Calculation of Energy Levels and B(E2) for ^{60}Ni and ^{60}Fe Isotopes by Using Nuclear Shell Model

Ali K. Hasan[1*], **Amer A. Salih**[2]

Abstract

In this study, the energy levels and the probability of electric transition B(E2) of the ^{60}Ni and ^{60}Fe nucleus were calculated using the OXBASH code within the fp shell and the use of the effective interaction f5pvh, gx1 and kb3. OXBASH is a computing code to perform a nuclear installation calculation based on a shell mode, Energy levels and the probability of electric transition is acceptable agreement with available experimental data.

Key Words: OXBASH Code, ^{60}Ni and ^{60}Fe Nucleus, Energy Levels, B(E2), fb Shell.

DOI Number: 10.14704/nq.2022.20.8.NQ44115 **NeuroQuantology 2022; 20(8): 1050-1055**

Introduction

Among the basic models proposed to describe the interaction between nucleons is the shell model, which is also called the independent particle model. The scientist (W. Elasser) was the first to present the idea of a closed nuclear shell in 1934, This model relies on the hypothesis that the nucleons rotate in their own orbits inside the nucleus and that their interaction with them is very weak and independently [1]. the nucleus consists of an inert core consisting of shells packed with neutrons and protons plus a certain number of additional nucleons called valence nucleons [2]. Some studies that were conducted on the properties of some nuclei showed that the nuclei are inside the nucleus they are linked in the form of orbits similar to the orbits of the electrons of the atom, and this has been called the shell structure or a level structure in which some shells are closed due to the stability of some nuclei [3]. The shell model is consistent with quantum mechanics as well as the Pauli exclusion principle. According to this model, an intermediate effort is taken that ensures the movement of the nucleons, Therefore he chooses a modified voltage which is the voltage of the harmonic oscillator [4]. The nucleus contains a large number of nucleons, each of which interacts strongly with all its neighboring nucleons, as this model treats the nucleus as a homogeneous system the shell model relied on a prototype called the independent particle model, in which it is assumed that the nucleons move freely inside the nucleus [5,6]. Several basic hypotheses were found for the shell model. The hypothesis of the existence of a nuclear potential in which all nucleons move, and which controls the movement of any nucleon individually is the voltage rate for all nucleons [7].

Theoretical Background

The region in which the calculations are performed represents the valence shell and its corresponding reaction coils. The shell model calculations of energy levels and the probability of electromagnetic transitions B (E2) can be performed. Nuclear crust model calculations have been successfully used to describe the low-level spectrum of proton and neutron nuclei located near closed crusts [8]. Since the closed- shell is treated as a vacuum because the nucleons do not change in the closed shell, and accordingly, the Hamiltonian, which controls the dynamics of valence nucleons, consists of two basic terms according to the following [9]:

Corresponding author: Ali K. Hasan
Address: [1*,2]Department of Physics, College of Education for Girls, University of Kufa, Iraq.
E-mail: [1*]alikh.alsinayyid@uokufa.edu.iq; [2]amer8adam8@gmail.com

$$H = \sum_{i} \varepsilon_i n_i + \sum_{ijkl} V_{ijkl} a_i^{\dagger} a_j^{\dagger} a_i a_j \ (1)$$

Where ε_i : the energy of a single particle, n_i: the number of nucleons in i, $a_i^{\dagger} a_j^{\dagger}$ represent the operators of construction, $a_i a_j$ represent the operators of demolition, and V_{ijkl} are the elements in the Two-Body Matrix Elements.

Nuclear Angular Momentum

Nuclear angular momentum is one of the important quantities, as it has an important role in the complex nuclear structure, as it affects all kinetic nuclear properties. Since the nucleus consists of nucleons (protons and neutrons), it moves in certain orbits around the center of mass [10]. Every single nucleon has a total the total angular momentum of a nucleus contains A nucleons, so the vector sum of the total angular momentum for all nucleons is

$$J = \sum_{i=1}^{A} j_i = \sum_{i=1}^{A} \ell_i + s_i \ (2)$$

$$j = l + s \ (3)$$

Where j represents the total angular momentum of the nucleus called spinning of the nucleus, and the orbital momentum, and s is total spin of nucleons. It is equal to [11].

$$J = \hbar\sqrt{J(J+1)} \ (4)$$

And self-spinning (s = ±1/2), and that the angular momentum is integers, and the nuclei whose mass number A is even, then their angular momentum is an integer, and for those whose mass number (A) is odd, the angular momentum is equitable Correct setting, The square of the angular momentum is [12].

$$P_J^2 = \hbar^2 J(J+1) \ (5)$$

J is called the total quantum number of angular momentum and sometimes it is called nuclear spin [13].

There are residual interactions between each of the particles in each case (secondary crusts) and that the remaining interactions are observed in the last secondary crust if it is partially filled. Its momentum is zero and its symmetry is positive, and the nucleons in the secondary shell give a coupling energy in which there are opposite values (Mj), where the magnetic quantum number takes one of the following possible values [14].

$$M_j = j, (j-1), \ldots, \frac{1}{2}, -\frac{1}{2}, \ldots, -(j-1), -j \ (6)$$

In some cases, nuclear properties are determined based on the number of nucleons outside the closed core, or on one particle (ie, a single nucleon) (J = j) or on two particles, and then ($J = j_1 + j_2$) [15]. Angular momentum plays an important role in classical mechanics, as it has a fundamental role in describing the molecular spin, the motion of electrons in atoms, and the motion of nucleons [16]. Therefore, the quantum theory of angular momentum is a prerequisite for the study of molecular, atomic and nuclear systems. The effect of angular momentum $\hat{J}$ can be known through ($\hat{J}_x$, $\hat{J}_y$, $\hat{J}_z$) which can be described by the following correlations [17]:

$$\left.\begin{array}{l} [\hat{J}_x , \hat{J}_y] = i\hbar \ \hat{J}_z \\ [\hat{J}_y , \hat{J}_z] = i\hbar \ {}_x\hat{J} \\ [\hat{J}_z , \hat{J}_x] = i\hbar \ {}_y\hat{J} \end{array}\right] \quad (7)$$

The effect of angular momentum for a single particle j is subject to the following exchange relationship [18]:

$$\underline{j} \times \underline{j} = i\underline{j}$$

If we assume that Ψ_{jm} is an eigenfunction of the angular operator of a single nickel, and that the properties are as follows [19]:

$$J^2 \Psi_{jm} = j(j+1)\hbar^2 \Psi_{jm} \ (8)$$

$$j_z \Psi_{jm} = m\Psi_{jm} \ (9)$$

j(j+1): It is the eigenvalue of the square of the angular momentum operator measured in units $\hbar$.

m: The eigenvalue of the angular momentum component in the direction of the z-axis is unit $\hbar$ and has a group between ($-j \le m \le j$).

The equations for the eigenvalues of the angular momentum operator can be written as [20]:

$$j_+ \Psi_{jm} = \sqrt{j(j+1) - m(m+1)}\Psi_{j,m+1} \ (10)$$

$$j_- \Psi_{jm} = \sqrt{j(j+1) - m(m-1)}\Psi_{j,m-1} \ (11)$$

In order to find out the states of the angular momentum of a system consisting of a number of particles from (J) to the (z) total component of the angular momentum of a wave function of a number of particles [21]:

$$\varphi_{JM} = \varphi_{j_1 m_1}(1)\varphi_{j_2 m_2}(2) \ldots \ldots\ldots \varphi_{j_n m_n}(n) \ (12)$$

$$M = \sum_{i} m_i \quad (13)$$

(M) is the sum of the components of angular momentum.

$$J_Z \varphi_M = (j_z)_i \varphi_M \quad (14)$$

$$J_Z \varphi_M = m_i \varphi_M \quad (15)$$

$$J_Z \varphi_M = M\varphi_M \quad (16)$$

Theorems by which the values of the total angular momentum of the nucleus can be calculated are [22].

1. When there are two identical nucleons in the same orbit for the single particle, i.e. ($j_1 = j_2$) and (j they are half integers), then their values are even, i.e.

$$J = 0, 2, 4, \ldots\ldots (2j - 1) \qquad (17)$$

2. When there are two nucleons, one in the (initial) statej_1 and the other in the (final) statej_2 i.e. ($j_2 \neq j_1$), the values of the total angular momentum are calculated from the following relationship:

$$J = j_1 + j_2, j_1 + j_2 - 1, \ldots\ldots |j_1 - j_2| \qquad (18)$$

3. When identical nucleons are in the orbit of the same single particle and (n>2) since n is the number of particles outside the closed shell, the cases of angular momentum are found in the following formulas [23]:

1) The greatest angular momentum I_M m arises from the order j^n and be.

$$J_M = n\left\{j - \frac{(n-1)}{2}\right\} \qquad (19)$$

2) There is no state of this arrangement j^n is given by

$$J = J_M - 1 \qquad (20)$$

3) There is only one state in the arrangement j^n is given by

$$J = J_M - 2 \qquad (21)$$

Where momentum, energy and symmetry are the quantities that can be measured according to the wave function for a particular case, therefore scientific experiments make more efforts to measure angular momentum and symmetry than they do on measuring any other quantities because they are considered one of the most quantities that help researchers in developing theories [24].

Energy Levels

1. ^{66}Ni Nucleus

By applying the shell model and using the OXBASH code, the ground state of the ^{60}Ni nucleus is a closed nucleus ^{56}Ni with four neutrons distributed in three orbits (($1f_{5/2}$, $2p_{3/2}$ and $2p_{1/2}$)) representing the model space. The expected states are formed through the presence of these nucleons in a model space, and to calculate their energy levels, the interaction (f5pvh) was used. Table (1) shows the comparison between the theoretical and experimental values available for the nucleus of ^{60}Ni [25].

Table 1. Is a comparison between the theoretical values of the energy levels relative to the ground state of the nickel ^{60}Ni nucleus with the experimental results using the f5pvh interaction

Theoretical values of f5pvh interaction		Experimental values	
J^+	E (MeV)	E (MeV)	J^π
0_1	0.0000	0.0000	0+
2_1	1.4040	1.3325	2+
2_2	2.1510	2.1586	2+
0_2	2.2510	2.2848	0+
4_1	2.4220	2.5058	4+
3_1	2.6460	2.6261	3+
4_2	2.8860	3.1199	4+
2_3	2.9250	3.1237	2+
2_4	3.0280	3.2692	2+
1_1	3.0800	3.1939	1+
3_2	3.0820	3.1860	(3+)
2_5	3.1010	3.3931	2+
1_2	3.1340	----	----
4_3	3.3630	3.6712	4+
0_3	3.5450	3.3178	0+
2_6	3.5780	3.7344	2+
3_3	3.6010	3.6195	3+
5_1	3.6720	----	----
0_4	3.7160	3.5877	0+
4_4	3.7490	3.7029	4+
6_1	3.8250	----	----
2_7	3.8290	3.8711	2+
3_4	3.8520	3.9252	2+,3+
1_3	3.8940	3.7980	1
2_8	3.8950	3.8874	2+
1_4	4.0510	4.0199	1+
4_5	4.0850	----	----
3_5	4.2000	----	----
2_9	4.2620	4.0064	2+
4_6	4.3770	----	----
1_5	4.4300	----	----
5_2	4.4940	----	----
3_6	4.5140	----	----
2_{10}	4.5200	----	----
0_5	4.5610	----	----
1_6	4.7100	----	----
0_6	4.8480	----	----
4_7	4.9050	----	----
5_3	4.9090	----	----
6_2	4.9180	----	----
3_7	5.0180	----	----
4_8	5.0850	----	----
3_8	5.3440	----	----
3_9	5.6300	----	----
6_3	5.6460	----	----
4_9	5.6690	----	----
1_7	5.6770	----	----
5_4	5.7550	----	----
0_7	5.7620	----	----
3_{10}	5.7750	----	----
4_{10}	5.8570	----	----
5_5	5.9810	----	----
0_8	6.1050	----	----
1_8	6.1130	----	----
1_9	6.2150	----	----

1_{10}	6.7320	----	----
0_{9}	7.3920	----	----
5_{6}	7.5510	----	----

Through Table (1) and comparing the results of this analogue, it is clear that:

1. A match was obtained for the ground state of the level 0^{+}_{1} when compared with the confirmed experimental value.
2. Good agreement was obtained at the levels (2_1, 2_2, 0_2, 4_1, 3_1, 4_2, 2_3, 2_4, 1_1 and 3_2). with energy values (1.4040, 2.1510, 2.2510, 2.4220, 2.6460, 2.8860, 2.9250, 3.0280, 3.0800, 3.0820) respectively in MeV, with the corresponding experimental values.
3. The total angular momentum and parity of the level (3+) with an uncertain value of (3.1860 MeV) were confirmed in practice by a close match of the theoretical values.
4. We found that there are theoretical energy levels with total angular momentum and parity, such as (1_2,5_1, 6_1, 4_5 and 3_5) that were not matched by any experimental value.
5. Found (29) new theoretical energy levels above the highest practical energy level (4.2620 MeV), were arrived at the energy (7.5510 MeV).

2. ^{60}Fe Nucleus

By applying the shell model and using the OXBASH code, the ground state of the ^{60}Fe nucleus is a closed nucleus ^{40}Ca with 20 nucleons distributed in fp shell. The expected states are formed through the presence of these nucleons in a model space, and to calculate their energy levels, the interactions (kb3, and gx1) were used. Table (2) shows the comparison between the theoretical and experimental values available for the nucleus of ^{60}Fe [25].

Table 2. Is a comparison between the theoretical values of the energy levels relative to the ground state of the Iron ^{60}Fe nucleus with the experimental results using the kb3, and gx1 interactions

Theoretical values of E(MeV)			**Experimental values**	
J^{+}	**kb3**	**gx1**	**E(MeV)**	J^{π}
0_1	0.0000	0.0000	0.0000	0+
2_1	0.8290	1.1520	0.8238	2+
0_2	1.5190	2.1510	1.9740	0+
4_1	1.9600	2.3420	2.1146	4+
2_2	1.9910	2.4320	2.6729	2+
3_1	2.2020	2.7230	2.7927	3+,4+
2_3	2.2670	2.7590	3.0389	2+
4_2	2.6180	2.8310	3.0720	4+
0_3	2.6240	3.1760	2.3562	0+
2_4	2.6550	2.7760	2.7559	2+
1_1	2.7620	3.0100	----	----
3_2	2.8200	2.9500	----	----
2_5	2.8440	3.2930	----	----
1_2	2.9160	3.4540	----	----
6_1	2.9830	3.1880	----	----
2_6	3.0410	3.7620	----	----
4_3	3.1000	3.1260	3.4986	(4+)
3_3	3.1970	3.4400	----	----
3_4	3.2400	3.7120	----	----
0_4	3.2690	3.3500	3.6980	0+
4_4	3.3130	3.5840	3.5200	(4+)
1_3	3.3590	3.7130	----	----
3_5	3.4730	4.0460	----	----
5_1	3.5270	3.8810	----	----
6_2	3.5480	4.1650	3.5201	6+
1_4	3.5760	4.0420	----	----
4_5	3.6350	3.9630	----	----
2_7	3.6940	3.9120	3.6350	2+
5_2	3.7550	4.2220	----	----
3_6	3.7830	4.3560	----	----
2_8	3.7860	4.0310	4.1760	2+
4_6	3.8980	4.1390	----	----
6_3	3.9000	4.2770	3.9045	(6+)
2_9	3.9050	4.2480	3.9299	2+
0_5	3.9320	3.8320	----	----
4_7	3.9420	4.7130	----	----
1_5	4.0150	4.2860	----	----
2_{10}	4.0290	4.4380	----	----
3_7	4.0880	4.6290	----	----
4_8	4.1560	4.8910	----	----
6_4	4.1730	4.7730	----	----
5_3	4.2050	4.4710	----	----
5_4	4.3350	4.8380	----	----
0_6	4.4030	4.5150	----	----
7_1	4.4090	5.1190	----	----
1_6	4.4520	4.3990	----	----
4_9	4.4900	4.9650	----	----
3_8	4.5110	4.8010	----	----
4_{10}	4.5510	4.9970	----	----
3_9	4.5580	4.8230	----	----
7_2	4.5640	5.1980	----	----
5_5	4.5890	4.9900	----	----
3_{10}	4.5910	4.8630	----	----
5_6	4.6510	5.2090	----	----
1_7	4.7070	4.7560	----	----

6_5	4.7190	5.0770	----	----
8_1	4.7230	4.8930	----	----
0_7	4.7820	5.0330	----	----
1_8	4.8400	5.0230	----	----
6_6	4.8690	5.2990	----	----
5_7	4.9370	5.3750	----	----
1_9	4.9680	5.0520	----	----
6_7	4.9990	5.4980	----	----
5_8	5.0210	5.4480	----	----
1_{10}	5.0940	5.2750	----	----
6_8	5.1310	5.5860	----	----
5_9	5.1510	5.6200	----	----
7_3	5.1510	5.4540	----	----
8_2	5.1730	5.5950	----	----
0_8	5.1750	5.3150	----	----
6_9	5.2480	5.6600	----	----
8_3	5.3560	5.7770	----	----
6_{10}	5.3670	5.7380	----	----
7_4	5.4440	5.5190	----	----
5_{10}	5.4450	5.8480	----	----
7_5	5.6370	5.7450	----	----
8_4	5.7400	5.9920	----	----
8_5	5.7660	6.0930	----	----
9_1	5.7970	5.6240	----	----
7_6	5.8290	5.9150	----	----
0_9	5.8580	5.7050	----	----
7_7	5.9170	5.9210	----	----
0_{10}	5.9740	6.1280	----	----
8_6	6.0550	6.3190	----	----
7_8	6.0700	6.1880	----	----
9_2	6.0820	6.2180	----	----
7_9	6.2500	6.3100	----	----
7_{10}	6.3500	6.6020	----	----
8_7	6.5130	6.4660	----	----
10_1	6.5620	6.2660	----	----
9_3	6.5780	6.6150	----	----
8_8	6.6040	6.5990	----	----
9_4	6.7270	6.9620	----	----
10_2	6.7870	6.6480	----	----
8_9	6.8780	6.6930	----	----
8_{10}	7.0200	6.9380	----	----
9_5	7.2310	7.2190	----	----
10_3	7.2580	7.3480	----	----
9_6	7.3540	7.3070	----	----
9_7	7.7400	7.6680	----	----
10_4	7.8140	7.8720	----	----
9_8	7.8290	7.7540	----	----
9_9	7.9260	7.8940	----	----
11_1	8.1130	7.8980	----	----
9_{10}	8.1270	8.0730	----	----
10_5	8.1830	8.1030	----	----
10_6	8.2590	8.1930	----	----
11_2	8.6340	8.4520	----	----
10_7	8.7380	8.2920	----	----

Through Table (2) and comparing the results of this analogue, it is clear that:

1. A match was obtained for the ground state of the level 0^+_1 for both interactions when compared with the confirmed experimental value.
2. Good agreement was obtained at the levels 2_1, 0_2, and 4_1. with energy values (0.8290, 1.1520), (1.5190, 2.1510), and (1.9600, 2.3420) MeV, respectively for both interactions, with the corresponding experimental values.
3. The total angular momentum and parity of the level (4+), (4+), and (6+) with an uncertain value of (3.4986, 3.5200, and 3.9045) MeV for both interactions, respectively, were confirmed in practice by a close match of the theoretical values.
4. We found that there are theoretical energy levels with total angular momentum and parity, such as (1_1, 3_2, 2_5, 1_2, 6_1, 2_6, 3_3, 3_4, 1_3, 3_5, 5_1, and 1_4) that were not matched by any experimental value.
5. Found (75) new theoretical energy levels above the highest practical energy level (3.9299 MeV), were arrived at the energy (8.7380 MeV).

Reduced Electric Quadrupole Transition Probability B(E2)

1. ^{60}Ni Nucleus

A ^{60}Ni nucleus was selected for the F5P model space, and table (3) shows the B(E2) values of ^{60}Ni isotope obtained for the f5pvh interaction with effective charges (e_p = 1.45 e and e_n = 1.45 e) in comparison with the available experimental values.

Table 3. Comparison of B (E2) values for ^{60}Ni nucleus with experimental values are available in e^{2}fm^4 units

Ji	**→**	**Jf**	**f5pvh B (E2) (e^{2}fm^4)**	**Exp. B (E2) (e^{2}fm^4)**
2^+_1	→	0^+_1	179.2	182.779
2^+_2	→	2^+_1	204.3	976.681
2^+_2	→	0^+_1	0.02566	3.070
4^+_1	→	2^+_2	29.67	2.651
4^+_1	→	2^+_1	70.84	76.739

3^+_1	→	2^+_2	218.4	10.604
3^+_1	→	2^+_1	0.01741	78.134
4^+_2	→	3^+_1	4.544	418.578
4^+_2	→	2^+_1	120.4	125.573
2^+_3	→	2^+_1	7.152	4.744

2. ^{60}Fe Nucleus

A ^{60}Fe nucleus was selected for the fp model space, and Table (4) shows the B(E2) values of ^{60}Fe isotope obtained for the kb3, and gx_1 interactions with effective charges (e_p = 1.6 e and e_n = 1.6 e) in comparison with the available experimental values.

Table 4. Comparison of B (E2) values for ^{60}Fe nucleus with experimental values are available in e^{2}fm^4 units

J_i^+	→	J_f^+	Theoretical B(E2)		Exp. B(E2)
			kb3	gx1	(e^{2}fm^4)
2_1	→	0_1	202.95	180.15	201.86
4_1	→	2_1	203.7	180.3	114.57
2_2	→	0_1	8.6115	7.1475	----
2_2	→	2_1	95.7	85.365	----
2_2	→	4_1	0.110385	23.385	----
4_2	→	2_1	22.065	1.39065	----
4_2	→	4_1	67.23	13.299	----
4_2	→	2_2	133.95	14.73	----
6_1	→	4_1	10.707	5.8545	----
6_1	→	4_2	4.746	2.5245	----

References

A.E. Green, D.L. Sellin, and A.S. Zachor, "Analytic independent-particle model for atoms" Physical Review, 184 (1), pp: 1-9, (1969).

A.B. Wieckowski, "Periodic table of nuclides based on the nuclear shell model", Romanian Journal of Physics 64, p: 303 (2019).

O. Sorlin, "Shell evolutions and nuclear forces", EPJ Web of Conferences 66, p: 01016 (2014).

M.G. Mayer, "On closed shells in nuclei", Physical Review 74 (3), pp: 235-239 (1948).

D. Radulov" Investigating the nuclear structure of the neutron rich odd mass Fe isotopes, in the beta-decay of their parent-Mn", Ph.D Thesis, University of Complutense, Madrid (2014).

S.Y. Hasan, "Study the properties of energy levels for an isotope 146Ba", M.Sc. Thesis, Babylon University (2000).

M.A. Preston, "Physics of the Nucleus", Addison-Wesley publishing company, Inc. London (1962).

M.H. Macfarlane and J.B. French, “Stripping reactions and the structure of light and intermediate nuclei", Reviews of Modern Physics 32 (3), p: 567 (1960).

J. lilly, "Nuclear Physics: Physics Principles and Applications", John Wiley & Sons (2013).

A.K. Muneeb,"Nuclear Physics", House of Books for Printing and Publishing Mosul (1996).

J.P. Elliott and A.M. Lane, "The nuclear shell-model", In Structure of Atomic Nuclei, Bau der Atomkerne, Springer, Berlin, Heidelberg 8 (39), pp: 241-410 (1957).

S.S.M. Wong, "Introductory Nuclear Physics", 2nd Ed., Willey-VCH Verlag GmbH & Co. KGaA, Weinheim (2004).

H.A. Enge, "Introduction to Nuclear Physics", Addison-Wesley (1983).

J.L. Basdevant, J. Rich and M. Spiro, "Fundamentals in Nuclear Physics", Springer-Verlay, New York (2005).

P.W.M. Glaudemans, G. Wiechers and P.J. Brussaard. "Shell model calculations on energy levels in the 2s1/2 1d3/2 shell (II)", Nuclear Physics 56, pp: 548-568 (1964).

A. Frank, P. Isacker, J. Jolie and P. Van Isacker, "Symmetries in Atomic Nuclei", Springer Science, Business Media, New York (2009).

C. Thibault, R. Klapisch, C. Rigaud, A.M. Poskanzer, R. Prieels, L. Lessard and W. Reisdorf," Direct measurement of the masses of 11Li and 26-32Na with an on-line mass spectrometer", Phys. Rev. C 12 (2) pp: 644-657 (1975).

K. Abdullah, "Fundamentals in Nuclear Physics", Noor Publishing, pp. 259 (2017).

K.T. Bainbridge, "The equivalence of mass and energy" Physical Review, 44(2), pp: 123 (1933).

C. Paola, "Introduction to Applied Nuclear Physics", Massachusetts Institute of Technology, Nuclear Science and Engineering Department, (2012).

J.M. Arias and M. Lozano, "The Hispalensis Lectures on Nuclear Physics", Springer Science & Business Media (2004).

M. Honma, T. Otsuka, B. A. Brown and T. Mizusaki," Shell-model description of neutron-rich pf-shell nuclei with a new effective interaction GXPF1", The European Physical Journal A-Hadrons and Nuclei 25(1), pp: 499-502 (2005).

W.F. Yong, L. Lei and L.Y. An, "Structure of the even-even 78–84Kr isotopes within SD-pair shell model", Chinese. Phys. C 34 (2), p: 270 (2010).

J.N. Gu, A. Vitturi, C.H. Zhang, P. Guazzoni, L. Zetta, G. Graw, M. Jaskola and G. Staudt," Homologous states and the structure of nuclei in the lead region", Physical Review C. 55 (5), pp: 2395-2406 (1997).

E. Browne and J.K. Tuli, "Nuclear data sheets for A= 60", Nuclear Data Sheets, 114(12), pp: 1849-2022 (2013).