SUFFICIENT CONDITION FOR THE EXISTENCE OF FIXED POINT FOR MULTI-VALUED MAPPINGS

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ABSTRACT

In this paper we follow the result ascertain the sufficient condition for existence of fixed point in the settings of fuzzy metric spaces.

Keywords : Fixed Point Theory, Metric Spaces, Fuzzy Mapping etc.

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Introduction

Many authors have expansively developed the theory of fuzzy sets and its applications since the inception of the concept of fuzzy set introduced by Zadeh (1965). Badard (1984), Chang, Cho, Lee, Jung and Kang (1997), Fang (1992), Grabiec (1988), Hadzic (1995), Jung, Cho and Kim (1994) Jung, Cho, Chang and Kang (1996), Sharma (2002), Mishra, Sharma and Singh (1994) have studied the fixed point theory with reference to different fuzzy metric spaces defined by the above cited authors. Bose and Sahani (1987), Butnariu(1982), Chang, Cho, B.S. Lee and G.M. Lee (1997) and Helipern (1981) proved fixed point theorems for fuzzy mappings.

Theorem 1.1. Let $(X, M, *)$ be a fuzzy metric space with $t * t \geq t$ for all $t \in [0, 1]$ and let T be a multi-valued mapping from X to $CB(X)$ such that

$$M_{\nabla}(T(x),T(y),qt) \geq \min\{M(x,y,t), M^{\nabla}(x,T(x),t), M^{\nabla}(y,T(y),t),$$
$$M^{\nabla}(x,Ty,(2-\alpha)t), M^{\nabla}(y,Tx,t)\} \quad (4.1)$$

(*i*) for all $x, y \in X$ and for all $t > 0$ where $0 < \alpha < 2$.

(ii) T is continuous at $x \in X$;

(iii) $\{xn\}$ has a subsequence $\{xn_k\}$ converging to x in X. Then T has a fixed point in X.

Proof.

Let $x_o \in X$ be arbitrary. Choose $x_1 \in X$ such that $x_1 \in T(x_0)$. Continuing this process we get $xn \in T(xn)$.

Since $\{xn_i\} \to x$ and T is continuous at x, $T(xn_i) \to x$.

Hence $\lim_{i\to\infty} M^{\nabla}(T(xn), T(x), t) = 1$ for all $t > 0$. Letting $\alpha = 1$ in (4.1)

we have

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$$M^{\nabla}\left(x_{n_i}, T(x), qt\right) \geq M_{\nabla}\left(T\left(x_{n_{i-1}}\right), T(x), qt\right)$$
$$\geq \min\left\{M\left(x_{n_{i-1}}, x, t\right), M\nabla\left(x_{n_{s-1}}, T\left(x_{n_{i-1}}\right|, t\right)\right.$$
$$\left. M^{\nabla}(x, T(x), t), M^{\nabla}\left(x_{n_{i-1}}, T(x), t\right), M^{\nabla}\left(x, T\left(x_{n_{2-1}}\right), t\right)\right\}$$
$$\geq \min\left\{M\left(x_{n_{i-1}}, x, t\right), M\left(x_{n_{i-1}}, x_{n_i}, t\right), M^{\nabla}(x, T(x), t)\right.$$
$$\left. M^{\nabla}\left(x_{n_{\Delta-1}}, T(x), t\right), M\left(x, x_{n_2}, t\right)\right\}$$

Letting $n_i \to \infty$

we get,

$$M^{\nabla}(x, T(x), qt) \geq M^{\nabla}(x, T(x), t).$$

Hence $x \in T(x)$.

Therefore T has a fixed point in X.

Theorem 1.2. Let $(X, M, *)$ be a fuzzy metric space and let T be a continuous mapping from X to $CB(X)$ such that

$$M_{\nabla}(T(x), T(y), qt) \geq \min\left\{M(x, y, t), M^{\nabla}(x, T(x), t), M^{\nabla}(y, T(y), t),\right.$$
$$\left. M^{\nabla}(x, T(y), (2-\alpha)t), M^{\nabla}(y, T(x), t)\right\}$$

(i) for all $x, y \in D$ and $0 < q < 1$, where D denotes an everywhere dense subset of X;

(ii) $\{x_n\}$ has a subsequence $\{x_{nk}\}$ converging to $x \in X$;

(iii) $\{x_n\}$ and $\{y_n\}$ are any two sequences in D such that $x_n \to x$ and $y_n \to y$, then $M(x_n, y_n, t) \to M(x, y, t)$ as $n \to \infty$ for all $t > 0$.

Then x is the fixed point of T in X.

Proof.

The proof will follow from Theorem (1.1) if we can show that (i) holds for any points $x, y \in X \backslash D$. Let $x, y \in X \backslash D$. Then there exist sequences $\{x_n\}, \{y_n\}$ in D such that $x_n \to x$, $y_n \to y$ in D. Since T is continuous, $T(x_n) \to T(x)$ and $T(y_n) \to T(y)$ as $n \to \infty$.

Now

$$M_{\nabla}\left(T\left(x_n\right), T\left(y_n\right), t\right) \geq \min\left\{M\left(x_n, y_n, t\right), M^{\nabla}\left(x_n, T\left(x_n\right), t\right), M^{\nabla}\left(y_n, T\left(y_n\right), t\right)\right.$$
$$\left. M^{\nabla}\left(x_n, T\left(y_n\right), (2-\alpha)t\right), M^{\nabla}\left(y_n, T\left(x_n\right), t\right)\right\}$$

Taking limit $n \to \infty$ and using condition (iii), we get

$$M_{\nabla}(T(x), T(y), qt) \geq \min\left\{M\left(x, y_i t\right), M^{\nabla}(x, T(x), t), M^{\nabla}(y, T(y), t)_i\right.$$
$$\left. M^{\nabla}\left(x_i T(y), (2-\alpha)t\right), M^{\nabla}\left(y_i T(x), t\right)\right\}.$$

Therefore, (i) holds for any points $x, y \in X \backslash D$. Hence $x \in T(x)$. Thus, T has a fixed point in X.

Theorem 1.3. Let $(X, M, *)$ be a fuzzy metric space and let the mapping $T: X \to CB(X)$ such that

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(i) $M_{\nabla}(T(x), T(y), \alpha t) \geq M(x, y, t)$ for all x, y in X, $0 < \alpha < 1$;

(ii) T is continuous at $x \in X$;

(iii) There exists a point $x \in X$ such that $\{x_n\}$ converging to x in X.

Then T has a fixed point in X.

Proof.

Define $x_n \in T(x_n - 1)$ for $n \in N$.

Since T is continuous at x and $x_n \to x$ as $n \to \infty$, $T(x_n) \to T(x)$ as $n \to \infty$.

Now,

$$M^{\nabla}\left(x_n, T(x), \alpha t\right) \geq M_{\nabla}\left(T\left(x_{n-1}\right), T(x), \alpha t\right)$$

$$\geq M\left(x_{n-1}, x, t\right)$$

$$= \quad 1 \text{ as } n \to \infty.$$

Hence, $x \in T(x)$. Thus T has a fixed point in X.

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