Coexistence Of The Fuzzy Topologies Produced by Strong Fuzzy Uniformity and Fuzzy Proximity

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Abstract
We define the term "fuzzy proximity" and examine some of its characteristics. In specifically, we demonstrate how a fuzzy closeness on set X naturally results in a fuzzy topology on the same set. In addition, the idea of a strong Fuzzy uniformity is proposed, which is a slightly modified version of Ying's idea of a Fuzzy uniformity. There are certain proven relationships between strong fuzzy uniformities, fuzzy proximities, and corresponding fuzzy topologies. In particular, we demonstrate the coexistence of the fuzzy topologies produced by strong fuzzy uniformity and fuzzy proximity.

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Introduction

In order to start the study of the so-called Fuzzy topology and fundamentally develop topology in the framework of fuzzy sets from a completely different angle, Ying [3] adopted the semantic technique of continuous valued logic. In a nutshell, a fuzzy topology on a set X assigns a degree of begin open—a state other than being absolutely open or not—to every crisp subset of X. Furthermore, Ying defined several fundamental characteristics of fuzzy uniform spaces in 1993 and popularised the idea of them ([4]). We present and explore the idea of fuzzy proximity within the context of fuzzy topology. As a result of the fuzzy proximity, a fuzzy topology is also introduced (Theorem 2.1). We offer a type of fuzzy uniformity that is stronger than the Fuzzy uniformity due to Ying and satisfies that the -level of it is a classical uniformity because the -level of the Fuzzy uniformity due to Ying may not be a classical uniformity (Counterexample 3.1) (Theorem 3.1). A Fuzzy topology that is caused by a strong Fuzzy homogeneity is also introduced (Theorem 3.2). There are certain proven relationships between strong fuzzy uniformities, fuzzy proximities, and corresponding fuzzy topologies. We specifically demonstrate the coincident existence of the fuzzy topology given by fuzzy closeness and the fuzzy topology induced by strong fuzzy homogeneity (Theorem3.5).

Preliminaries

In this section we present the fuzzy logical and corresponding set theoretical notations due to Ying [3]. For any formulae ϕ, the symbol $[\phi]$ means the truth value of ϕ, where the set of truth values is the unit interval [0, 1]. We write
$\models \phi$ if $[\phi] = 1$ for any interpretation. The original formulae of fuzzy logical and corresponding set theoretical notations are:
(1) (a) $[\alpha] = \alpha(\alpha \in [0, 1])$; (b) $[\phi \wedge \psi] := \min([\phi], [\psi])$; (c) $[\phi \to \psi] := \min(1, 1 - [\phi] + [\psi])$
(2) If $\tilde{A} \in F(X)$, then $[x \in \tilde{A}] := \tilde{A}(x)$.

(3) If X is the universe of discourse, $[\forall x\phi(x)] := \inf_{x\in X}[\phi(x)]$. In addition the following derived formulae are given,
(1) $[\neg\phi] := [\phi \to 0] := 1 - [\phi]$;
(2) $[\phi \vee \psi] := [\neg(\neg\phi \wedge \neg\psi)] := \max([\phi], [\psi])$;
(3) $[\phi \leftrightarrow \psi] := [\phi \to \psi] \wedge [\psi \to \phi]$;
(4) $[\exists x\phi(x)] := [\neg\forall x\neg\phi(x)] := \sup_{x\in X} [\phi(x)]$;
(5) if $\tilde{A}, \tilde{B} \in F(X)$, then
(a) $[\tilde{A} \subseteq \tilde{B}] := [\forall x(x \in \tilde{A} \to x \in \tilde{B})] := \inf_{x\in X} \min(1, 1 - \tilde{A}(x) + \tilde{B}(x))$;
(b) $[\tilde{A} \equiv \tilde{B}] := [\tilde{A} \subseteq \tilde{B}] \wedge [\tilde{B} \subseteq \tilde{A}]$,
where F(X) is the family of all fuzzy sets in X.

We do often not distinguish the connectives and their truth value functions and state strictly our results on formalization as Ying did ([3, 4]). We give now the following definitions and results in Fuzzy topology which are useful in the rest of the present paper.

Definition 1.1.([3]) Let X be a universe of discourse, P (X) the family of all subsets of X and $\tau \in F(P(X))$, i.e., $\tau : P(X) \to [0, 1]$ satisfy the following conditions:

(1) $\models X \in \tau$, $\models \varphi \in \tau$;
for any A, B, $\models (A \in \tau) \wedge (B \in \tau) \to A \cap B \in \tau$;
for any $\{A_\lambda : \lambda \in \Lambda\}$, $\models \forall\lambda(\lambda \in \Lambda \to A_\lambda \in \tau) \to (\bigcup_{\lambda\in\Lambda} A_\lambda) \in \tau$.
Then τ is called a Fuzzy topology and (X, τ) is a Fuzzy topological spaces.

Remark 1.1.([3]) The conditions in Definiton 1.1 may be rewritten respec- tively as follows:
(1) $\tau(X) = 1$, $\tau(\varphi) = 1$;
(2) for any A, B, $\tau(A \cap B) \geq \tau(A) \wedge \tau(B)$;
(3) for any $\{A_\lambda : \lambda \in \Lambda\}$, $\tau(\bigcup_{\lambda\in\Lambda} A_\lambda) \geq \bigwedge_{\lambda\in\Lambda}\tau(A_\lambda)$.

Definition 1.2.([3]) The family of Fuzzy closed sets, denoted by $F \in F(P(X))$, is defined as $A \in F := X \sim A \in \tau$, where $X \sim A$ is the complement of A.

Definition 1.3.([3]) A fuzzy set $\tilde{A} \in F(X)$ is called normal if there exists $x \in X$ such that $\tilde{A}(x) = 1$.

Definition 1.4.([4]) Let X be a set and $u \in FN(P(X \times X))$, i.e., $u : P(X \times X) \to [0, 1]$ and normal. If for any U , $V \subseteq X \times X$, (FU1) $\models (U \in u) \to (\Delta \subseteq U)$;
(FU2) $\models (U \in u) \to (U^{-1} \in u)$;
(FU3) $\models (U \in u) \to (\exists V)((V \in u) \wedge (V \circ V \subseteq U))$; (FU4) $\models (U \in u) \wedge (V \in u) \to (U \cap V \in u)$;
(FU5) $\models (U \in u) \wedge (U \subseteq V) \to (V \in u)$,
then u is called a Fuzzy uniformity and (X, u) is called a Fuzzy uniform space.

Fuzzy Proximity Space

In this section the concept of Fuzzy proximity spaces is established and some of its properties are discussed. Also, a Fuzzy topology induced by the Fuzzy proximity is introduced.

Definition 2.1. Let X be a set and $\delta \in F(P(X) \times P(X))$, i.e., $\delta : P(X) \times P(X) \to [0, 1]$. If for any A, B, $C \in P(X)$, the following axioms are satisfied:
(FP1) $\models \neg(X, \varphi) \in \delta$;
(FP2) $\models (A, B) \in \delta \leftrightarrow (B, A) \in \delta$;
(FP3) $\models (A, B \cup C) \in \delta \leftrightarrow (A, B) \in \delta \vee (A, C) \in \delta$; (FP4) for every A, $B \subseteq X$ there exists $C \subseteq X$ such that
$\models ((A, C) \in \delta \vee (B, X \sim C) \in \delta) \to (A, B) \in \delta$;
(FP5) $\models \{x\} \equiv \{y\} \leftrightarrow (\{x\}, \{y\}) \in \delta$,

then δ is called a Fuzzy proximity on X and (X, δ) is called a Fuzzy proximity space.

Theorem 2.1. Let (X, δ) be a Fuzzy proximity space. Then, we have

(1) |= (A, B) ∈ δ ∧ B ⊆ C → (A, C) ∈ δ;

(2) |= (A ∩ B) /= φ → (A, B) ∈ δ;

(3) |= ¬δ(A, φ).

Proof. (1) If [B ⊆ C] = 0, then the result holds. Suppose that [B ⊆ C] = 1.
Then, we have δ(A, C) = δ(A, B ∪ C) = δ(A, B) ∨ δ(A, C) ≥ δ(A, B).

(2) If [A ∩ B /= φ] = 0, then the result holds. Suppose [A ∩ B φ] = 1. Then there exists x ∈ A ∩ B. Thus, we obtain 1 = δ({x}, {x}) = δ({x}, {x}) ∧ [{x}⊆ A] ≤ δ({x}, A) = δ(A, {x}) = δ(A, {x}) ∧ [{x}⊆ B] = δ(A, B).

(3) δ(A, φ) = δ(φ, A) = δ(φ, A) ∧ [A ⊆ X] ≤ δ(φ, X) = 0. Hence, [¬(A, φ) ∈ δ] = 1.

Proposition 2.1. For every α ∈ (0, 1], δa is a proximity on X, where δα is the α-level of δ, i.e., δα = {(A, B) : δ(A, B) ≥ α}.

Proof. Let α be a fixed value in (0,1].

(P1) Since by (FP1) δ(X, φ) = 0, then we have δ(X, φ) < α. So, (X, φ) δα.

(P2) Suppose (A, B) ∈ δα. Then δ(A, B) ≥ α and by (FP2) δ(A, B) = δ(B, A) ≥ α. Hence, we obtain (B, A) ∈ δα.

(P3) Using (FP3) we have (A, B ∪ C) ∈ δα if and only if δ(A, B ∪ C) ≥ α if and only if δ(A, B) ∨ δ(A, C) ≥ α if and only if δ(A, B) ≥ α or δ(A, C) ≥ α if and only if (A, B) ∈ δα or (A, C) ∈ δα.

(P4) Let (A, B) /∈ δα. Then we have δ(A, B) < α and by (FP4) there exists C ∈ P (X) such that δ(A, B) ≥ δ(A, C) ∨ δ(B, X ∼ C). Hence, δ(A, C) ∨

δ(B, X ∼ C) < α which implies that δ(A, C) < α and δ(B, X ∼ C) < α. So, (A, C) /∈ δα and (B, X ∼ C) /∈ δα.

(P5) Suppose x = y. Then we have [{x} ≡ {y}] = 1. So, by (FP5) we have δ({x}, {y}) = 1. Hence, ({x}, {y}) ∈ δc.

Definition 2.2. Let (X, δ) be a Fuzzy proximity space. For each α ∈ (0, 1] and A ⊆ X, we define the interior operation induced by δα, denoted by intδα :P (X) → P (X), as follows:

$$\mathrm{int}_{\delta\alpha}(A) = \bigcup_{B\in P(X),(B,X\sim A)*=\delta a} B,$$

Proposition 2.2. For every α ∈ (0, 1], the family τδα = {A : A ⊆ X and intδα (A) = A} is a topology on X.

Proof. Let α be a fixed value in (0,1].

Since intδα (X)= B∈P (X),(B,φ)∗= δα B = X and intδα (φ) = B∈P (X),(B,X)∗∈δα B = φ, then X ∈ τδα and φ ∈ τδα .

Let A, C ∈ τδα . Then, we obtain that

A∩C = intδα (A) ∩ intδα (C) = B∈P (X),(B,X∼A) δα B ∩ G∈P (X),(G,X∼C)∗∈δα G

= B∩G∈P (X),(B∩G,X∼A∩C) δα (B ∩G)= H∈P (X),(H,X∼A∩C) δα H = intδα(A∩C).

Hence, A ∩ C ∈ $\tau_{\delta\alpha}$.

Let {Aλ : λ ∈ Λ}⊆ τδα . Now, λ∈Λ Aλ = λ∈Λ intδα (Aλ) ⊆ intδα (λ∈Λ Aλ), because intδα is monotone (Indeed, If A ⊆ C, then $int_{\delta\alpha}$ (A) = $\bigcup_{B\in P(X),(B,X\sim A)*\in\delta_\alpha}$ B ⊆ $\bigcup_{B\in P(X),(B,X\sim C)*\in\delta_\alpha}$ B = intδα(C)). Also, intδα($\bigcup_{\lambda\in\Lambda}$ Aλ) ⊆ $\bigcup_{\lambda\in\Lambda}$ Aλ because intδα(A)= $\bigcup_{B\in P(X),(B,X\sim A)*\in\delta_\alpha}$ B = $\bigcup_{B\in P(X),B\cap(X\sim A)=\varphi}$ B = $\bigcup_{B\in P(X),B\subseteq A}$ B = A for any A ∈ P (X). Hence U $A_\lambda \in \Lambda_{\delta\alpha}$

α = τδ(A) ∧ τδ(B).

Theorem 2.2 Let {Aλ : λ ∈ Λ}⊆ P (X). Then we have

$$\tau\delta(\bigcup_{\lambda\in\Lambda} A\lambda) = \sup_{\alpha\in(0,1],\ \bigcup_{\lambda\in\Lambda} A_\lambda\in\tau_{\delta_\alpha}} \alpha \geq \sup_{\alpha\in(0,1],A_\lambda\in\tau_{\delta_\alpha},\lambda\in\Lambda} \alpha = \inf_{\lambda\in\Lambda} \sup_{\alpha\in(0,1],A_\lambda\in\tau_{\delta_\alpha}} \alpha$$

$$= \inf_{\lambda\in\Lambda} \tau\delta(A\lambda).$$

Definition 2.3. Let (X, δ) and (Y, δ∗) be a Fuzzy proximity spaces. A unary fuzzy predicate PC ∈ F(Y X), i.e., PC : Y X → [0, 1] and Y X is the set of all functions from X to Y , is called a Fuzzy proximal continuity and is defined as follows: f ∈ PC := ∀(A, B)((A, B) ∈ δ → (f (A),f (B)) ∈ δ∗). Intuitively, the degree to which f is proximal continuous is

$$PC(f) = \inf_{(A,B)\subseteq P(X)\times P(X)} \min(1, 1 - \delta(A, B) + \delta*(f(A),f(B))).$$

Theorem 2.2.

(1) |= ∀f (f ∈ PC → ∀α(α ∈ (0, 1] → (1 − α) ∨ Hα(f))), where

$$H\alpha(f) = \begin{cases} 1 & \text{if } f : (X, \delta_\alpha) \to (Y, \delta^*_{PC(f)-(1-\alpha)}) \text{ is proximal continuous when} \\ 0 & \text{otherwise} \end{cases}$$

(2) if PC(f) = 1, then |= ∀f (f ∈ PC ↔ ∀α(α ∈ (0, 1] → (1 − α) ∨ Hα(f)));

(3) |= ∀f (∀α(α ∈ (0, 1] → (1 − α) ∧ Hα(f)) → f ∈ PC).

Proof. (1) Suppose PC(f) = γ. Then for each (A, B) ∈ P (X) × P (X) such that (A, B) ∈ δa we have, 1 − δ(A, B) + δ∗(f (A),f (B)) ≥ γ, δ∗(f (A),f (B)) ≥ γ + δ(A, B) − 1 ≥ γ + α − 1 = γ − (1 − α). If γ < 1 − α, then γ ≤ (1 − α) ∨ Hα(f) and if γ ≥ 1 − α, then (1 − α) ∨ Hα(f) = (1 − α) ∨ 1 ≥ γ. Hence, infα∈(0,1]((1 − α) ∨ Hα(f)) ≥ γ.

If PC(f) = 1, then PC(f) ≥ infα∈(0,1]((1 − α) ∨ Hα(f)). Using (1) above we obtain that PC(f) = infα∈(0,1]((1 − α) ∨ Hα(f)).

If 1−α ≤ γ, then (1−α)∨Hα(f) ≤ γ and if 1−α > γ, then (1−α)∧0 ≤ γ. Hence, infα∈[0,1)((1 − α) ∧ Hα(f)) ≤ γ.

Corollary 2.1. PC(f) = 1 if and only if for each α ∈ (0, 1], f : (X, δα) → (Y, δα∗) is proximal continuous.

Proof. From Theorem 2.2 (2), PC(f) = 1 = infα∈(0,1]((1 − α) ∨ Hα(f)). Then for each α ∈ (0, 1], (1−α)∨Hα(f) = 1 implies Hα(f) = 1. This implies that f : (X, δα) → (Y, δα∗) is proximal continuous. Conversely, if for each α ∈ (0, 1] then function f : (X, δα) → (Y, δα∗) is proximal continuous, then Hα(f) = 1. So PC(f) = infα∈(0,1]((1 − α) ∨ 1) = 1.

Strong Fuzzy Uniform Spaces

In this section we give a conuter example to illustrate that there exists some α-level of the Fuzzy uniformity in sense of Ying [4] which is not a uniformity. Then we introduce and study a type of Fuzzy uniformity stronger than Ying's one with respect to which each α-level is a uniformity. Besides, a Fuzzy topology induced by the strong Fuzzy uniformity is introduced. Further, the connections between the Fuzzy proximity, the strong Fuzzy uniformity and the corresponding Fuzzy topologies are considered.

Counterexample 3.1. Consider the subsets U and Vα of [0, 1]×[0, 1] defined as follows: U = [0, 1] × [0, 1] − {(0, 1)} and Vα = ∆ ∪ {(η, δ) : η ∈ (α, 1) and δ ∈ (0, 1]}∪{(δ, ζ) : δ ∈ (0, 1] and ζ ∈ (α, 1)} for each α ∈ (0, 1). Define the Fuzzy uniformity in sense of Ying on [0,1] as follows:

u(H) = 1, if H ∈ {[0, 1] × [0, 1], U,U −1,U ∩ U −1};
1 − α, if H = Vα or if H ⊇ Vα and H /⊇ Vα∗ where α > α∗; 0, otherwise

We note that Vα = Vα−1, if α1 ≥ α2. Then Vα 2 ⊆ Vα 1 and Vα ⊆ U ∩U −1. Also, for each α ∈ (0, 1), Vα ◦Vα ⊆ U ∩ U −1 because (0, 1) Vα ◦Vα and (1, 0) Vα ◦Vα.
For each H ⊇ Vα and H /⊇ Vα∗, where α < α∗ we have u(H) = u(H−1). Now, the 1-level of u denoted by u1 = {[0, 1] × [0, 1], U,U −1,U ∩ U −1}. There is no subset G ∈ u1 such that G ◦ G ⊆ U . So u1 is not uniformity.
In the following we introduce a Fuzzy uniform space stronger than Ying's one.

Definition 3.1. Let X be a set and u ∈ FN (P (X×X)), i.e., u : P (X×X) → [0, 1] and normal. If for any U, V ⊆ X × X, (FU1) |= (U ∈ u) → (∆ ⊆ U);
(FU2) |= (U ∈ u) → (U −1 ∈ u);
(FU3)∗ |= (U ∈ u) → (∃V)((V ∈ H ⊂ P (X × X)) ∧ (V ∈ u) ∧ (V ◦ V ⊆ U);
where ⊂ stands for "a finite subset of"; (FU4) |= (U ∈ u) ∧ (V ∈ u) → (U ∩ V ∈ u);
(FU5) |= (U ∈ u) ∧ (U ⊆ V) → (V ∈ u),
then u is called strong Fuzzy uniformity and (X, u) is called strong Fuzzy uniform space.

Remark 3.1. In Counterexample 3.1 u is a Fuzzy uniformity in the sense of Ying but it is not a strong Fuzzy uniformity.

Theorem 3.1. Let (X, u) be a strong Fuzzy uniform space. Then for each α ∈ (0, 1], the α-level of u denoted by uα is a classical uniformity on X.

Proof. Let α ∈ (0, 1]. Since u is normal, then there exists U ∈ P (X × X) such that u(U) = 1 ≥ α. Thus U ∈ uα and so uα /= φ.
(U1) Let U ∈ uα. Then u(U) ≥ α and so from (FU1), we have [∆ ⊆ U] = 1. (U2) Let U ∈ uα. Then from condition (FU2), u(U −1) ≥ u(U) ≥ α. Then U −1 ∈ uα.
(U3) Let U ∈ uα. Then from condition (FU3)∗, supV ∈H⊂P (X×X)(u(V)∧[V ◦V ⊆ U]) ≥ u(U) ≥ α.
Then there exists V• ∈ H such that u(V•) ∧ [V• ◦ V• ⊆ U] ≥ α. Hence, we obtain that V• ∈ uα and [V• ◦ V• ⊆ U] = 1.
(U4) Let U , V ∈ uα. From condition (FU4), u(U ∩ V) ≥ u(U) ∧ u(V) ≥ α. So, U ∩ V ∈ uα.
(U5) Let U ∈ uα and [U ⊆ V] = 1. Using (FU5) u(V) ≥ u(V) ∧ [U ⊆ V] = u(U) ≥ α. So, V ∈ uα.

Theorem 3.2. Let (X, u) be a strong Fuzzy uniform space. The fuzzy set τu ∈ F(P (X)), defined by: τu(A) = supα∈(0,1],A∈τuα α, is a Fuzzy topology. It is called the Fuzzy topology induced by the strong Fuzzy uniformity u.

Proof. (1) Since X, φ ∈ τuα for each α ∈ (0, 1], then we have τu(X) =

sup α = 1 and τu(φ) = sup α = 1.
α∈(0,1],X∈τuα α∈(0,1],φ∈τuα
(2) τu(A∩B)= sup α ≥ sup α∧ sup α = τu(A)∧τu(B).
α∈(0,1],A∩B∈τuα α∈(0,1],A∈τuα α∈(0,1],B∈τuα
(3) τu(λ∈Λ Aλ) = sup α ≥ sup α = inf sup α =
α∈(0,1], λ∈Λ Aλ∈τ α∈(0,1],Aλ∈τuα,λ∈Λ λ∈Λ α∈(0,1],Aλ∈τuα
inf τu(Aλ). uα

λ∈Λ

Theorem 3.3. Let δuα be the proximity induced by the uniformity uα. Then the mapping δu : P (X) P (X) [0, 1], defined by δu(A, B)= sup α, is
α∈(0,1],(A,B)∈δuα
a Fuzzy proximity. It is called the Fuzzy proximity induced by the strong Fuzzy uniformity u.

Proof.

(FP1) δu(X, φ) = supα∈(0,1],(X,φ)∈δuα α = 0.

(FP2) It is clear that δu(A, B) = δu(B, A), because (A, B) ∈ δuα if and only if (B, A) ∈ δuα .

(FP3) δu(A, B C)= sup α = sup α
α∈(0,1],(A,B∪C)∈δuα α∈(0,1],(A,B)∈δuα or (A,C)∈δuα
= sup α ∨ sup α = δu(A, B) ∨ δu(A, C).
α∈(0,1](A,B)∈δuα α∈(0,1],(A,C)∈δuα

(FP4) Assume (A, B) /∈ δuα . Then there exists C ∈ P (X) such that (A, C) /∈ δuα and (B, X ∼ C) /∈ δuα . So, for every C ∈ P (X) such that (A, C) ∈ δuα or (B, X ∼ C) ∈ δuα implies (A, B) ∈ δuα . Therefore, δu(A, B) =
sup α ≥ sup α = sup α ∨
α∈(0,1],(A,B)∈δuα α∈(0,1],(A,C)∈δuα or α∈(0,1],(A,C)∈δuα
sup (B,X∼C)∈δuα
α = δu(A, C) ∧ δu(B, X ∼ C).
α∈(0,1],(B,X∼C)∈δuα

(FP5) Suppose [{x} ≡ {y}] = 1. Then x = y. So, ({x}, {y}) ∈ δuα for any α ∈ (0, 1]. Therefore, δu({x}, {y}) = sup α = 1. Again, assume [{x} ≡ {y}] = 0. Then x α∈(0,1],({x},{y})∈δuα
y. So, ({x}, {y}) /∈ δuα for any
α ∈ (0,]. Hence, δu(x , y) = { } { } sup
α∈(0,1],({x},{y})∈δuα
α = 0.

Theorem 3.4. The mapping uδ : P (X × X) → [0, 1] defined by uδ(U) =
sup α is a strong Fuzzy uniformity. It is called the strong Fuzzy
α∈(0,1],U ∈uδα
uniformity induced by the Fuzzy proximity δ.

Proof. (FU1) If uδ(U) = supα∈(0,1],U∈uδα α > 0, then there exists α• ∈ (0, 1] such that α• > 0 and U ∈ uδα• . So, ∆ ⊑ U . Hence, [∆ ⊑ U] = 1 ≥ uδ(U).

(FU2) uδ(U) = sup α = sup α = uδ(U −1).
α∈(0,1],U ∈uδα α∈(0,1],U −1∈uδα

(FU3)∗ uδ(U) = sup α. Now for each U ∈ uδα there exists V ∈ P (X ×
α∈(0,1],U ∈uδα
X) such that V V U and V ⊆ uδα . So, uδ(U) = sup α ≤
α∈(0,1],U ∈uδα
sup sup α = sup uδ(V).

V ∈H⊂P (X×X),V ◦V ⊆U α∈(0,1],V ∈uδα V ∈H⊂P (X×X),V ◦V ⊆U

(FU4) uδ(U ∩ V) = sup α sup≥ α sup α ≥ ∧

α∈(0,1],U ∩V ∈uδα α∈(0,1],U ∈uδα ,V ∈uδα α∈(0,1],V ∈uδα

sup α = uδ(U) ∧ uδ(V).

α∈(0,1],V ∈uδα

(FU5) Suppose [U ⊴ V] = 1. Then uδ(U) = sup α sup ≤ α =
α∈(0,1],U ∈uδα α∈(0,1],V ∈uδα
uδ(V).

Theorem 3.5. Let (X, u) be a strong Fuzzy uniform space, (X, δ) be a Fuzzy proximity space, uδ be the strong Fuzzy uniformity induced by δ and δu be the Fuzzy proximity induced by u. Then |= τuδ ≡ τδu.

Proof. τuδ (A) = supα∈(0,1],A∈τuδα α = supα∈(0,1],A∈τδuα α = τδu (A).

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