The radio D-distance in harmonic mean number of a modern graphs.

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Abstract:

A radio D-distance in harmonic mean labelling of a connect graph G is an injective map f from the vertex set $V(G)$ to the N such that for two distinct vertices u and v of G, $d^D(u,v) + \left\lceil \frac{2f(u)f(v)}{f(u)+f(v)} \right\rceil \geq diam^D(G) + 1$. The radio D-distance harmonic mean number of $f, rh^D n(f)$ is the maximum number assigned to any vertex of G On Radio D-distance harmonic mean number of some modern graphs.

Key words: Mangolian Tent graph, Diamond graph, Umbrella graph.
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Introduction:

A graph $G = (V, E)$ we mean a finite undirected graph without loops or multiple edges. The order and size of G are denoted by p and q respectively. Graph labelling was introduced by Alexander Rosa in 1967. Radio mean labelling was introduced by S.Somasundaram and R.Ponraj in 2004. Harmonic mean labelling was introduced by S.Somasundaram and S.S.Sandhya in 2012.

The concept of D-distance was introduced by D.Reddy Babu et al. The concept of radio D-distance was introduced by T.Nicholas and K.John Bosco in 2017.

Radio labelling (multi-level distance labelling)can be regarded as an extension of distance-two labelling which is motivated by the channel assignment problem introduced by Hale[6]. Chartrand et al. [2] introduced the concept of radio labelling of graph. Chartrand et al.[3] gave the upper bound for the radio number of path. The exact value for the radio number of path and cycle was given by Liu and Zhu [10]. However Chartrand et al.[2] obtained different values than Liu and Zhu [10]. They found the lower and upper bound for the radio number of cycle. Liu [9] gave the lower bound for the radio number of tree. The exact value for the radio number of Hypercube was given by R. Khennoufa and O. Togni [8]. M. M. Rivera et al. [20].

The concept of D-distance was introduced by D. Reddy Babu et al. [17,18,19]. If u, v are vertices of connected graph G, the D-length of a connected $u-v$ path s is defined as

$l^D(s)=l(s)+\deg(v)+\deg(u)+\sum\deg(w)$

where the sum runs over all international vertices w of s and $l(s)$ is the length of the path. The D-distance $d^D(u,v)$ between two vertices u,v of a connected graph. G is defined a $d^D(u,v)=\min\{l^D(s)\}$ where the minimum is taken over all $u-v$ path s in G. In other words,

$d^D(u,v)=\min\{l(s)+\deg(v)+\deg(u)+\sum\deg(w)$

where the sum runs over all intermediate vertices w in s and minimum is taken over all $u-v$ path s in G.

In this paper we introduce the concept the radio D-distance in harmonic mean number. A radio D-distance in harmonic labelling is a one to one mapping f from $V(G)$ to $\mathbb{N}$ satisfying condition.

$$d^D(u,v)+\left\lceil\frac{2f(u)f(v)}{f(u)+f(v)}\right\rceil\geq diam^D(G)+1.$$

For every $u,v\epsilon V(G)$. The span of a labelling f is the maximum integer that f maps to a vertex of G. The radio D-distance harmonic mean number of G, $rh^Dn(G)$ is the lowest span taken over all radio D-distance mean labelling of the graph G. The condition is called radio D-distance mean condition. In this paper we determine the radio harmonic mean number of some well-known graphs. The function $f:V(G)\rightarrow\mathbb{N}$ always represents injective map unless otherwise stated.

Theorem: 1.1

The radio d-distance in harmonic mean number of a mangolian tent graph

$$rh^Dn(MT_n)\leq 3n+7,\qquad if\ n\geq 3.$$

Proof:

Let $V(MT_n)=\{u_i,v_j\quad i=0,1,2,\dots,n;j=1,2,\dots,n\}$ be the vertex set and $E(MT_n)=\{u_0u_k,v_iv_{i+1},u_kv_k,u_iu_{i+1},\ \ i=1,2,\dots,n-1\,;j=1,2,\dots,n-1\,;k=1,2,\dots,n\}$ be the edge set. Some distance are $d^D(u_0,u_1)=n+4$, $d^D(u_n,v_1)=n+11$, $d^D(u_0,u_n)=n+4$; $d^D(v_1,v_2)=6$, $d^D(u_i,u_{i+1})=9, 2\leq i\leq n-2$; $d^D(u_i,v_i)=8,\ 2\leq i\leq n-2$, $d^D(v_i,v_{i+1})=7, 2\leq i\leq n-2$, $d^D(u_2,v_{n-1})=n+14$.

Then the $diam^D(MT_n)=n+14$. Then the radio D-distance in harmonic mean condition implies that

$$d^D(u,v)+\left\lceil\frac{2f(u)f(v)}{f(u)+f(v)}\right\rceil\geq diam^D(MT_n)+1$$

Fix $f(u_0)=n+7$

For $d^D(u_0,u_1)$

$$d^D(u_0,u_1)+\left\lceil\frac{2f(u_0)f(u_1)}{f(u_0)+f(u_1)}\right\rceil\geq n+15$$

$$1+n+3+\left\lceil\frac{2(n+7)f(u_1)}{(n+7)+f(u_1)}\right\rceil\geq n+15$$

$$\Rightarrow \left\lceil \frac{2(n+7)f(u_1)}{(n+7)+f(u_1)} \right\rceil \geq 11$$
$$\Rightarrow 2(n+7)f(u_1) \geq 10[(n+7)+f(u_1)]$$
$$\Rightarrow 2(n+7)f(u_1) - 10f(u_1) \geq 10(n+7)$$
$$\therefore\ f(u_1) = n+8$$

For $d^D(u_1,u_2)$

$$d^D(u_1,u_2) + \left\lceil \frac{2f(u_1)f(u_2)}{f(u_1)+f(u_2)} \right\rceil \geq n+15$$
$$1+7+\left\lceil \frac{2(n+8)f(u_2)}{(n+8)+f(u_2)} \right\rceil \geq n+15$$
$$\Rightarrow \left\lceil \frac{2(n+8)f(u_2)}{(n+8)+f(u_2)} \right\rceil \geq n+7$$
$$\Rightarrow 2(n+8)f(u_2) \geq (n+6)[(n+8)+f(u_2)]$$
$$\Rightarrow 2(n+8)f(u_2) - (n+6)f(u_2) \geq (n+6)(n+8)$$
$$\therefore\ f(u_2) = n+9$$

Since

$$f(u_i) = n+7+i, 3 \leq i \leq n$$
$$\therefore f(u_n) = 2n+7$$

Let $d^D(u_n,v_1)$

$$d^D(u_n,v_1) + \left\lceil \frac{2f(u_n)f(v_1)}{f(u_n)+f(v_1)} \right\rceil \geq n+15$$
$$3+3+3+2+n+\left\lceil \frac{2f(u_n)f(v_1)}{f(u_n)+f(v_1)} \right\rceil \geq n+15$$
$$\Rightarrow \left\lceil \frac{2(n+7)f(v_1)}{(2n+7)+f(v_1)} \right\rceil \geq 4$$
$$\Rightarrow 2(n+7)f(v_1) \geq 3[(2n+7)+f(v_1)]$$
$$\Rightarrow 2(n+7)f(v_1) - f(v_1) \geq 3(2n+7)$$
$$\therefore\ f(v_1) = 2n+8$$

For $d^D(v_1,v_2)$

$$d^D(v_1,v_2) + \left\lceil \frac{2f(v_1)f(v_2)}{f(v_1)+f(v_2)} \right\rceil \geq n+15$$
$$6+\left\lceil \frac{2f(v_1)f(v_2)}{f(v_1)+f(v_2)} \right\rceil \geq n+15$$

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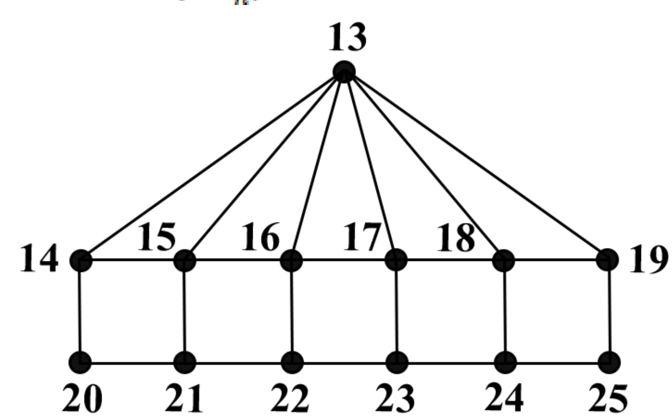
$\Rightarrow \left\lceil \frac{2(2n+8)f(v_2)}{(2n+8)+f(v_2)} \right\rceil \geq n+8$
$\Rightarrow 2(2n+8)f(v_2) \geq (n+9)[(2n+8)+f(v_2)]$
$\Rightarrow 2(2n+8)f(v_2) - (n+9)f(v_2) \geq (n+9)(2n+8)$
$\therefore\ f(v_2) = 2n+9$
For $d^D(u_1, v_1)$
$d^D(u_1, v_1) + \left\lceil \frac{2f(u_1)f(v_1)}{f(u_1)+f(v_1)} \right\rceil \geq n+15$
$1+5+\left\lceil \frac{2f(u_1)f(v_1)}{f(u_1)+f(v_1)} \right\rceil \geq n+15$
$\Rightarrow \left\lceil \frac{2(n+8)(2n+8)}{(n+8)+(2n+8)} \right\rceil \geq n+9$
$f(v_i) \geq 2n+7+i, \quad 3 \leq i \leq n$
$\therefore\ f(v_n) = 3n+7$. The greatest radio number is $3n+7$.
Hence $rh^D n(MT_n) \leq 3n+7$.



$rh^D n(MT_6) = 25$

Theorem: 1.2
The radio D-distance in harmonic mean number of a diamond graph $rh^D n(D_n) \leq 4n+4,\ n \geq 3$.

Proof:
Let $V(D_n) = \{u_i, v_j \quad i = 0,1,2,\ldots,n; j = 1,2,\ldots,n\}$ be the vertex set and $E(D_n) = \{u_0u_k, v_jv_{j+1}, v_0v_k, u_iu_{i+1}, \ \ i = 1,2,\ldots,n-1\,; j = 1,2,\ldots,n-1\,; k = 1,2,\ldots,n\}$

be the edge set. Some distance are $d^D(u_0,u_1) = n+4$; For $d^D(u_n,v_1) = n+11$,For $d^D(v_1,v_2) = 6, d^D(v_n,v_{n+1}) = n+4, d^D(u_i,u_{i+1}) = 9$; $d^D(v_i,v_{i+1}) = 9$; $d^D(u_0,v_{n+1}) = 2n+11; 2 \leq i \leq n-1,$

Then the $diam^D(D_n) = 2n+11$. Then the radio D-distance in harmonic mean condition implies that

$$d^D(u,v) + \left\lceil \frac{2f(u)f(v)}{f(u)+f(v)} \right\rceil \geq diam^D(D_n) + 1$$

Fix $f(u_0) = 2n+3$

For $d^D(u_0,u_1)$

$$d^D(u_0,u_1) + \left\lceil \frac{2f(u_0)f(u_1)}{f(u_0)+f(u_1)} \right\rceil \geq 2n+12$$

$$1+n+3+\left\lceil \frac{2f(u_0)f(u_1)}{f(u_0)+f(u_1)} \right\rceil \geq 2n+12$$

$$\Rightarrow \left\lceil \frac{2(2n+3)f(u_1)}{(2n+3)+f(u_1)} \right\rceil \geq n+8$$

$$\Rightarrow 2(n+3)f(u_1) \geq (n+7)[(2n+3)+f(u_1)]$$

$$\Rightarrow 2(n+3)f(u_1) - (n+7)\, f(u_1) \geq (n+7)(2n+3)$$

$$\therefore\ f(u_1) = n+4.$$

For $d^D(u_1,u_2)$

$$d^D(u_1,u_2) + \left\lceil \frac{2f(u_1)f(u_2)}{f(u_1)+f(u_2)} \right\rceil \geq 2n+12$$

$$1+7+\left\lceil \frac{2f(u_1)f(u_2)}{f(u_1)+f(u_2)} \right\rceil \geq 2n+12$$

$$\Rightarrow \left\lceil \frac{2(2n+4)f(u_2)}{(2n+4)+f(u_2)} \right\rceil \geq 2n+4$$

$$\Rightarrow 2(n+4)f(u_2) \geq (2n+3)[(2n+4)+f(u_2)]$$

$$\Rightarrow 2(n+4)f(u_2) - (2n+3)\, f(u_2) \geq (2n+3)(2n+4)$$

$$\therefore\ f(u_2) = 2n+5.$$

$$f(u_i) = 2n+3+i,\ 3 \leq i \leq n.$$

$$\therefore\ f(u_n) = 3n+3.$$

For $d^D(u_n,v_1)$

$$d^D(u_n,v_1) + \left\lceil \frac{2f(u_n)f(v_1)}{f(u_n)+f(v_1)} \right\rceil \geq 2n+12$$

$$3+9+n+\left\lceil \frac{2f(u_n)f(v_1)}{f(u_n)+f(v_1)} \right\rceil \geq 2n+12$$
$$\Rightarrow \left\lceil \frac{2(3n+3)f(v_1)}{(3n+3)+f(v_1)} \right\rceil \geq n$$
$$\Rightarrow 2(3n+3)f(v_1) \geq (n-1)[(3n+3)+f(v_1)]$$
$$\Rightarrow 2(3n+3)f(v_1)-(n-1)\, f(v_1) \geq (n-1)(3n+3)$$
$$\therefore\ f(v_1)=3n+4.$$

For $d^D(v_1,v_2)$

$$d^D(v_1,v_2)+\left\lceil \frac{2f(v_1)f(v_2)}{f(v_1)+f(v_2)} \right\rceil \geq 2n+12$$
$$1+3+4+\left\lceil \frac{2(3n+4)f(v_2)}{(3n+4)+f(v_2)} \right\rceil \geq 2n+12$$
$$\Rightarrow \left\lceil \frac{2(3n+4)f(v_2)}{(3n+4)+f(v_2)} \right\rceil \geq 2n+4$$
$$\Rightarrow 2(3n+4)f(v_2) \geq (2n+3)[(3n+4)+f(v_2)]$$
$$\Rightarrow 2(3n+4)f(v_2)-(2n-3)\, f(v_2) \geq (2n+3)(3n+4)$$
$$\therefore\ f(v_2)=3n+5.$$
$$\therefore\ f(v_i)=3n+3+i, \quad 3\leq i\leq n.$$

Since $f(v_n)=4n+3$.

For $d^D(v_n,v_{n+1})$

$$d^D(v_n,v_{n+1})+\left\lceil \frac{2f(v_n)f(v_{n+1})}{f(v_n)+f(v_{n+1})} \right\rceil \geq 2n+12$$
$$1+n+3+\left\lceil \frac{2f(v_n)f(v_{n+1})}{f(v_n)+f(v_{n+1})} \right\rceil \geq 2n+12$$
$$\Rightarrow \left\lceil \frac{2f(v_n)f(v_{n+1})}{f(v_n)+f(v_{n+1})} \right\rceil \geq n+8$$
$$\Rightarrow \left\lceil \frac{2(4n+3)f(v_{n+1})}{(4n+3)+f(v_{n+1})} \right\rceil \geq n+8$$
$$\Rightarrow 2(4n+3)f(v_{n+1}) \geq (n+7)[(4n+3)+f(v_{n+1})]$$
$$\Rightarrow 2(4n+3)f(v_{n+1})-(n-7)\, f(v_{n+1}) \geq (n+7)(4n+3)$$
$$\therefore\ f(v_{n+1})=4n+4.$$

∴ The radio d-distance in harmonic mean number of a diamond graph

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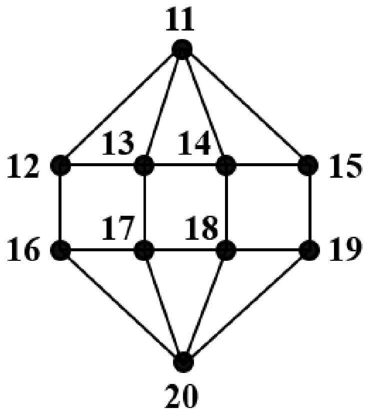
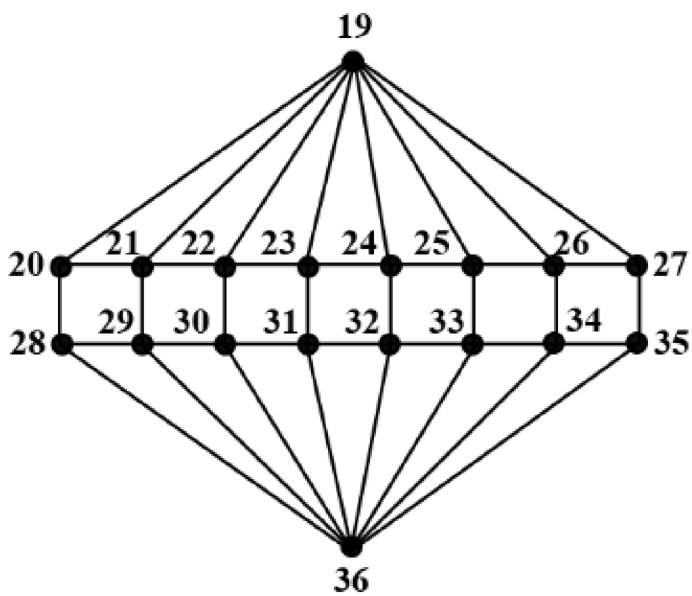
$rh^D n(D_n) \leq 4n + 4. if\ n \geq 3$.

Hence the proof:



$rh^D n(D_4) = 20$



$rh^D n(D_8) = 36$

Theorem: 1.3

The radio d-distance in harmonic mean number of a umbrella graph

$rh^D n(u_n) \leq 2n + 14, if\ n \geq 3.$

Proof:

Let $V(u_n) = \{u_i, v_j, \quad i = 1,2, \ldots, n: j = 1, \ldots, 5\}$ be the vertex set and

$E(u_n) = \{v_j v_{j+1}, u_k v_1, u_i u_{i+1}, \quad i = 1,2, \ldots, n - 1; j = 1,2,3,4; k = 1,2, \ldots, n\}$ be the edge set. Some distance are

$d^D(u_1,u_2) = 6,\ d^D(u_2,v_1) = n+5,\ d^D(u_n,v_1) = n+4,\ \ d^D(v_1,v_2) = n+4,\ d^D(v_2,v_5) = n+16$

. Then the $diam^D(u_n) = n+16$.

Then the radio d-distance harmonic mean condition implies that

$$d^D(u,v) + \left\lceil \frac{2f(u)f(v)}{f(u)+f(v)} \right\rceil \geq diam^D(u_n) + 1$$

Fix $f(u_1) = n+10$

For $d^D(u_1,u_2)$

$$d^D(u_1,u_2) + \left\lceil \frac{2f(u_1)f(u_2)}{f(u_1)+f(u_2)} \right\rceil \geq n+17$$

$$1+2+3 + \left\lceil \frac{2f(u_1)f(u_2)}{f(u_1)+f(u_2)} \right\rceil \geq n+17$$

$$\Rightarrow \left\lceil \frac{2(2n+10)f(u_2)}{(n+10)+f(u_2)} \right\rceil \geq n+11$$

$$\Rightarrow 2(n+10)f(u_2) \geq (n+10)[(n+10)+f(u_2)]$$

$$\Rightarrow 2(n+10)f(u_2) - (n+10)\,f(u_2) \geq (n+10)(n+10)$$

$$\therefore\ f(u_2) = n+11$$

$$f(u_i) = n+9+i, \quad 3 \leq i \leq n.$$

$$\therefore\ f(u_n) = 2n+9.$$

For $d^D(u_n,v_1)$

$$d^D(u_n,v_1) + \left\lceil \frac{2f(u_n)f(v_1)}{f(u_n)+f(v_1)} \right\rceil \geq n+17$$

$$1+2+n+1 + \left\lceil \frac{2(2n+9)f(v_1)}{(2n+9)+f(v_1)} \right\rceil \geq n+17$$

$$\Rightarrow \left\lceil \frac{2(2n+9)f(v_1)}{(2n+9)+f(v_1)} \right\rceil \geq 13$$

$$\Rightarrow 2(n+9)f(v_1) \geq 12[(2n+9)+f(v_1)]$$

$$\Rightarrow 2(n+9)f(v_1) - 12\,f(v_1) \geq 12(2n+9)$$

$$\Rightarrow\ f(v_1) = 2n+10$$

For $d^D(v_1,v_2)$

$$d^D(v_1,v_2) + \left\lceil \frac{2f(v_1)f(v_2)}{f(v_1)+f(v_2)} \right\rceil \geq n+17$$

$$1+2+n+1+\left\lceil\frac{2f(v_1)f(v_2)}{f(v_1)+f(v_2)}\right\rceil \geq n+17$$

$$\Rightarrow \left\lceil\frac{2(2n+10)f(v_2)}{(2n+10)+f(v_2)}\right\rceil \geq 13$$

$$\Rightarrow 2(n+10)f(v_2) \geq 12[(2n+10)+f(v_2)]$$

$$\Rightarrow 2(n+10)f(v_2) - 12\,f(v_2) \geq 12(2n+10)$$

$$\Rightarrow\; f(v_2) = 2n+11$$

For $d^D(v_2,v_3)$

$$d^D(v_2,v_3)+\left\lceil\frac{2f(v_2)f(v_3)}{f(v_2)+f(v_3)}\right\rceil \geq n+17$$

$$1+4+\left\lceil\frac{2f(v_2)f(v_3)}{f(v_2)+f(v_3)}\right\rceil \geq n+17$$

$$\Rightarrow \left\lceil\frac{2(2n+11)f(v_3)}{(2n+11)+f(v_3)}\right\rceil \geq n+12$$

$$\Rightarrow 2(n+11)f(v_3) \geq (n+11)[(2n+11)+f(v_3)]$$

$$\Rightarrow 2(n+11)f(v_3) - (n+11)(v_3) \geq (n+11)(2n+11)$$

$$\Rightarrow\; f(v_3) = 2n+12$$

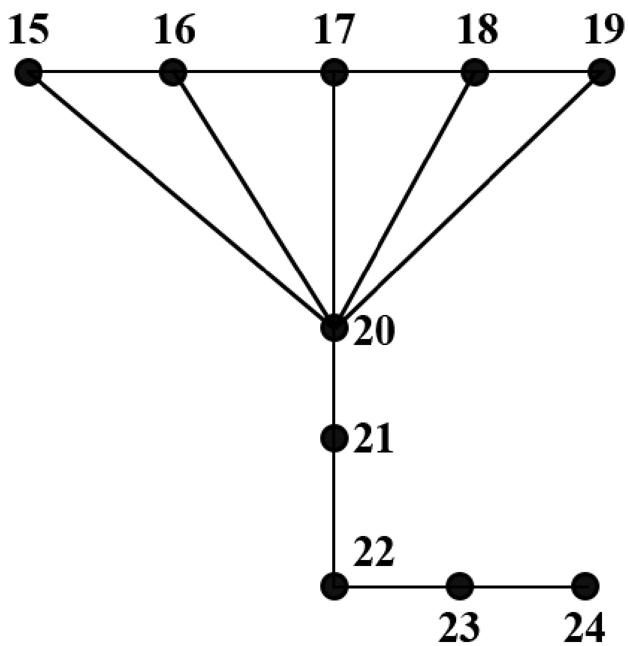
$$f(v_i) = 2n+9+i, \quad i = 3,4,5.$$

$$\therefore\; f(v_5) = 2n+14.$$

The radio d-distance in harmonic mean number of a umbrella graph is $2n+14$.

$\therefore rh^D n(u_n) \leq 2n+14, if\ n \geq 3.$

Hence the proof.



$rh^D n(u_5) = 24$

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