# Extended Tor-Cube: A New Scalable Hybrid Interconnection Network for Massive Computing

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***Abstract:*** *The Interconnection networks are backbone of massive computing systems. They also involve message communication among the processing elements. To make the computation and communication faster the interconnection topology is always designed with some innovations. The current paper introduces a new hybrid interconnection topology called the Extended Tor Cube (ETC) for high end computing system. As compared to the other interconnection networks ETC is found to be more attractive in terms of topological parameters such as diameter, cost, average node distance, time cost effectiveness factor and message traffic density etc. It helps to improving the node packing density for high performance computing. Our proposed network is extremely scalable with sufficiently reduced diameter and also robust in nature. The suggested topology is hierarchical and easily expandable architecture. The various performance metrics show that the proposed topology is a better candidate for parallel processing and massive computing than its predecessors. The scalability with increasing dimension for the new network are also presented.*



***Keywords:*** *Cost effectiveness, Fault tolerance, Packing Density, Message traffic density, Robust, Reliability, Routing*

## I. INTRODUCTION

The interconnection network, which establishes the means of data interchange among numerous isolated processing units, is essential to the architecture of high performance computing systems[1-3]. The interconnection network of these systems plays a key role in their overall performance. Thus, in order to create a highly dependable high performance computing system, it is necessary to ensure that the interconnection network being utilised is highly reliable, meaning that it should continue to function well for a set amount of time even when links or processing units fail. Additionally, these systems should utilize interconnection networks economically. The cost of the connectivity network can be calculated by multiplying its degree with diameter. In other words, it may be claimed that designing such systems requires the deployment of an interconnection network that is both extremely reliable and cost-effective. Hypercube and its variants [8-15], Torus [20], Mesh etc are some examples of interconnection network. Due to its characteristics, including its tiny diameter, strong connectivity regularity, symmetry, recursive construction, partition ability, fault tolerance, and dependability, the hypercube (HC) has been used. Hypercube has undergone numerous modifications in the past, either to increase its dependability or lower its price. Researchers have suggested a variety of hierarchical connectivity networks using the hypercube, including extended hypercubes, hierarchical crossing cubes, and hierarchical cube networks [4-7]. The inherent scale constraint of the hypercube and its derivatives, however, is the fundamental issue. Mesh and torus are the two popular topologies that offer a great level of scalability. In the fields of scientific calculations, flow dynamics, structural analysis, etc., these networks are discovered to be commonly used. The architecture of a massively parallel computer system cannot use a mesh network due to its higher link complexity. It is additionally challenging to deploy on actual parallel machines due to the network's high cost and difficulties in setup and maintenance [16-19,21-22]. Torus is more dependable and has higher connection than mesh. With topological criteria including dimension, packing density, average node distance, and message traffic density, the Tor cube [23] is seen as a bit improved network. It is still difficult to create a network that is scalable, affordable, highly dependable, fault resistant, and has good connectivity with less complex technology. This inspires us to suggest a new hybrid interconnection network called Extended TOR-Cube (ETC) which inherits the properties of the popular interconnection topology Tor Cube [23]. Most of the significant Tor Cube features are utilised by the suggested topology (ETC). We examine the various design elements of the Extended TOR-Cube (ETC) and evaluate its topological characteristics in comparison to those of other significant interconnection topologies. The Extended Tor-Cube (ETC) is a hierarchical, expandable structure which retains the positive features of the k-cube at different levels of hierarchy. With the practice of network controllers, it has better routing properties. It has compact diameter and average distance.

However, some of the key considerations that must be made in the design of any interconnection architecture are reliability, fault tolerance, and other performance metrics including cost, cost-effectiveness, and time-cost-effectiveness. This demand prompts the current study to suggest the Extended TOR-Cube, a new hierarchical fault-tolerant network (ETC). This paper is prepared as follows: The Segment II presents the background of the current work. and the construction of the proposed network. In segment III the topological parameters are offered by using graph theoretical symbolizations. In Segment IV a detail comparison with the predecessor networks is done. Then Segment V concludes the paper with a scope for further research.

## II. PROPOSED INTERCONNECTION TOPOLOGY: EXTENDED TOR-CUBE (ETC)



The extended Tor Cube interconnection topology is an undirected graph where the processing components are represented by the vertices and the bidirectional links are represented by the edges. Some of the crucial traits of the Tor cube and Extended hypercube are carried over into the new structure. The Fig.1 demonstrates the structure of TC topology[23]. The basic building The basis of the Tor Cube is an n-dimensional Hypercube. An n-dimensional Tor- Cube is created by connecting additional links from a torus interconnection network to the base hypercube interconnection network. e.g. A 3D hypercube (HC) is initially taken for a 3D TC (Fig. 1), and further 4 links from the torus interconnection topology are then added to it. The additional links are joined to the torus pattern's neighboring and opposing nodes. The Extended Tor Cube in k dimensions is a hierarchical structure with numerous labels. As shown in Figure 2, it may be described as an ETC (k, l) labelled graph that can be built recursively with two unique types of vertices named Network Controller (NC) and Processing Elements (PE). The PEs execute computational job where as NCs are accountable for communication job. The NC is located at the highest level and PEs at the 0th or lowest level. The basic unit ETC (k,1) comprise of k-dimensional TC with one NC, has some level of hierarchy. The NC at the initial level and the k-TC at the 0th level is shown in Fig 2. In addition, there is k-TC of $2^k$ NCs at the (l-1)th level. Again it can be seen that 2k NCs form a k tor cube at the (l-1)th level. Any ETC(k,l) can be recursively constructed from the basic unit ETC(k,1).As for instance, the NCs of individually ETC(k,2) can be built from ETC(k,1) and this process can be repetitive hierarchically to build the required dimensions of ETC. The ETC(3,2) built from eight numbers of ETC(3,1)s. The NC's at the (l-1)th level of ETC(k,1) are addressed by 0.The k-TC at the lth level be made up of of 2k NCs have address as 00,01….0M,where M=2k-1,The address of the NC precedes the node address of PEs. Thus the PEs associated to the NC's 0i(0 ≤i ≤m) have addresses 0i0,0i1,…0iM .

In an ETC, the NCs serve as the communication processor for both local communication between two different basic modules and global message transfer between several tiers. However, the two nodes of the same basic module do not communicate using NCs. Any two nodes of the ETC(k,l) are connected by (k+1) parallel pathways, one path resulting from the NC and k paths supplied by the k edges of the TC. The suggested structure is scalable, making it ideal for extensive parallel processing (Fig. 2).

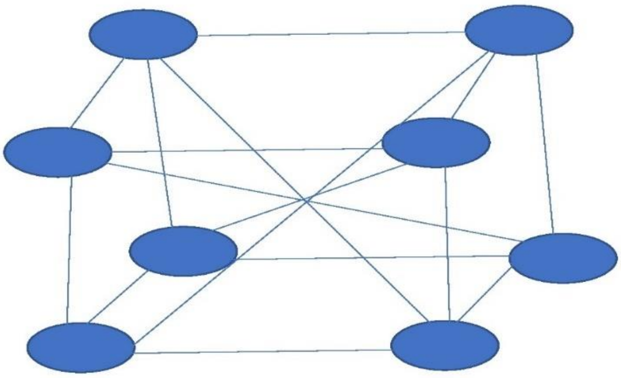

**Fig. 1 TOR Cube**

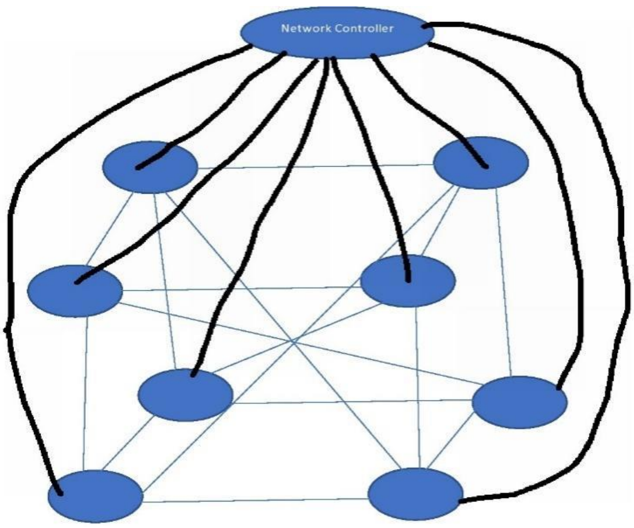


**Fig. 2 Extended TOR Cube**



## III. TOPOLOGICAL PROPERTIES OF EXTENDED TOR-CUBE INTERCONNECTION NETWORK

The Extended Tor Cube is observed as an undirected graph *G (N, D),* where , N is the node set representing the processing elements and D denotes the set of edges representing the connecting links between the processing elements. The different topological properties of Extended Tor Cube are offered in this Section.

### A. Degree

The degree of a network is defined as the degree of the node which has a maximum number of links attached to that node.

**Proposition 1:** Node degree of Processing Element = n+2

Node degree of Network Controller at first level = $2^n$+n+2

Node degree of Network Controller at jth level where (1<j<k) = $2^n$+n+3

Node degree of Network Controller at kth level = $2^{n-1}$

**Proof : (i)** Since in an ETC(*n,k*) the PEs are at the lowermost or zeroth level of hierarchy. Individually PEs belonging to ETC(*n,*1) is directly linked to *n*-neighboring PEs at $0^{th}$ level and two NCs at its next advanced level that is $1^{st}$ level. Thus the degree of PE in ETC(n,k) is n+2 .

**(ii)** Individually NC at level 1 is associated to $2^n$ PEs at its just lower level, *n* NCs at its own level and two NCs at its next higher level that is at level 2. This shows its degree to $2^n + n + 2$.

**(iii)** Now NCs at level j (1<j<k) is linked to $2^n$+1 NCs at its just lower level. n NCs at its level and two NCs at its next higher level. thus the degree of NCs at level j (1<j<k) is defined as ($2^n + n + 3$).

**(iv)** Now NCs at highest level that is kth level of hierarchy is linked to two $2^n$ NCs at its just lower level. Hence the degree of NCs at highest level is defined as $2*2^n = 2^{n+1}$



### B. Node

The node of an interconnection topology is central unit of the graph. The total number of nodes indicate the size and complexity of a network.

**Proposition 2:** The total number of nodes of n-dimensional Extended Tor Cube is $\mathbf{2^{nk} + (2^{nk} - 1) \ / \ (2^n - 1)}$.

**Proof:** At $k^{th}$ level , number of NC = 1

At k-$1^{th}$ level, number of NC = $2^{n\{k-(k-1)\}} = 2^n$

At k-$2^{th}$ level , number of NC = $2^{n\{k-(k-2)\}} = 2^n * 2^n = 2^{2n}$

At { k- (k-1) }$^{th}$ level, number of NC = $2^{n\{k-(k-(k-1))\}} = 2^{n(k-1)}$

Hence At level 1 , number of NC = $2^{n(k-1)}$

At $0^{th}$ level that is $(k-k)^{th}$ level , number of PE = $2^{n\{k-(k-k)\}} = 2^{nk}$

Hence total number of NC = $2^0 + 2^n + 2^{2n} + 2^{n(k-1)}$

$= \sum_{(j=1)}^{k} 2^{n(k-j)} = (2^{nk} - 1) \ / \ (2^n - 1)$

Suppose N = Number of PE = $2^{nk}$

M = Number of NC = $(2^{nk} - 1) \ / \ (2^n - 1)$

Hence total number of nodes = Number of PE + Number of NC

= N + M

= $2^{nk} + (2^{nk} - 1) \ / \ (2^n - 1)$

### C. Links

link designated as communication edge between two processing elements in a topology. Two nodes are said to be adjacent if they attached with the same link.

**Proposition 3 :** The total number of links of Extended Tor Cube is

$$[\ (n+2)\,2^{nk} + (2^n+n+1)\ 2^n\ \{(2^{n(k-2+1)}-1)\ /\ (2^n-1)\ \} + 2^n\ ]\ /\ 2$$

**Proof :** If L is the number of links then according to Handshaking Lemma of Graph Theory

2L = Sum of degree of Processing Elements i.e PE

= Sum of degree of NC at level j (1<j<k) + Sum of degree of NC at level k

$= (n+2)\,2^{nk} + (2^n+n+1)\,(2^n + 2^{2n} + 2^{3n} + \ldots\ldots + 2^{n(k-1)}) + 2^n$

$= (n+2)\,2^{nk} + (2^n+n+1)\ 2^n\ (1 + 2^n + 2^{2n} + \ldots\ldots + 2^{n(k-2)}) + 2^n$

$= (n+2)\,2^{nk} + (2^n+n+1)\ 2^n\ \{(2^{n(k-2+1)}-1)\ /\ (2^n-1)\ \} + 2^n$

So $L = [\ (n+2)\,2^{nk} + (2^n+n+1)\ 2^n\ \{(2^{n(k-2+1)}-1)\ /\ (2^n-1)\ \} + 2^n\ ]\ /2$

**Theorem 1 : If** N , M , L are the number of PEs , number of NCs and number of links of Extended Tor Cube (n,k)

Then the average degree of Extended Tor Cube (n,k) can be defined as follows :

**$d = 2L\ /\ (N+M)$**

Where $N = 2^{nk}$ , $M = (2^{nk}-1)\ /\ (2^n-1)$ and

$L = [\ (n+2)\,2^{nk} + (2^n+n+1)\ 2^n\ \{(2^{n(k-2+1)}-1)\ /\ (2^n-1)\ \} + 2^n\ ]\ /\ 2$

**Hence** $d = [\ (n+2)\,2^{nk} + (2^n+n+1)\ 2^n\ \{(2^{n(k-2+1)}-1)\ /\ (2^n-1)\ \} + 2^n\ ]\ /\ [\ 2^{nk} + (2^{nk}-1)\ /\ (2^n-1)\ ]$

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### D. Diameter

The diameter of graph S is defined as the maximum of the minimum distances between any two different vertices of S. The diameter is considered as the most important performance metric to evaluate the efficiency of any network . It should be remain small for an efficient network.

**Proposition 4 :** The diameter of Extended Tor Cube ETC(n,k) is **$(n-1) + 2(k-1)$.**

**Proof:** Considering two nodes, the source node *u* and the destination node *v*, they are either in the same TORCUBE(n,k) or in different TORCUBE(n,k) .

**Case 1:** Suppose *u* and *v* are in the same TORCUBE(n,k). Then the distance between *u* and *v is (n-1).*

**Case 2:** Suppose *u* and *v* are in different TORCUBE(n,k) then we have to find out neighboring node *w* in the same with *u*, which can be connected with *v* by links through *NC*s. From the definition of diameter, *w* communicating with *v* must go through the topmost *NC*. Thus, the shortest distance between *w* and *v* is $2(l-1)$. Hence the distance between *u* and *v* is at most $(n-1) + 2(k-1)$.

### E. Cost

The cost of any network is obtained by multiplying the degree with the diameter of the network. This is the another important performance metric for evaluation the efficiency of any topology.

Mathematically,

$$Cost = degree \times diameter$$

**Proposition 5:** The cost of Extended Tor Cube can be computed as follows :

Cost = Degree * Diameter

$= (n+1) * \{(n-1) + 2(k-1)\}$

As the degree of Extended Tor Cube is (n+1) and the diameter of the Extended Tor Cube is $(n-1) + 2(k-1)$.

## F. Average Distance

The average distance is most important performance metric for evaluation the efficiency of any topology. The average distance of any network is the sum of distances of all nodes from a source node over the entire number of nodes.

**Preposition 6: (I)** The local average distance in ETC(n,k) is

$$1/2^n \ [(l_1)_n + \sum_{k=2}^{n-1} ( k \times (l_1)_n)]$$

**(II)** The global average distance in ETC(n,k) is $d \square \sum dN_d \ /(N \square M)$

Where $N_d$ = No of processor at distance $d$ from the source node

$N$ = Total number of PCs = $2^{nk}$

$M$ = Total number of NCs = $2\,(2^{nk} - 1) / (2^n - 1)$



**Proof : I.** The average distance is an important parameter to measure the actual performance of the network. The average distance of any topology can be defined as the sum of distances of every nodes from a source node over the number of nodes . In any communication network average distance is categorized into two category that is local and global. In the ETC(n,k) network the lower level is a Tor cube. So, the average distance in the basic module is same as that of the tor cube and it is $1/2^n \ [(l_1)_n + \sum_{k=2}^{n-1} ( k \times (l_1)_n)]$

In the ETC(n,k) network the global average distance is $d \square \sum dN_d \ /(N \square M)$

where $N = 2^{nk}$

$M = 2\,(2^{nk} - 1) / (2^n - 1)$

$$\sum dN_d = 2k\,(n \square 1) \square 2\,(2^n \square (n \square 1))\,(k \square 1) \square (2^n \square (n \square 1)) \quad \square 2k\, 2^{n(k \square 1)} \square (2k \square 1)\, n\, 2^{n(k \square 1)} \square 2 \sum_{j=1} n \square 2^{jn} \square 2k \square 1 \square j \square \square$$

$$\square 2^n \square \square n \square 1 \square 2^{jn} \square 2k \square j \square \square \square dN_d$$

$$\sum dN_d = 2 \square n \square 2(2^n \square (n \square 1)) \quad \text{for } k=1$$

## G. Density of Message Traffic

The message traffic density is another important factor to measure the actual performance of the network. For an efficient network message traffic density should be kept minimum. The message traffic density can be represented by P/Q.

Where P = Average Distance * Total number of nodes

Q = Total number of links

**Preposition 7:** The message traffic density of Extended Tor Cube (n,k) is defined as follows :

□ □ $dN_t / L$

**Proof :** Suppose every node is transferring a message to another node situated at z distance and assume that every communicating links can accommodate such a traffic, □ is nothing but a better metric to evaluate message traffic in the network .

The density of message traffic in Extended Tor Cube or ETC(n,k) is □ □ $dN_t / L$

where d is the average distance , $N_t$ = N + M and L is the number of links

As we know that , N = Number of PE = $2^{nk}$

M = Number of NC = $(2^{nk} – 1) / (2^n – 1)$

Hence total number of nodes = Number of PE + Number of NC

= N + M

= $2^{nk} + ((2^(nk) - 1))/(2^n - 1)$

As we know that L is the total number of links which can be defined as follows :

$[ (n+2) 2^{nk} + (2^n+n+1) 2^n ((2^(n(k-2+1)-1))/(2^n-1) + 2^n ] / 2$

Hence □ □ $dN_t / L$

$= \{ d * 2^{nk} + (2(2^(nk) - 1))/(2^n - 1) \} / [ \{ (n+2) 2^{nk} + (2^n+n+1) 2^n ((2^(n(k-2+1)-1))/(2^n-1) + 2^n \} / 2 ]$



### H. Cost Effectiveness Factor

Any parallel algorithm design must consider both time- and cost-effectiveness in order to be successful. A parallel algorithm's cost effectiveness takes into account not only the cost of the processors but also the cost of the communication lines. It accounts for the cost of the full multiprocessor as well as how much the parallel method uses the processors.

**Preposition 7:** The cost effectiveness of Extended Tor Cube (n,k) is defined as follows :

$1/ [1+ □ * \{[ (n+2) 2^{nk} + (2^n+n+1) 2^n \{ (2^{n(k-2+1)} - 1 ) / (2^n - 1) \} + 2^n ] / 2 \} / \{ 2^{nk} + (2^{nk} – 1) / (2^n – 1) \}]$

**Proof :** The cost effectiveness of Extended Tor Cube (n,k) is derived as follows :

**CEF(p)= 1/ ( 1 + □ * g(p) )**

Where □ is the message traffic density of Extended TORCUBE(n,k)

P is the number of processors

g(p) is the ratio of number of links to the number of processors

In general the number of links is a function of the number of nodes that is E= f(p).

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The total number of processors is given by as follows :

Total number of processors = Number of PE + Number of NC

$$= N + M$$

$$= 2^{nk} + (2^{nk} - 1) \ / \ (2^{n} - 1)$$

And the total number of links in Extended TORCUBE(n,k) is given by as follows :

$$[\ (n+2)\, 2^{nk} + (2^{n}+n+1)\ \ 2^{n}\ \{ (2^{n(k-2+1)} - 1)\ /\ (2^{n} - 1)\ \} + 2^{n}\ ]\ /\ 2$$

As $g(p) = L / P$

Where L is the total number of links in Extended TORCUBE(n,k)

P is the total number of processors

So $g(p) = \{[\ (n+2)\, 2^{nk} + (2^{n}+n+1)\ \ 2^{n}\ \{ (2^{n(k-2+1)} - 1)\ /\ (2^{n} - 1)\ \} + 2^{n}\ ]\ /\ 2\ \} \mathbf{/} \{ 2^{nk} + (2^{nk} - 1)\ /\ (2^{n} - 1) \}$

As we know that

$$CEF(p) = \frac{1}{1+\rho g(p)}$$

$$= 1/\ [1+ □ * \{[\ (n+2)\, 2^{nk} + (2^{n}+n+1)\ \ 2^{n}\ \{ (2^{n(k-2+1)} - 1)\ /\ (2^{n} - 1)\ \} + 2^{n}\ ]\ /\ 2\ \} \mathbf{/} \{ 2^{nk} + (2^{nk} - 1)\ /\ (2^{n} - 1) \}]$$

### I. *Factor of Time Cost Effectiveness*

The time it takes to solve an issue is a parameter for this measurement. The TCEF believes that a delayed solution is in no way advantageous. For Extended Tor Cube or ETC(n,k) the TCEF is expressed as follows:

$$TCEF(p) = \{1 + □ \} /\ (\ 1 + □\ *\ \ g(p)\ \mathbf{+}\, □ / p\ )$$

**Preposition 8 :** The time cost effectiveness factor in ETC(n,k) is expressed as follows :

$$\{1 + □ \} / \mathbf{[}1+ □ * \{[\ (n+2)\, 2^{nk} + (2^{n}+n+1) 2^{n}\{ (2^{n(k-2+1)} - 1) / (2^{n} - 1) \} + 2^{n}\ ]\ /\ 2\ \} \mathbf{/}\{ 2^{nk} + (2^{nk} - 1)\ /\ (2^{n} - 1)\}$$

$$+ □ / \{ 2^{nk} + (2^{nk} - 1)\ /\ (2^{n} - 1) \} \mathbf{]}$$

**Proof :** The *Time Cost Effectiveness Factor* in ETC(n,k) is derived as follows :

$$TCEF(p) = \{1 + □ \} /\ (\ 1 + □\ *\ \ l(p)\ \mathbf{+}\, □ / p\ )$$

Where □ is the density of message traffic in ETC(n,k)

P is the number of processors

l(p) is the number of links over the number of processors

□ is the ratio of cost of penalty to cost of processors

In general the number of links is a function of the number of nodes that is E= f(p).

The total number of processors is given by as follows:

Total number of processors = Number of PE + Number of NC

$$= N + M$$

$$= 2^{nk} + (2^{nk} - 1) \ / \ (2^{n} - 1)$$

And the total number of links in ETC(n,k) is given by as follows :

$$[\ (n+2)\,2^{nk} + (2^{n}+n+1)\ 2^{n}\ \{ (2^{n(k-2+1)} - 1) / (2^{n} - 1) \} + 2^{n} ]\ / 2$$

As l(p) = L / P

Where L is the total number of links in ETC(n,k)

P is the total number of processors

So $l(p) = \{[\ (n+2)\,2^{nk} + (2^{n}+n+1)\ 2^{n}\ \{ (2^{n(k-2+1)} - 1) / (2^{n} - 1) \} + 2^{n} ]\ / 2 \} / \{ 2^{nk} + (2^{nk} - 1) / (2^{n} - 1) \}$

As we know that TCEF(p) = $\{1 + \square \} / (1 + \square * l(p) + \square / p)$

$$= \{1 + \square \} / [1 + \square * \{[\ (n+2)\,2^{nk} + (2^{n}+n+1)2^{n}\{ (2^{n(k-2+1)} - 1) / (2^{n} - 1) \} + 2^{n} ]\ / 2 \} / \{ 2^{nk} + (2^{nk} - 1) / (2^{n} - 1)\}$$

$$+ \square / \{ 2^{nk} + (2^{nk} - 1) / (2^{n} - 1) \} ]$$

## IV. RESULTS AND DISCUSSIONS

This section estimates different parameters of Extended Tor Cube(ETC) and a comparison is made with other networks like hyper cube(HC), crossed cube(CC), leafy cube(LC) and tor cube(TC). For comparison of parameters, the n and k values are chosen such that n is varying from 2 keeping k fixed at 3 and 4. Different topological properties are compared in Table-I. The parameters that are considered for comparison are node, edges, degree, diameter, average distance, message traffic density, bisection width and cost. From the Table-I, it can be observed that the degree of ETC is more as compared to hyper cube(HC), crossed cube(CC), leafy cube(LC) and tor cube(TC). More the degree reflects, the better is the connectivity among the nodes or the processing elements. As far as the diameter is considered, the ETC offers the shortest diameter as compared to the Leafy Cube. The degree, diameter, and cost are the three desirable and important properties of any interconnection network, where one always needs a higher degree network with the low diameter and low cost. From the Table-I, it can be observed that the proposed topology ETC offers the higher cost as compared to other prescribed network but ETC exhibits scalability property which is the demand of today's high performance computing. The proposed network is more reliable than the parent networks as it has more node disjoint paths with increase values of n. The average distance and fault tolerance of the proposed network ETC are also proved better than its predecessor hyper cube(HC), crossed cube(CC), leafy cube(LC).



**Table I: Comparison of Attributes of Various Network with Dimension**

| Parameter | HC | CC | LC | TC | ETC |
|---|---|---|---|---|---|
| Node | $2^{n}$ | $2^{n}$ | $2^{n} + n2^{n}$ | $2^{n}$ | $2^{nk} + (2^{nk} - 1) / (2^{n} - 1)$ |
| Edges | $n2^{n-1}$ | $2^{n-1}$ | $n2^{n-1} + n2^{n}$ | $2L_{n-1}+2^{n-1}$ | $[\ (n+2)\,2^{nk} + (2^{n}+n+1)$ |

| | | | | | |
|---|---|---|---|---|---|
| | | | | | $2^n \{ (2^{n(k-2+1)} - 1) / (2^n - 1) \} + 2^n ] / 2$ |
| Degree | $n$ | $n$ | $2n$ | $n+1$ | n+2 |
| Diameter | $n$ | $\left\lceil \frac{n+1}{2} \right\rceil$ | $n+2$ | $n-1$ | $(n-1) + 2(k-1)$. |
| Average Distance | $\frac{n}{2}$ | $\frac{n}{2}$ | $\frac{n}{2}$ | $\left(\frac{1}{2^n}\right) \sum_{k=1}^{n-1} L_k$ | $1/2^n$ [ $(l_1)_n + \sum_{k=2}^{n-1}( k \times (l_1)_n$ )] |
| Message Traffic Density | 1 | 1 | (n+1)/3 | $\frac{\left(\frac{1}{2^n}\right) \sum_{k=1}^{n-1} L_k 2^n (n+1)}{2} \times Ln - 1 + 2^{n-1}$ | {d $2^{nk}$+ $(2(2^(nk) - 1))/(2^n - 1)$}/[{ (n+2) $2^{nk}$ +($2^n$+n+1) $2^n$ $((2^(n(k-2+1)-1))/(2^n-1)$ + $2^n$ } / 2 ] |
| Bisection Width | $\frac{2^n}{2}$ | $\frac{2^n}{2}$ | $\frac{2^n}{2}$ | $\frac{2^n}{2}$ | $2^{n-1}$ |
| Cost | $n^2$ | $n\left\lceil \frac{n+1}{2} \right\rceil$ | $2n(n+2)$ | $n^2-1$ | *(n+2){(n-1)+2(k−1)}* |



The following graphs are drawn to show the relationship between the basic topological parameters like degree, node, diameter and cost with respect to dimension of various networks like Extended hyper cube (EHC), Extended crossed cube (ECC), Extended Varietal Hypercube(EVH) and tor cube(TC). From the Figure-I, it can be observed that the degree of ETC is more as compared to TC due to addition of network controllers. From the Figure-II, it can be observed that the total number of nodes of ETC is extremely large with increase in values of n as compared to TC which proved that ETC can pack more number of nodes as compared to TC. In the comparison the extended crossed cube and extended hypercube are omitted because they have equal number of nodes. In the similar manner, Figure-III shows a relationship between diameter and network dimension that takes diverse network topologies into account. The figure demonstrates that the ETC has improved value of diameter in comparison to the TC network with increased node count. A graph is also designed between cost of the network with respect to network dimension of various network topologies and offered in the Figure IV. The figure displays that the ETC has a reduced cost than EVH but higher than that of EHC, ECC. Message communication will be faster with reduced diameter as compared to other contemporary networks.

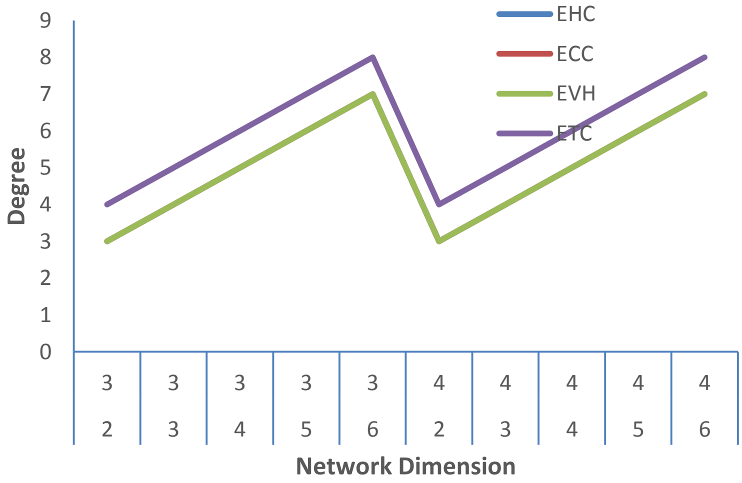


**Figure I: Comparison of Degree wrt Network Dimension**

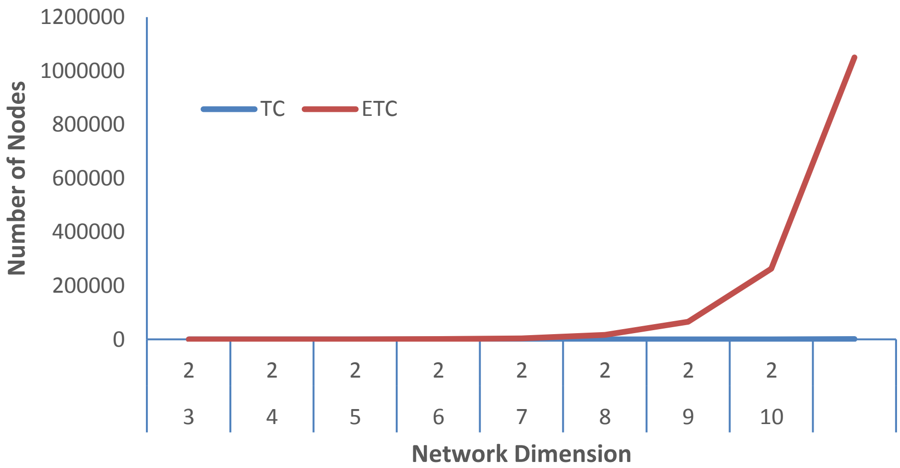


**Figure II: Comparison of Total Number of Nodes wrt Network Dimension**

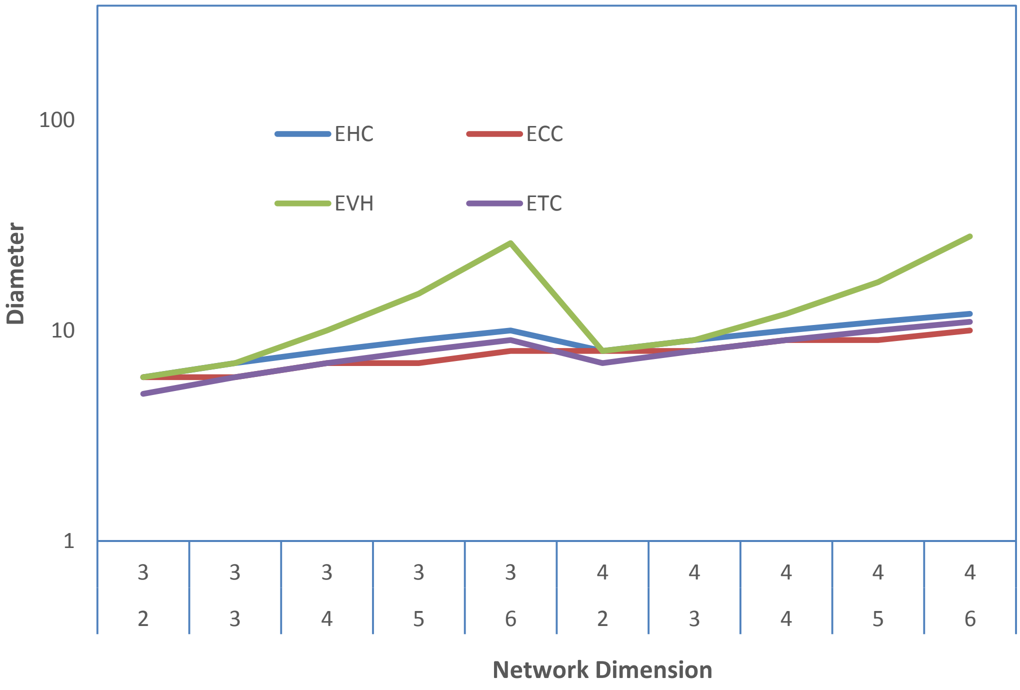


**Figure III: Comparison of Diameter wrt Network Dimension**

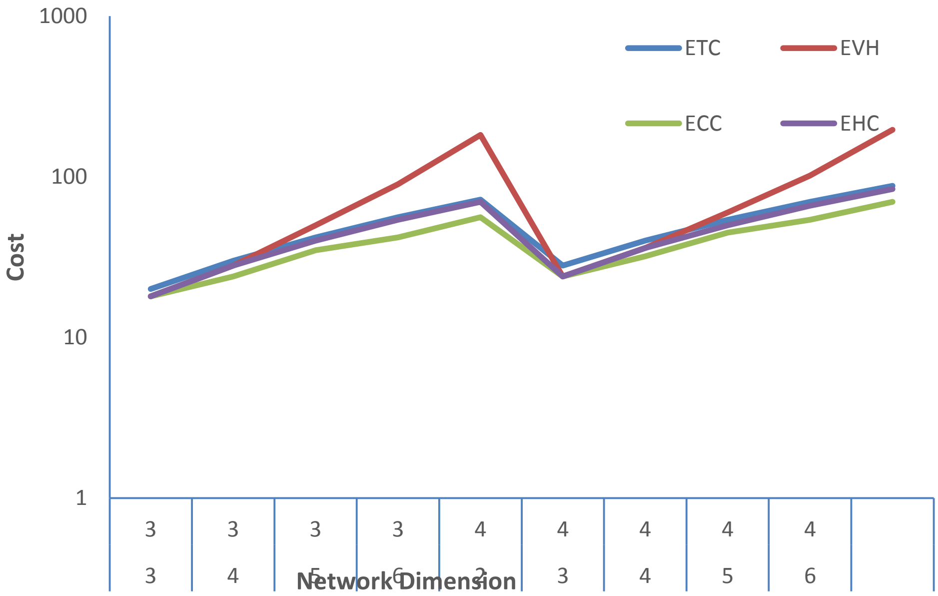




**Figure I: Comparison of Diameter wrt Network Dimension**

## V. CONCLUSIONS & FUTURE SCOPE

The current research work proposes a new hierarchical and expandable network ETC or Extended Tor Cube. It is a hierarchical recursive network and node fault resistant. The addition of network controllers help in faster communication and make the processing elements free for computational work. It has improved fault tolerance, dependability, and other performance metrics like cost, cost-effectiveness, and time-cost-effectiveness. The majority of the appealing qualities of EH and TC are utilized by the Extended Tor Cube network. Here, the main concentration is on its topological characteristics. However, in future works will be done to examine the robustness, fault tolerance, and other dynamic performance metrics for the proposed network. This network can introduce routing, broadcasting, implementation of parallel algorithms and evaluation of performance in the presence of faulty link for effective message transmission. It is possible to use an optimal routing technique for ETC network topology.

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