Remarks on Computability and Stability

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ABSTRACT

Previously, it has been suggested that the Gödel-type non-algorithmic process may be associated with consciousness. This paper discusses the halting problem and the non-computability of self-referential consciousness. Specifically, this discussion outlines the distinguishability between the halting and looping of computation, or the semantic understanding between true and false, that may be associated with the non-computability of consciousness. The nonlocality intrinsically possessed in quantum theory is one of the striking features of nature that has received a lot of attention. In particular, Bell-type inequalities have been not only extremely powerful but practical tools in quantum information technology. A numerical approach is used to describe different cases of Bell-CHSH inequalities, particularly with different numbers of the choices of measurement bases. It is shown that there are variations of inequalities that are relatively stable against measurement errors.

Key Words: Algorithms, Consciousness, Nonlocality, Numerical Methods

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Introduction

The Copenhagen interpretation of quantum theory indicates that an observation in quantum theory involves a state vector, a wavefunction, and an observable or a reference frame to measure the wavefunction (Peres, 1997). If the logic of the Copenhagen interpretation is taken to the extreme, it may be considered that the whole situation consists of the wavefunction of the observed universe and the collection of observables that measure the wavefunction. In (Song, 2007), unlike the previous attempts of connecting consciousness or memory function with wavefunctions, it was argued that consciousness ought to be represented by a collection of observables or reference frames to measure the wavefunction of the universe.

A concept that is often associated with the study of consciousness is self-reference, i.e., when the object refers to itself. To consider the structure of self-referential logic in computation, one may consider the case when the input is the machine that is performing the computation. In fact, we know that the process of the computer computing the computer itself leads to an infinite recursion as shown in Fig. 1 (i) (Kaufmann, 1987). This process may also be viewed as the cyclical model (Song, 2017a; Reynolds, 1994) or the strange loop (ii) (Hofstadter, 2007)).

Halting Problem

One of the important applications of self-reference in theoretical computer science is the halting problem (Turing, 1936). The halting problem considers if there exists a Turing machine (TM) such that given an arbitrary input, it decides if the computation would halt, denoted as h, or not, denoted as l. That is, the halting problem asks if it is possible to distinguish between halting and looping, or to understand the semantic difference between the two. The proof proceeds using contraction, therefore, by supposing there exists such a TM, which we will

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call A. Then, since A decides the given Turing machine W_i, with the input W_j, one may come up with the following:

$$A(W_i, W_j) = h/l$$

	W_1	W_2	W_3	W_4	…
W_1	**h/l**	h/l	h/l	h/l	
W_2	h/l	**h/l**	h/l	h/l	…
W_3	h/l	h/l	**h/l**	h/l	
W_4	h/l	h/l	h/l	**h/l**	
⋮		⋮			

Let us now consider the following particular TM, which we will call B, with an input W_k:

$B(W_k) \equiv$ If $A(W_k, W_k)$ halts (loops),
then B loops (halts) (1)

Since A computes on all TM's, B must be included in the table, which we will call W_d. However, we run into a problem if the diagonal element $A(W_d, W_d)$ halts, according to the table above, then W_d loops, according to the definition of B in (1). Similarly, if $A(W_d, W_d)$ loops, W_d halts.

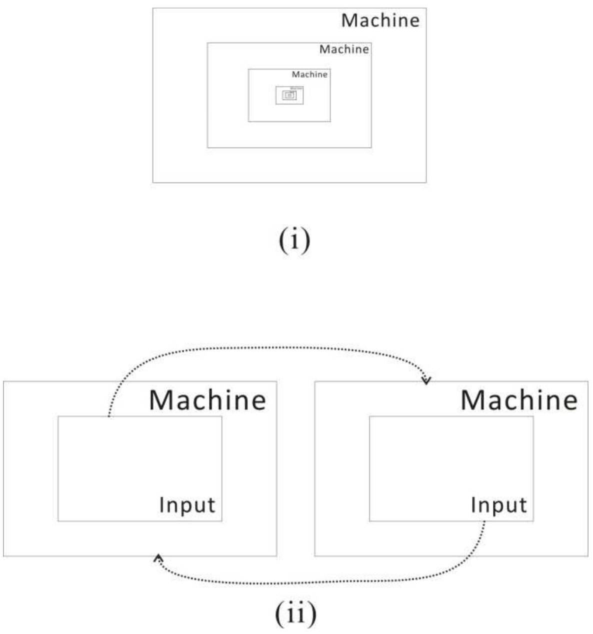

Figure 1. Self-referential process of computing machines: (i) A machine computing itself may have infinitely recursive processes. (ii) The same process may be considered analogous to the cyclical model

The halting problem discussed above used a technique known as "diagonalization," introduced by Cantor in 1891. In fact, Cantor showed that discrete natural and continuous real numbers can't be put into one-to-one correspondence. The classical computing machine may be processing within the set of natural numbers. Although the discrete computer, composed of bits, can compute by halting or looping, it is not able to distinguish between the two. That is, the semantics involving halting and looping is not computable because understanding or being aware of the infinite recursion involving the machine itself, as in Fig. 1, requires the discrete computation to be embedded in continuity. It is interesting to note that this argument is consistent with our previous discussion on associating the semantics of language with universal grammar since classical computers cannot generate continuous semantics (Song, 2017b).

While discrete classical computers were not able to compute the self-aware process as shown in Fig. 1, even continuous quantum computers are unable to compute self-referential consciousness, either. Quantum computers are continuous in the sense that there exists a continuity between the state $|0\rangle$ and $|1\rangle$ (Hardy, 2001; Nielsen *et al.*, 2000). Based on the assumption of identifying consciousness with observables, non-computability of consciousness in quantum physics may be examined as follows: in the case of self-observation, the wavefunction, which is the object being observed, must be the same as the observable or the reference frame in observing the object. In such a case, the symmetry that exists in quantum theory no longer holds. That is, the active approach (the Schrödinger picture, where the object is the dynamic part) and the passive approach (the Heisenberg picture, where the reference frame or the observable is the dynamic part) are no longer equivalent. This then leads to non-computability of the self-observation phenomenon (Song, 2007).

Numerical Simulation of Stability

Since the theoretical discussion on the nonlocal aspect implied in quantum theory has been put forward (Einstein *et al.*, 1935; Bell, 1964; Clauser *et al.*, 1969), there have been numerous experiments confirming the nonlocal nature implied by quantum theory (Aspect *et al.*, 1980). In particular, with more advanced techniques and precision, the distance of this spooky action at a distance has increased. In 1998, Tittel *et al.*, provided a confirmation of nonlocality over the distance of ~10km (Tittel *et al.*, 1998). More recently, research teams successfully generated entanglement over ~100km (Yin *et al.*, 2012) and ~1,200km using a satellite (Yin *et al.*, 2017).

Even with numerous experiments confirming the nonlocal nature in quantum theory, there still remain debates on its contradictory aspect with respect to relativity. That is, more puzzlement is added when this superluminal influence cannot be used to signal (Ghirardi *et al.*, 1980; Gisin, 1989). Although there are some opinions that seek to consider a situation of peaceful coexistence between quantum theory and relativity, others consider the situation unsettled. It is also noted that the long distance influencing as indicated by quantum theory has been shown to be useful in establishing shared secret keys between distant parties (Ekert, 1991).

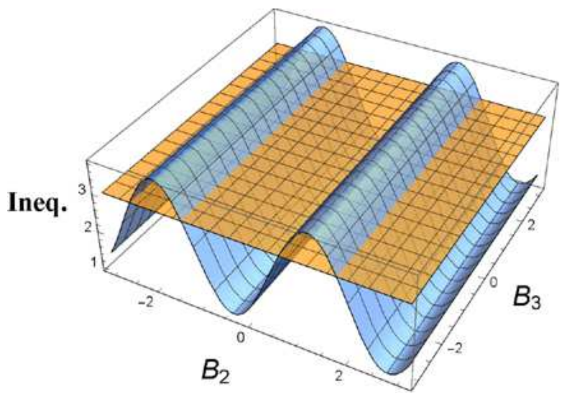

Figure 2. The figure shows the violation of inequalities with the +3 plane, with respect to B_2 and B_3, where A_1, A_2, B_1 are fixed at 0, 0.8, and 2, respectively.

In the following we wish to numerically examine different cases of Bell-type inequalities, particularly with additional choices of measurement bases. Given the maximally correlated state $|\psi\rangle_{Bell} = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$, if A_i and B_j denote the choice of measurement bases at each end, the average of probabilities yields the following correlation function (Bell, 1964: Clauser *et al.*, 1969),

$$\omega(A_i, B_j) = -\cos(2(A_i - B_j)) \tag{2}$$

We wish to consider a case where $i = 1,2$ and $j = 1,2,3$, and the locality assumption may yield the following inequality relations,

$$|\,\omega(A_1,B_1) + \omega(A_1,B_2) + \omega(A_2,B_1) - \omega(A_2,B_2) - \omega(A_1,B_3)\,| \leq 3 \tag{3}$$

It could be numerically checked that quantum theory yields outcomes that violate the inequalities. For instance, in Table 1, some examples of numerical values of A_1, A_2, B_1, B_2, and B_3 that violate (3) are shown. In order to see the nonlocal aspects from a different angle, Fig. 2 considers the case when A_1 = 0, A_2 = 0.8 and B_1 = 2 and the inequality violations are provided with respect to B_2 and B_3 in the range of $(-\pi, \pi)$.

Table 1. Numerical examples that violate the locality condition in (3) are provided for bases choices at one end, A_1 and A_2, and the other end, B_1, B_2, B_3.

Ineq.	A_1	A_2	B_1	B_2	B_3
3.21545	0.2	-0.8	0.8	-1.	-1.
3.09439	0.4	-0.8	1.	-1.	-1.
3.3421	0	0.8	-1.	0.8	-1.
3.50547	0.2	0.8	-1.	1.	-1.
3.41083	0.2	0.6	-1.	1.	-0.8
3.65011	0	-0.8	1.	-1.	-0.6
3.2134	0.6	0.2	-1.	-0.2	-0.6
3.10936	-1.	-0.2	0.8	0.2	-0.6
3.15879	0.4	-0.8	1.	-0.8	-0.6

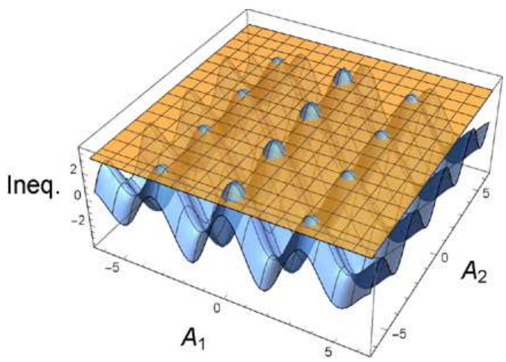

Figure 3. The nonlocality is shown with the inequality condition in (3) as a function of A_1 and A_2, whereas B_1, B_2, B_3 are assumed to be fixed at 2, 1.2, -0.8

In Fig. 3, a different approach is shown when the inequalities are given with respect to A_1 and A_2, where B_1, B_2, B_3 are fixed at 2, 1.2, and -0.8, respectively. In actual experiments involving measurements, there is always inaccuracy with the choice of basis. Fig. 4 considers such cases and numerically shows the violation of inequalities when a deviated measurement is made with respect to ε_1 and ε_2, where $B_2 - \varepsilon_1$ and $B_3 - \varepsilon_2$, given A_1 =0, A_2 =0.8, B_1 =2, B_2 =1.2 and B_3 =-0.8. Similarly, Fig. 5 shows the case when $A_1 - \delta$ and $B_1 - \varepsilon$, and the inequalities are provided with respect to δ and ε. It shows that there is a substantial range of tolerance in the choice of measurement bases that still yield nonlocality. A similar result is obtained in Fig. 6, with the error defined as $A_2 - \delta$ and $B_2 - \varepsilon$, whereas the values of A_1, A_2, B_1, B_2, B_3 are assumed the same as in Fig. 5.

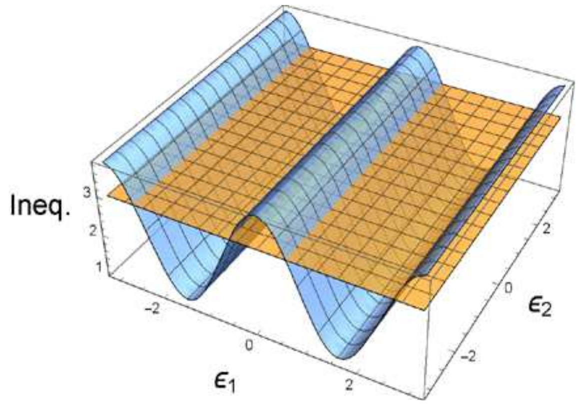

Figure 4. It shows the level of nonlocality where there exist errors in measurement choices, with $B_2 - \varepsilon_1$, $B_3 - \varepsilon_2$, given A_1 =0, A_2 =0.8, B_1 =2, B_2 =1.2, and B_3 =-0.8.

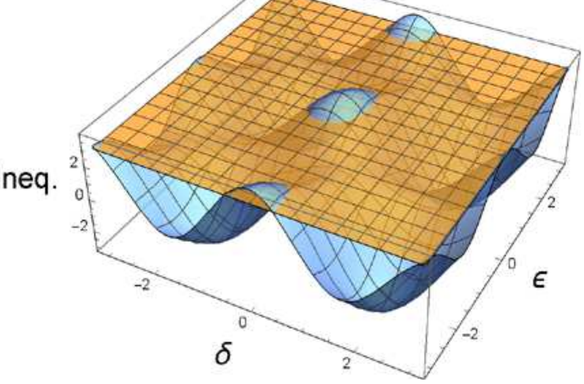

Figure 5. Given A_1 =0, A_2 =0.8, B_1 =2, B_2 =1.2, and B_3 =-0.8, similar to Fig. 4, errors in the measurement basis are considered, with $A_1 - \delta$ and $B_1 - \varepsilon$, where the violation of locality conditions is indicated.

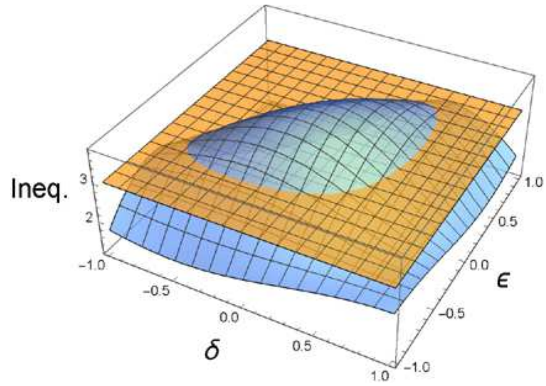

Figure 6. The violation of inequalities are numerically examined given $A_2 - \delta$ and $B_2 - \varepsilon$, where A_1 =0, A_2 =0.8, B_1 =2, B_2 =1.2, and B_3 =-0.8. It shows the nonlocality is relatively stable with respect to the errors of A_2 and B_2.

Let us consider another inequality condition as follows:

$$|\,\omega(A_1,B_1)+\omega(A_1,B_2)-\omega(A_1,B_3)+\omega(A_2,B_1) \\ -\omega(A_2,B_2)+\omega(A_2,B_3)\,|\le 4 \qquad (4)$$

Similar to (3), it may be numerically checked that there are a wide range of bases that violate the inequalities in (4). For example, Table 2 provides some examples of measurement bases at each end that violates the inequalities. Moreover, Fig. 7 shows the case of inequalities being violated with respect to A_1 and A_2, whereas B_1, B_2, B_3 are fixed at -0.2, 0.6, -1, respectively. Similar to (3), the inequalities in (4) are also stable with respect to errors in the measurement bases choices. Fig. 8 shows the inequalities with respect to δ and ε, where $A_1 - \delta$ and $B_1 - \varepsilon$ are examined with A_1 =1.9, A_2 =0.8, B_1=-0.2, B_2=0.6, B_3=-1.

Table 2. Some examples of measurement choices A_1, A_2 and B_1, B_2, B_3 that violate the inequalities in (4)

Ineq.	A_1	A_2	B_1	B_2	B_3
4.29803	0.8	-1.	-0.1	-0.9	0.8
4.05228	0.9	-0.8	-0.3	-0.9	0.8
4.40456	1.	-1.	-0.1	-0.9	0.8
4.11159	0.7	-1.	-0.1	-0.9	0.8
4.32852	-1.	1.	0.1	0.8	-0.6
4.20687	-1.	0.9	0.	0.8	-0.6

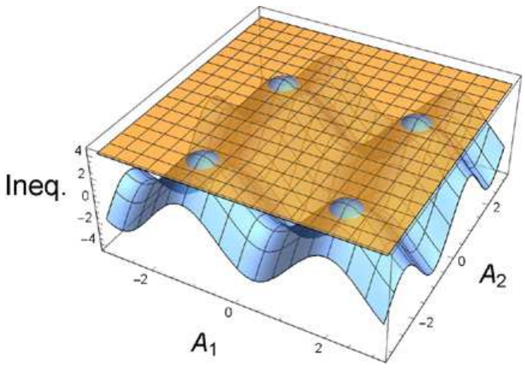

Figure 7. The figure shows the violation of inequalities in (4) as a function of A_1 and A_2, whereas B_1, B_2, B_3 are fixed at -0.2, 0.6, -1, respectively

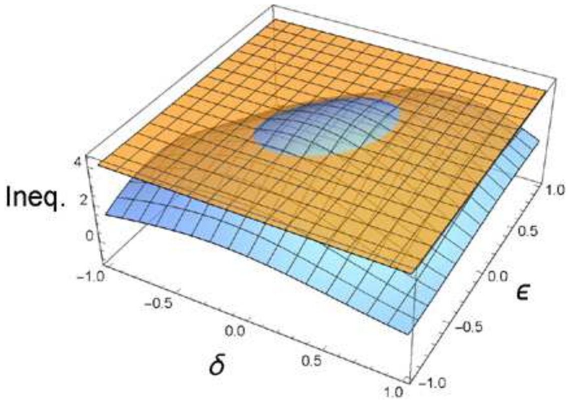

Figure 8. Stability of nonlocality is provided where A_1 - δ and B_1 - ε are considered, with A_1 =1.9, A_2 =0.8, B_1 =-0.2, B_2 =0.6, B_3 =-1

Remarks

Nonlocality and entanglement have been some of the most important parts in recent developments of quantum information technology. In particular, Bell-type inequalities have yielded an important clue in our understanding of quantum nature. In this paper, we have numerically considered different versions of inequalities, particularly with emphasis on inaccurate choices of measurement bases at each end. It was shown that inequalities not only provide striking aspects of nature but are relatively stable against errors in the process of measurement.

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