function to express the discontinuous nature.

In this paper, firstly we show the strategy on the application of dynamical theory and quantum theory to psychology, and then the method is applied to the statistical investigations on the creative attitudes and the related factors to give the dynamical modeling of creativity.

Methods

Strategy: procedure for dynamical modeling of creativity

In this section, we explain our strategy on the procedure for dynamical modeling of creativity. First, we summarize the general dynamical systems for the modeling. General dynamical systems are expressed in the following ordinary differential equations:

$$\begin{cases} \dfrac{dq_1(t)}{dt} = F_1(q_1, \cdots, q_m; t) \\ \vdots \\ \dfrac{dq_m(t)}{dt} = F_m(q_1, \cdots, q_m; t) \end{cases}$$

where q_i are dynamical variables, m is number of degree of freedom, $F_i(q_1, \cdots, q_m; t)$ are generally nonlinear functions of $q_1, \cdots, q_m$. When F_i depend on time t explicitly, the dynamical system contains external forces. We assume that the temporal change of creativity can be described by the dynamical systems.

Now, we focus on the Newtonian mechanics which is a prototype of the dynamical theory, and is regarded as the classical approximation of quantum theory in the limit of Planck's constant → 0 (Dirac, 1958; Feynman, 1948). We imitate Newtonian mechanics for the interpretations of variables and their relations. In particular, we utilize the concepts of the momentum which have similarity in the attitudes of individuals in the simplified model of creativity, as described later. We call our model "kinetic model of creativity" hereafter.

In Newtonian mechanics, the equation of motion on object is expressed as

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$$\frac{d\boldsymbol{p}}{dt} = \boldsymbol{F},$$

where $\boldsymbol{p}$ is the momentum vector, $\boldsymbol{F}$ is the force vector. The equation of motion is the fundamental relation to connect momentum and force which are the basic concepts in Newtonian mechanics. Correspondingly, we regard this equation as the fundamental relation in the kinetic model of creativity.

One of the most significant features of creativity, on the other hand, is discontinuous nature such as "aha" experiences which relates to sudden appearance of idea (Bowden and Jung-Beeman, 2003). "Aha" experiences have been treated both in the quantum approach (Goswami, 1996; 1999) and the nonlinear dynamical approach (Schuldberg, 1999) in the studies of creativity. In the quantum approach, the sudden appearance of idea has been explained as quantum leap. In nonlinear dynamical approach, on the other hand, it has been explained as bifurcation. Thus, mathematical expression of discontinuity is needed to build a model of creativity.

The fundamental function to express discontinuity is the Heaviside's step function θ (Heaviside, 1893; 1894) :

$$\theta(x) = \begin{cases} 1 & (x \geq 0) \\ 0 & (x < 0) \end{cases}$$

The differential of the Heaviside's step function is the Dirac delta function:

$$\frac{d\theta(x)}{dx} = \delta(x)$$

Therefore the delta function can be used to express the discontinuous nature. In the development of quantum mechanics, Dirac introduced the delta function (Dirac, 1927). The Dirac delta function $\delta(x)$ is singular: the value of delta function is zero at all domains except for one point, but integrated value of $\delta(x)$ is 1. Such singular function with discrete property is indispensable for formalization of quantum mechanics.

There are many representations on the delta function by elementary functions, and one of the representations is the Fourier series in the following:

$$\delta(x) \sim \frac{1}{2\pi} + \frac{1}{\pi}(\cos x + \cos 2x + \cos 3x + \cdots).$$

In this representation, the trigonometric functions in Fourier series are regarded as harmonic waves in the brain, and those harmonic waves with various wavelengths generate resonant coherently (Jibu *et al.*, 1994; 1996). Such interpretation corresponds to the quantum leap in "Quantum Brain Dynamics" (Globus *et al.*, 2004; Jibu and Yasue, 1995; Riccicardi *et al.*, 1967; Stuart *et al.*, 1979; Vitiello, 2003).

In the first step of modeling beyond the qualitative approach, we build a mathematical model on the development of creativity with linear approximation, focusing on creative attitudes and related parameters. Then, it is needed to find variables which dominantly control the development of creativity. Accordingly, we use the factor analysis based on the statistical investigations, as described later.

Linear approximate model of creative attitudes

In the following section, we construct a kinetic model to clarify the process on the development of creativity. The present study is consisted of two phases, with the statistical analysis first and the modeling of the development of creativity second. A sample of students is used in the first part to provide the large sample size necessary for the factor analysis. The second phase of the modeling is done with assumptions which are extracted from the results of statistical analysis.

Statistical investigations for modeling

Participants and materials

The experimental group (n=434) was comprised of lower secondary school students in Japan. Students were asked to fill out two questionnaires, relating to the creative attitude and experiences done so far. The special educational activity which sets training of creativity was not performed at the lower secondary school.

A questionnaire on the creative attitudes was prepared based on 24 items of creative attitudes proposed by Onda (Onda, 1994), which are shown in Table 2. The creative attitudes given by Onda have many similar contents to the ones given by Davis (Davis, 1999), especially relating to independence and originality. The 24 items of the questionnaire were answered in the Likert-style 5-point scale format ranging from 5(strongly agree) to 1(strongly disagree).

Another questionnaire asking on primary experiences, which were done so far by the students, was prepared. The contents of the questionnaire of primary experiences, which were usually done in their childhood, were selected to 130 terms. Some of the questions are shown in Table 1. In order to clarify the qualitative differences in "experience", three type answers were prepared as follows:

(a) "I had done so far."
(b) "I had done so far, and impressed or performed eagerly".
(c) "I had not done."

Table 1. A part of the questionnaire of experiences

The lists indicated below are the activities which children sometimes do. Put Y marks on the activities that you actually have done so far. Put ◎ marks on the activities that you were impressed or performed eagerly. Put × marks on the activities that you have never done.

( ) 1 I caught some insects without an insect net.
( ) 2 I caught some insects from inside of trees and dead trees.
( ) 3 I used food as traps to catch some insects.
( ) 4 I looked for some insects to catch in winter.
( ) 5 I enjoyed catching some beetles and stag beetles.
( ) 6 I ate larvae.
( ) 7 I enjoyed catching some lizards and snakes.
( ) 8 I listened to wild birds singing and observed them.
( ) 9 I searched and checked footmarks, traces of eaten food and droppings of animals.
( ) 10 I looked for tunnels of mice and moles.
・
・
・
( ) 120 I made pinhole cameras, telescopes and kaleidoscopes.
( ) 121 I tried plant dyeing.
( ) 122 I checked acidity and alkalinity with juice from vegetable leaves and flowers.
( ) 123 I made some musical instruments by using something I can make sound with.
( ) 124 I played by making thread telephones.
( ) 125 I played by making whistles.
( ) 126 I played with batteries and miniature bulbs.
( ) 127 I played with magnets.
( ) 128 I played by causing static electricity (friction electricity).
( ) 129 I played by making electromagnets.
( ) 130 I played by making water mills.

Results

The factor analysis was done (Toyoshima and Niwase, 2000) by using a questionnaire on the creative attitudes. The principal factor method and Varimax rotation were utilized.

Two chief factors of the creative attitudes were abstracted by factor analysis. The Eigenvalue of the third factor was 0.72. Since the Eigenvalue of the third factor was less than the value of 1, it was adapted till the second factor. Two factors explaining 64.5% of the total variance were obtained. The detailed results of the factor analysis and the creative attitudes related to the chief factors are shown in Table 2. As the reliability on the measure of this creative attitudes' survey, Cronbach's alpha (Cronbach, 1951) was calculated. The overall alpha coefficient was 0.79 and internal consistency estimates was checked.

Table 2. Results of the factor analysis

Contents	1st factor	2nd factor
The 1st factor: "effort and durability		
・**I work perseveringly.**	**0.652**	0.128
・**I try for difficult things by myself, instead of depending on others.**	**0.585**	0.162
・**I finish up what I have begun once by myself.**	**0.570**	0.091
・**Even if I fail, I don't give up.**	**0.523**	0.270
・**I endure the present pain for a future target.**	**0.496**	0.094
・**I pursue until I'm convinced.**	**0.416**	0.223
・I concentrate on my work and do it quickly.	0.348	0.207
・I challenge to more difficult things than easier ones.	0.313	0.279
・If I notice the fault of my way of thinking or behavior, I'll correct it immediately.	0.300	0.041
・Even if I distract the mind, I can cool down immediately	0.259	0.012
・I will not be distracted by the environment.	0.255	0.038
The 2nd factor: "independence and originality		
・**I act vigorously with my will.**	0.203	**0.596**
・**I sometimes surprise others by doing novel things.**	0.052	**0.558**
・**I'm interested in various hobbies.**	0.191	**0.505**
・**Once I decide to do something, I carry it out immediately.**	0.140	**0.456**
・**I'm not influenced by others' words as I have my own thoughts.**	0.312	**0.397**
・**Even if I expect opposition, I can express my own opinions clearly.**	0.267	**0.393**
・When I find out something unexpectedly, I become excited and give a shout.	-0.011	0.390
・I carry out my tasks quickly without worrying about mistakes and failure.	-0.012	0.353
・I sometimes lose track of time when I concentrate on something.	0.066	0.324
・Even if the environment and the situation change, I can adopt myself to them immediately.	0.142	0.308
・I usually try to think the best way of using my everyday tools and daily behavior.	0.258	0.303
・I sometimes forget myself in listening to and observing others.	0.083	0.288
・I often try to find out strange things and unknown things.	0.196	0.260
Eigenvalue	4.023	0.109
Contribution ratio	0.506	0.139
Accumulated contribution ratio	0.506	0.645

The first factor was labeled "efforts and durability" as it relates to accomplish tenaciously by oneself and never give up without minding difficulties. The second factor was labeled "independence and originality" as it relates to have own view and not to align with others.

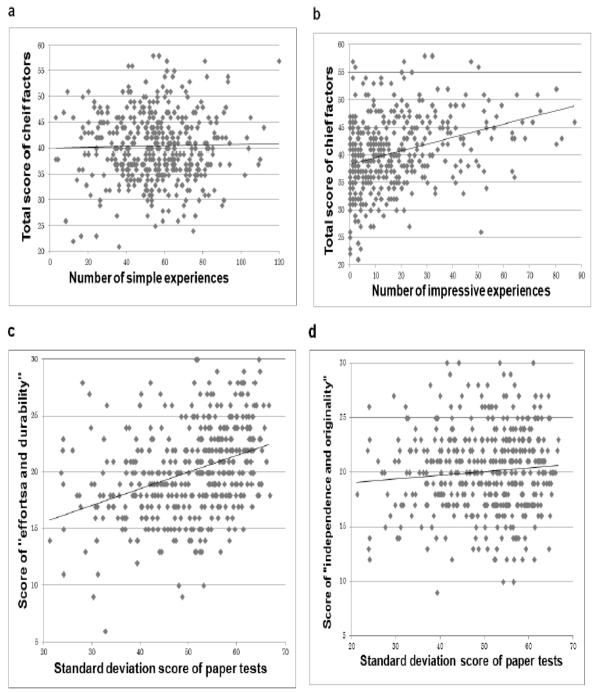

Figure 2. Relationship among the creative attitudes, experiences and paper tests
Total score on the chief factors of creative attitudes versus the number of a, simple experiences b, impressive experiences. The score of c, "efforts and durability" d, "independence and originality" versus the standard deviation score of paper tests.

Table 3. Correlation coefficients of the two chief factors of creative attitudes and experiences, scholastic ability

	efforts and durability	independence and originality
Number of total experience	0.222	0.236
Number of simple experience	0.041	-0.008
Number of impressive experience	0.235	0.31
Standard deviation score of paper tests	0.347	0.071

Figures 2**a** and 2**b** show relationships between the total scores of two chief factors and the number of experiences. "Impressive experience" is the one which was impressed or was performed eagerly but "simple experience" is not. A significant positive relationship between the two chief factors and "impressive experience" was found, but not for "simple experiences". Scholastic ability judged by paper tests, which was done to check the knowledge learnt in school, on the other hand, showed a significant positive relationship with "efforts and durability" but insignificant relationship with "independence and originality" (Figures 2**c**, 2**d** & Table 3)

The results can be summarized as follows:
(1) Two chief factors of the creative attitudes, that are "efforts and durability" and "independence and originality," are abstracted by factor analysis.
(2) There exists a significant positive relationship between the two chief factors and "impressive experience".
(3) Scholastic ability judged by paper tests shows a significant positive relationship with "efforts and durability" but insignificant relationship with "independence and originality".

Model

Imitating the impulse-momentum theorem in Newtonian mechanics, we construct a kinetic model to describe temporal changes of the creative attitudes $q_i(t)$. The creative attitudes have been extracted from the people who gave creative results, and then it will lead to creative results in the future. So, the creative attitudes can be considered to correspond to the vigor of growth of creativity. In Newtonian mechanics, "the vigor of motion" is expressed by "momentum". Then, "creative attitudes" can be supposed to correspond to the "momentum" in Newtonian mechanics. Also, something which can change the creative attitude can be supposed to correspond to the "force" in Newtonian mechanics. According to Newtonian mechanics, time derivative of the momentum-like quantity $q_i(t)$ is proportional to the force. Therefore, the fundamental equation on the present modeling should be differential equation such as Newton's equation of motion.

Based on the results obtained by statistical investigation, the changes of the creative attitude are considered with the following assumptions, which are schematically shown in Figure 3.

1. Creative attitude can be expressed by a momentum-like vector ($\boldsymbol{q}(t)$) as a function of time (t), consisting of two components of "effort and durability" ($q_1(t)$) and "independence and originality" ($q_2(t)$).
2. Tests to check knowledge or fixed works, given as "extrinsic motivation", are regarded as an external force ($G(t)$) which increases the magnitude of "effort and durability".
3. Impressed experiences or enthusiastic experiences, on the other hand, are regarded as

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an internal force ($E(t)$) which increases the magnitude of both the "effort and durability" and "independence and originality".

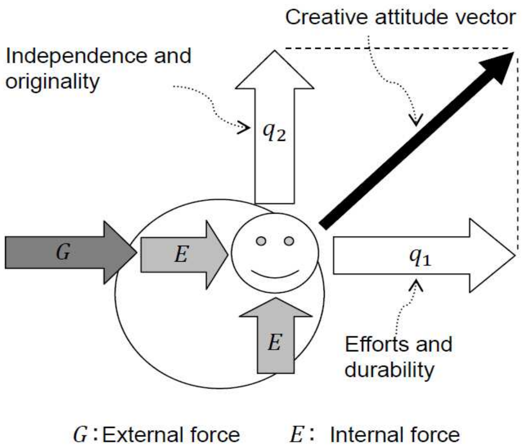

Figure 3. Creative attitude vector and related forces
Creative attitude vector consists of two components of "effort and durability" and "independence and originality", which are extracted by factor analysis. External force ($G(t)$), which can increase merely the magnitude of "effort and durability", is given by extrinsic motivations such like tests or assignments, which are aimed to reach the goal. On the other hand, internal force ($E(t)$), which is regarded as impressed experiences or emotional experiences, can increase both the magnitude of "effort and durability" and "independence and originality".

Thus "creative attitude vector" is expressed as $\boldsymbol{q}(t) = q_1(t)\boldsymbol{e_1} + q_2(t)\boldsymbol{e_2}$. Where $\boldsymbol{e_1}$ and $\boldsymbol{e_2}$ are the basis vectors. So, in this model, creative attitudes are time dependent due to the effects of external force ($G(t)$) and internal force ($E(t)$).

Now, equations to describe a temporal change of the creative attitude are considered with the above assumptions. According to Newtonian mechanics, time derivative of the momentum-like quantity is proportional to the force. Thus, the fundamental equation on the model is

$$\frac{dq_i(t)}{dt} = K_i(t),$$

where $K_i(t)$ corresponds to the force to change $q_i(t)$ and it can be expressed by the following equations:

$$K_1(t) = a_1 E(t) + b_1 G(t) - \lambda_1 q_1(t),$$
$$K_2(t) = a_2 E(t) - \lambda_2 q_2(t).$$

In the right hand sides of $K_i(t)$, the higher order terms of $q_i(t)$ are ignored by the present linear approximation. Where, a_1, a_2 and b_1 are parameters related to the individuality. $-\lambda_1 q_1(t)$ and $-\lambda_2 q_2(t)$ are the terms of resistance such like various temptations in life, which weaken the creative attitudes. λ_i is personal susceptibility on the resistance. This resistance corresponds to the one of the air, which is proportional to velocity in Newtonian mechanics. The form of this resistance was introduced by Newton in "Principia".

We assume that the value of external force $G(t)$ is constant as a first approximation, since "extrinsic motivation" is usually not time-dependent within a fixed period of time. On the other hand, the internal force $E(t)$ is considered to be impulse as impressive experiences shake the heart shockingly, which may induce a sudden appearance of an idea. Here we adapt the Dirac delta function to show these effects, as mentioned above.

$$E(t) = E_0 + \sum_{j=1}^{n} E_j \delta(t - t_j),$$

where n is the number of impressive experiences, t_j are the times when the impressive experience is given, and E_j the absolute value. E_0 is an initial value of $E(t)$, and is set up as constant.

Results of calculation

By solving the equations to describe the temporal change on the creative attitude of "effort and durability" ($q_1(t)$) and "independence and originality" ($q_2(t)$), we obtain the following general solutions:

$$q_1(t) = q_1(0)e^{-\lambda_1 t} + \frac{a_1 E_0}{\lambda_1}(1 - e^{-\lambda_1 t})$$
$$+a_1 \sum_{j=1}^{n} E_j \theta(t - t_j) e^{-\lambda_1 (t - t_j)}$$
$$+b_1 e^{-\lambda_1 t} \int_0^t e^{\lambda_1 t'} G(t') dt',$$

$$q_2(t) = q_2(0)e^{-\lambda_2 t} + \frac{a_2 E_0}{\lambda_2}(1 - e^{-\lambda_2 t})$$
$$+a_2 \sum_{j=1}^{n} E_j \theta(t - t_j) e^{-\lambda_2 (t - t_j)}.$$

Behaviors of the attitude vector $\boldsymbol{q}(t) = q_1(t)\boldsymbol{e_1} + q_2(t)\boldsymbol{e_2}$ calculated in the various conditions of the forces G and E by the above equations are shown in Figure 4. In all calculations, we assume that the attitude vector takes an initial value $\boldsymbol{q}(0)$ at the time of $t = 0$.

Figure 4**a** shows the case of $G \neq 0, E = 0$. In this case, the creative attitude vector is tilting to the direction of "effort and durability" as no force exists to increase the creative attitude of "independence and originality".

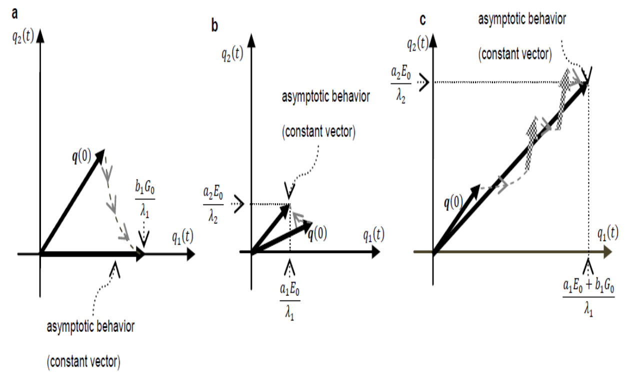

Figure 4. Changes of vector $\boldsymbol{q}(t)$ in three different conditions
a: $E(t) = 0, G(t) = G_0(= const.)$If no impressed experiences were given, a creative attitude vector will direct to q_1.
b: $E(t) = E_0(= const.), G(t) \cong 0$
(Setting of parameters : $q_1(0) > \frac{a_1E_0}{\lambda_1}, q_2(0) < \frac{a_2E_0}{\lambda_2}$.)
c: $E(t) = E_0 + \sum_{j=1}^{n} E_j\delta(t - t_j), G(t) = G_0(= const.)$
(Setting of parameters: $n = 2, q_1(0) < \frac{a_1E_0+b_1G_0}{\lambda_1}, q_2(0) < \frac{a_2E_0}{\lambda_2}$.) It corresponds to the case where both E and G are effective and the value of a_2E_0/λ_2 is high. This is an ideal case for the growth of a creative attitude vector.
 : Impact vector of the impressed experience ($n = 2$)

Figure 4**b** shows the case of $E \neq 0, G \neq 0$ and G is very weak. In this case, it is difficult to increase "effort and durability" as the force of G is too weak. Figure 4**c** shows the case that both the forces of E and G exist. The arrows of thin color indicate impressed experiences, which improve the creative attitude like a pulse, but the influences from the impressed experiences are decreasing gradually with increasing the time due to the resistance from the circumstances. If both of E and G are given in a balanced state, the growth of the creative attitude vector would be in a balanced state. We can see that "independence and originality" is increased by the impact force by the impressed experiences.

One should note that it is impossible to be $q_1(t) = 0, q_2(t) \neq 0$, as shown in Figure 5 under all the conditions, since "independence and originality" inevitably accompanies with "effort and durability", as described in the assumption 3.

Comparison with actuality

According to the correspondence with Newtonian mechanics, there should exists $\boldsymbol{x}(t)$ which satisfy $\boldsymbol{q}(\boldsymbol{t}) \propto d\boldsymbol{x}(t)/dt$ or $\boldsymbol{x}(t) \propto \int_0^t \boldsymbol{q}(t')dt'$, where $\boldsymbol{x}(t)$ corresponds to a position vector. If we trace a point of a position vector $\boldsymbol{x}(t)$ along the time, a trajectory can be drawn, as shown in

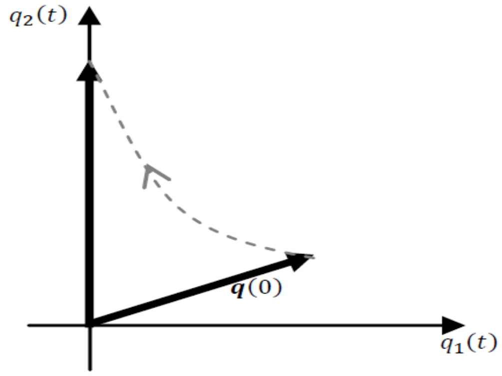

Figure 5. Impossible case in asymptotic behavior
When $E = 0$, $G(t) \neq 0$ it will be $q_1(t) \neq 0, q_2(t) = 0$ asymptotically. But it is impossible to be $q_1(t) = 0, q_2(t) \neq 0$ in any cases. (Refer to general solutions of equations of motion.) The "independence and originality" inevitably accompanies with the "effort and durability", as is described in the assumption 3.

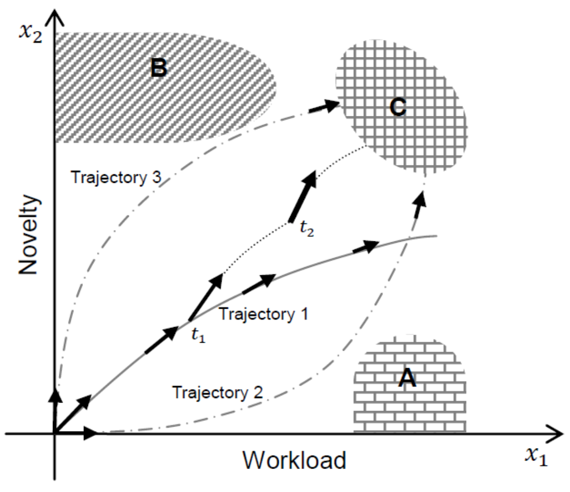

Figure 6. Trajectories of the position vector $x(t)$
$\boldsymbol{x}(t)$ satisfies $\boldsymbol{x}(t) \propto \int_0^t \boldsymbol{q}(t')dt'$ or $\boldsymbol{q}(\boldsymbol{t}) \propto d\boldsymbol{x}(t)/dt$, where $\boldsymbol{q}(t)$ is the creative attitude vector shown by arrows. The x_1-axis and the x_2-axis corresponds to "workload" and "novelty" respectively. t_1 and t_2 are the time when impressive experience shakes the heart shockingly. Area **A** is "slave-like region" where "independence and originality" is prohibited. Area **B** is the one where the arrival is impossible since "independence and originality" inevitably accompanies with "effort and durability" which induce some movement along the x_1-axis. Creative works are generally done in area **C**, where both the attitudes are highly activated and well-balanced.

Figure 6. The x_1-axis is integration of the momentum-like quantity of "effort and durability" and should correspond to the work done. Then, we define the name of x_1-axis as "workload". On the other hand, the x_2-axis is integration of the momentum-like quantity of "independence and originality" and it should reflect the degree of novelty. Then, we define the name of x_2-axis as "novelty". The accumulation ($\boldsymbol{x}(t)$) should reflect the results of creativity.

The area **A** in Figure 6 is the one where the component of "effort and durability" distinctly exists, but the component of "independence and originality" is very small. We call the area **A** as "slave-like region" since one reaches this area when "independence and originality" is prohibited. The area **B** in Figure 6, on the other hand, is the one where the arrival is difficult since "independence and originality" inevitably accompanies with "effort and durability". The success region of the creative work is the area **C** in Figure 6 where well-balanced values are taken. Impressive experiences are needed to enter the success region.

An example of the calculated result of trajectory moving to the area **C** is shown in Figure 6. Impressed experiences are given at times t_1 and t_2. The effects of impressed experiences on the trajectory are shown by dashed curves. The changes are suddenly given by the force of impressed experience, which remarkably increases "independence and originality" as mentioned in Figure 4. So, the trajectory deviates upwards. Also, we show trajectories 2 and 3, as other examples, moving to the area **C**. In these trajectories, we omitted the detailed changes induced by the impressed experiences. In trajectory 2, the component of "independence and originality" is small at first, but it increases with time due to the impressed experiences. In trajectory 3, the component of "independence and originality" is large at first, but it decreases with time due to the lack of the impressed experiences. These examples show that the internal force E of impressed experiences is necessary as well as external force G to give creative achievement.
Thus, our model reproduces actuality well.

Conclusion

In this paper, we proposed a mathematical model on the development of creativity. The present model relates not only to chaos theory but also to quantum theory, reflecting the complicated phenomena of creativity. To construct a specific model, we investigated the creative attitudes and the related factors based on the statistical surveys for lower secondary school students. We abstracted two chief factors of the creative attitudes; that are "efforts and durability" 1) and "independence and originality". Both of the two chief factors are shown to be influenced by impressed experiences which correspond to internal force $E(t)$. The internal force $E(t)$ contains discontinuous effects which relates to sudden appearance of idea, expressed by the Dirac delta function. Impressed experiences or emotional experiences causing the discontinuity should be a key factor on the development of creativity as described by Dunsmoor recently (Dunsmoor *et al.*, 2015).

The present linear approximate model must be the first step to build the predictive model of creativity. In other words, it is the first one of mathematical model in the complicated phenomena of creativity. It is interesting to remember that the Bohr model is approximate model of hydrogen in the quantum mechanics.

Note 1) One of the chief factors "effort and durability" in our model relates to "GRIT" that is defined as "perseverance and passion for long-term goals" and is clarified to be the important factor for the success in life (Duckworth, 2016; Duckworth and Gross, 2014).

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