COINCIDENCE AND COMMON FIXED POINTS OF HYBRID PAIR OF MAPPINGS

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Abstract

In this paper we define a new weak contractive condition for a hybrid pair of mappings and prove the existence of coincidence and common fixed points. Our results generalize various known results in the literature.

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Introduction :

The aim of this paper is to define a generalized weak contractive condition and give some new coincidence and fixed point theorems under hybrid contractive conditions. Further we extend the results regarding the coincidence and common fixed points of mappings, one is contractive with respect to other to a hybrid pair of mappings. Our results generalize many known theorems in the paper.

'**Definition 1.1**. Let $(X, M, *)$ be a fuzzy metric space. The mappings $f : X \to X$ and $T : X \to CB(X)$ are said to be (IT) commuting at $x \in X$ if $M_{\nabla}(fTx, Tfx, t) = 1$, whenever $fx \in Tx$.

Definition 1.2. Let $(X, M, *)$ be a fuzzy metric space. The mappings $f : X \to X$ and $T : X \to CB(X)$ are said to satisfy the property $(E.A)$ if there exists a sequence $\{x_n\} \in X$, some $t \in X$ and $A \in CB(X)$ such that $\lim_{n\to\infty} fx_n = t \in A = \lim_{n\to\infty} Tx_n$.

Definition 1.3. Let $(X, M, *)$ be a fuzzy metric space and $T : X \to CB(X)$. The mapping $f : X \to X$ is said to be T weakly commuting at $x \in X$ if $ffx \in Tfx$.

Definition 1.4. Let $(X, M, *)$ be a fuzzy metric space and f, g, $F, G : X \to X$. The mappings pair (f, F) and (g, G) are said to satisfy the common property (E.A) if there exist two sequences $\{x_n\}, \{y_n\} \in X$ and some x_0 in X such that $\lim_{n\to\infty} Gy_n = \lim_{n\to\infty} Fx_n = \lim_{n\to\infty} gy_n = \lim_{n\to\infty} fx_n = x_0 \in X$.

Definition 1.5. Let $(X, M, *)$ be a fuzzy metric space, $f, g : X \to X$ and $F, G : X \to CB(X)$. The mappings pair (f, F) and (g, G) are said to satisfy the common property $(E.A)$ if there exist two sequences $\{xn\}, \{yn\} \in X$, some $x_0 \in X$ and $A, B \in CB(X)$ such that $\lim_{n\to\infty} Fx_n = A$, $\lim_{n\to\infty} Gyn = B$,

$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gy_n = x_0 \in A \cap B$.

Theorem 1.6. Let *f, g* be two self-mappings of a fuzzy metric space (*X, M,* *) and let *F, G* be two mappings from *X* to *CB*(*X*) such that

(1) (*f, F*) and (*g, G*) satisfy the common property (*E.A*);

(2) for all *x, y* in *X* with *x*≠ *y* ,

$$M_{\nabla}(Fx,Gy,t) > \min\left\{M(fx,gy,t), \frac{M^{\nabla}(fx,Fx,t)+M^{\nabla}(gy,Gy,t)}{2}, \frac{M^{\nabla}(fx,Gy,t)+M^{\nabla}(gy,Fx,t)}{2}\right\}_i$$

If *f X* and *gX*are closed in *X*, then

a) *f* and *F* have a coincidence point;
b) *g* and *G* have a coincidence point;
c) *f* and *F* have a common fixed point provided that *f* is *F* -weakly commuting at *v* and *ffv* = *fv* for *v* ∈*C*(*f, F*);
d) *g* and *G* have a common fixed point provided that *g* is *G*-weakly commuting at *v* and *ggv* = *gv* for *v* ∈*C*(*g, G*);
e) *f, F, g* and *G* have a common fixed point provided that both (*c*) and (*d*) are true.

Proof.

Since (*f, F*) and (*g, G*) satisfy the common property (*E.A*), there exist two sequences {*xn*} and {*yn*} in *X*, *u* ∈*X* and *A, B* ∈ CB(*X*) such that

$$\lim_{n\to\infty} Fx_n = A \text{ and } \lim_{n\to\infty} Gy_n = B, \lim_{n\to\infty} fx_n = \lim_{n\to\infty} gy_n = u \in A \cap B.$$

By virtue of *f* (*x*) and *g*(*x*) being closed, we have *u* = *fv*and *u* = *gw*for some *v, w* ∈*X*. Now we shall show that *fv*∈*Fv*and *gw*∈*Gw*.

The condition (2) implies that

$$M_{\nabla}\left(Fx_{n_1}Gw,t\right) > \min\left\{M\left(fx_n,gw,t\right), \frac{M^{\nabla}\left(fx_{n_1}Fx_{n_1}t\right)+M^{\nabla}(gw,Gw,t)}{2}, \frac{M^{\nabla}\left(fx_{n_2}Gw,t\right)+M^{\nabla}\left(gw,Fx_{n_1}t\right)}{2}\right\}.$$

Taking the limit as *n* → ∞, we get

$$M_{\nabla}(A,Gw,t) \geq \min\left\{M(fv,gw,t), \frac{M^{\nabla}(fv,A,t)+M^{\nabla}(gw,Gw,t)}{2}, \frac{M^{\nabla}(fv,Gw,t)+M^{\nabla}(gw,A,t)}{2}\right\}.$$

Since *u* = *fv*= *gw*, we obtain

$$M_{\nabla}\left(A_1Gw,t\right) \geq \min\left\{1, \frac{M^{\nabla}\left(u,A_1,t\right)+M\nabla(u,Gw,t)}{2}, \frac{M^{\nabla}(u,Gw,t)+M\nabla(u,A,t)}{2}\right\}.$$

That is,

$$M_{\nabla}(A,Gw,t) \geq \frac{1+M^{\nabla}(u,Gw,t)}{2}, \text{ since } u \in A.$$

$$\Rightarrow \qquad 2M_{\nabla}(A,Gw,t) \geq \left\{1+M^{\nabla}(u,Gw,t)\right\} \tag{4.2}$$

But from (4.1.2), we have $M_{\nabla}(A,Gw,t) \leq M^{\nabla}(u,Gw,t)$.

That is,

$$2M_{\nabla}(A,Gw,t) \leq 2M^{\nabla}(u,Gw,t) \tag{4.3}$$

Combining the inequalities (4.2) and (4.3) we get

$$1+M^{\nabla}(u,Gw,t) \leq 2M_{\nabla}(A,Gw,t) \leq 2M^{\nabla}(u,Gw,t).$$

This implies $1+M^{\nabla}(u,Gw,t) \leq 2M^{\nabla}(u,Gw,t).$

$$\Rightarrow \qquad M^{\nabla}(u,Gw,t) \geq 1$$

$$\Rightarrow \qquad M^{\nabla}(u,Gw,t) = 1.$$

On the other hand, by condition (2), we have

$$M_{\nabla}\left(Fv,Gy_{n,}t\right) > \min\left\{M\left(fv,gy_{n_2},t\right), \frac{M^{\nabla}(fv,Fv,t)+M^{\nabla}\left(gy_{n_1}Gy_{n_2},t\right)}{2}, \frac{M^{\nabla}\left(fv,Gy_{n_1}t\right)+M^{\nabla}\left(gy_{n,}Fv_2t\right)}{2}\right\}.$$

Taking limit as $n \to \infty$, we get

$$M_{\nabla}(Fv,B,t) \geq \min\left\{M(fv,gw,t), \frac{M^{\nabla}(fv,Fv,t)+M^{\nabla}(gw,B,t)}{2}, \frac{M^{\nabla}\left(fv,Gy_{n,},t\right)+M^{\nabla}\left(gy_n,Fv,t\right)}{2}\right\}.$$

That is,

$$M_{\nabla}(Fv,B,t) \geq \min\left\{1, \frac{M^{\nabla}(fv,Fv,t)+1}{2}, \frac{1+M^{\nabla}(gw,Fv,t)}{2}\right\}.$$

Similarly, we obtain

$M_{\nabla}(fv, Fv, t) = 1$, which implies that $fv \in Fv$. Thus f and F have a coincidence point v, g and G have a coincidence point w. This ends the proof of (*a*) and (*b*).

By virtue of condition (*c*), we get $ffv \in Ffv$. Thus $u = fu \in Fu$. This proves (*c*). Similarly (*d*) can be proved. Then (*e*) follows immediately.

Corollary 1.7. Let f be a self-mapping of a fuzzy metric space $(X, M, *)$ and let F be a mapping from X to CB(X) such that

(1) (f, F) satisfies the property (*E.A*);

(2) for all x, y in X with $x \neq y$,

$$M_{\nabla}(Fx, Fy, t) > \min\left\{M(fx, fy, t), \frac{M^{\nabla}(fx, Fx, t)+M^{\nabla}(fy, Fy, t)}{2}, \frac{M^{\nabla}(fx, Fy, t)+M^{\nabla}(fy, Fx, t)}{2}\right\}$$

If $f X$ is a closed subset of X, then
(a) f and F have a coincidence point;
(b) f and F have a common fixed point provided that f is F -weakly commuting at v and $ffv = fv$, for $v \in C(f, F)$.

Proof.

Let $F = G$ and $f = g$ in Theorem (1.6), then the result follows.

Corollary 1.8. Let f be a self-mapping of a fuzzy metric space $(X, M, *)$ and let F and G be two mappings from X to $CB(X)$ such that

(1) (f, F) and (f, G) satisfy the common property $(E.A)$;

(2) for all x, y in X with $x \neq y$,

$$M_{\nabla}(Fx, Gy, t) > \min\left\{M(fx, fy, t), \frac{M^{\nabla}(fx, Fx, t)+M^{\nabla}(fy, Gy, t)}{2}, \frac{M^{\nabla}(fx, Gy, t)+M^{\nabla}(fy, Fx, t)}{2}\right\}.$$

If $f X$ is a closed subset of X, then
(a) f, G and F have a coincidence point;
(b) f, G and F have a common fixed point provided that f is both F -weakly commuting and G-weakly commuting at v and $ffv = fv$, for $v \in C(f, F)$.

Proof.

Let $f = g$. Then the result follows.

If both F and G are single valued mappings in Theorem (1.6), then we have the following corollary.

Corollary 1.9. Let f, g, F and G be four self-mappings of a fuzzy metric space $(X, M, *)$ such that

(1) (f, F) and (f, G) satisfy the common property (EA);

(2) for all x, y in X with $x \neq y$,

$$M_{\nabla}(Fx, Gy, t) > \min\left\{M(fx, gy, t), \frac{M^{\nabla}(fx, Fx, t)+M^{\nabla}(gy, Gy, t)}{2}, \frac{M^{\nabla}(fx, Gy, t)+M^{\nabla}(gy, Fx, t)}{2}\right\}$$

If $f X$ and gX are closed subsets of X, then

(a) f and F have a coincidence point;
(b) g and G have a coincidence point;
(c) f and F have a common fixed point provided that f is F -weakly commuting at v and $ffv = fv$ for $v \in C(f, F)$;

(d) g and *G* have a common fixed point provided that *g* is *G*-weakly commuting at *v* and *ggv* = *gv* for $v \in C(g, G)$;
(e) f, F, g and *G* have a common fixed point provided that both (*c*) and (*d*) are true.
The next theorem involves a continuous function $\phi : [0, 1] \to [0, 1]$ satisfying the following conditions:
(*A*1) ϕ is non-increasing on [0, 1] and
(*A*2) $\phi(t) > t$ for each $t \in (0, 1)$.

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