Fixed Point Results related to soft set

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Abstract:
In The present paper some basic definitions of soft sets are reviewed also a new extended result is established for random variable in soft s- metric spaces.

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1.Introduction& Preliminaries

Numerous questions of physical world lead to nonlinear problems. Few of them are- behavior of plastic materials, moments of viscous fluids, chemical reaction; processes in nuclear reactors, nonlinear oscillation in physics, chemistry and biology, gravitational effect of masses in the content of general relativity etc. Tease nonlinear problems can be reduced to nonlinear operator equations. Fixed point theorems constitute an important tool for proving the existence of solutions to such equations. The Banach contraction principle is the most celebrated fixed point theorem and has been generalized in various direction. Fixed point problem for contractive mapping in metric spaces with partial order have been studied by many authors]). In 1999, Molodstov initiated a novel concept of soft set theory as new mathematical tool for dealing with uncertainty. Application of as soft set theory in other disciplines and real life problems are now catching momentum. Maji et al. [15] gave first practical application of soft sets in decision making problem. Jungck [10] introduced the notion of fixed point for commuting map. Further, Jungck [11] introducing the notion of compatible mapping which is generalization of commuting maps. Pant [18] gave the concept of non-compatible maps. Aamri and Moutawakil [2] introduced E.A. property which is generalization of non-compatible mapping in metric spaces and prove some common fixed-point theorems for non-compatible mapping under strict contractive conditions.

Preliminaries

Definition 2.1. [14] Let A be a set of parameters and E be an initial universe. Let P(E) denote the power set of E. A pair (F,A) is called a soft set over E, where F is a mapping given by F : A → P(E).

Definition 2.2. [15] Let (F,A) and (G,A) be two soft sets over a common universe E.

(i) (F,A) is said to be null soft set, denoted by $\emptyset$, if for all $\lambda \in A$, $F(\lambda) = \emptyset$. (F,A) is said to an absolute soft set denoted by $\tilde{E}$, if for all $\lambda \in A$, $F(\lambda) = E$.

(ii) (F,A) is said to be a soft subset of (G,A) if for all $\lambda \in A$, $F(\lambda) \subseteq G(\lambda)$ and it is denoted by (F,A) $\tilde{\subseteq}$(G,A). (F,A) is said to be a soft upper set of (G,A) if (G,A) is a soft subset of (F,A). We denote it by (F,A) $\tilde{\supseteq}$ (G,A). (F,A) and (G,A) is said to be equal if (F,A) is a soft subset of (G,A) and (G,A) is a soft subset of (F,A).

(iii) The intersection (H,A) of (F,A) and (G,A) over E is defined as H(λ) = F(λ) ∩ G(λ) for all λ ∈ A. We write (F,A) $\tilde{\cap}$ (G,A) = (H,A).
(iv) The union (H,A) of (F,A) and (G,A) over E is defined as H(λ) = F(λ)∪G(λ) for all λ ∈ A. We write (F,A) $\tilde{\cup}$ (G,A) = (H,A).
(v) The cartesian product (H,A) of (F,A) and (G,A) over E denoted by (H,A) = (F,A)$\tilde{\times}$ (G,A), is defined as H(λ) = F(λ) × G(λ) for all λ ∈ A.
(vi) The difference (H,A) of (F,A) and (G,A) over E denoted by (F,A)$\tilde{/}$ (G,A) = (H,A), is defined as H(λ) = F(λ)\G(λ) for all λ ∈ A.
(vii) The complement of (F,A) is defined as $(F,A)^c = (F^c,A)$, where F^c : A → P(E) is a mapping given by $F^c(\lambda)$ = B\F(λ) for all λ ∈ A. Clearly, we have $\widetilde{E^c}$ = Φ and $\Phi^c = \tilde{E}$.

Definition 2.3: [7,9] Let A be a set of parameters and R be the set of real numbers and B(R) be the collection of all non-empty bounded subsets of R. Then a mapping F : A → B(R) is called a soft real set, denoted by (F,A). If specifically (F,A) is a singleton soft set, then after identifying (F,A) with the corresponding soft element, it will be called a soft real number. The set of all soft real numbers is denoted by R(A) and the set of non-negative soft real numbers by R(A)∗.
Let $\tilde{r}$ and $\tilde{s}$ be two soft real numbers. Then the following statements hold:

- $\tilde{r} \tilde{\leq} \tilde{s}$, if $\tilde{r}(\lambda) \leq \tilde{s}(\lambda)$, $\forall \lambda \in A$,
- $\tilde{r} \tilde{<} \tilde{s}$, if $\tilde{r}(\lambda) < \tilde{s}(\lambda)$, $\forall \lambda \in A$,
- $\tilde{r} \tilde{\geq} \tilde{s}$, if $\tilde{r}(\lambda) \geq \tilde{s}(\lambda)$, $\forall \lambda \in A$,
- $\tilde{r} \tilde{>} \tilde{s}$, if $\tilde{r}(\lambda) > \tilde{s}(\lambda)$, $\forall \lambda \in A$,

Definition 2.4: [9] Let X be a non-empty set and A be non-empty a parameter set. A mapping d:$\mathbb{SE}(\tilde{X}) \times \mathbb{SE}(\tilde{X}) \to R(A)^*$ is said to be a soft metric on the soft set $\tilde{X}$ if d satisfies the following conditions: 387

(i) $d(\tilde{x},\tilde{y}) \tilde{\geq} \tilde{0}$, for all $,\tilde{x},\tilde{y} \in \tilde{X}$.
(ii) $d(\tilde{x},\tilde{y})$ = ¯0 if and only if $\tilde{x} = \tilde{y}$.
(iii) $d(\tilde{x},\tilde{y}) = d(\tilde{y},\tilde{x})$ for all $\tilde{x},\tilde{y} \tilde{\in} \tilde{X}$.
(iv) $d(\tilde{x},\tilde{y}) \tilde{\leq} d(\tilde{x},\tilde{z}) + d(\tilde{z},\tilde{y})$ for all $\tilde{x},\tilde{y},\tilde{z} \tilde{\in} \tilde{X}$.

The soft $\tilde{X}$ with a soft metric d on $\tilde{X}$ is said to be a soft metric space and denoted by $(\tilde{X},d,A)$ or $(\tilde{X},d)$.

Definition 2.5: [20] Let X be a nonempty set. An S-metric on X is a function $S: X \times X \times X \longrightarrow [0,\infty)$ that satisfies the following conditions holds for all $x, y, z, a \in X$.
(1) $S(x,y,z) \geq 0$,
(2) $S(x,y,z) = 0$ iff $x = y = z$,
(3) $S(x,y,z) \leq S(x,x,a) + S(y,y,a) + S(z,z,a)$.
The pair (X,S) is called a S-metric space.

Definition 2.6: [20] Let (X,S) be a S-metric space for $r > 0$ and $x \in X$, we define the open ball $B_S(x,r)$ and the closed ball $B_S[x,r]$ with centre x and radious r as follows:
$B_S(x,r) = \{y \in X: S(y,y,x) < r\}$,
$B_S[x,r] = \{y \in X: S(y,y,x) \leq r\}$.
The topology induced by the S-metric is the topology generated by the base of all open ball in X.

Definition 2.7: [20] Let (X,S) be a S-metric space.
(i) A sequence $\{x_n\} \subset X$ converges to $x \in X$ if $S(x_n,x_n,x) \longrightarrow 0$ as $n \longrightarrow \infty$. That is, for each $\varepsilon > 0$, there exists $n_0 \in \mathbb{N}$, such that for all $n \geq n_0$, we have, $S(x_n,x_n,x) < \varepsilon$.We write for $\lim_{n\to\infty} x_n \longrightarrow x$.
(ii) A sequence $\{x_n\} \subset X$ Cauchy sequence if $S(x_n,x_n,x_m) \longrightarrow 0$ as $n,m \longrightarrow \infty$. That is, for each $\varepsilon > 0$, there exists $n_0 \in \mathbb{N}$, such that for all $n,m \geq n_0$, we have, $S(x_n,x_n,x_m) < \varepsilon$.
(iii) The S-metric space (X,S) is complete if every Cauchy sequence is convergent.
(iv) Let τ be the set of all A$\subset X$ with $x \in A$ and there exists $r > 0$ such that $B_S(x,r) \subset A$. Then τ is a topolpgy on X (induced by S-metric).

Definition 2.8:[13]Let(X,S) and (X',S') be two S-metric space. A function $f:(X,S) \longrightarrow (X',S')$ is said to be continuous at point $a \in X$ if for every sequence $\{x_n\}$ in X with $S(x_n,x_n,a) \longrightarrow 0$, $S'(f(x_n),f(x_n),f(a)) \longrightarrow 0$. We say that f is continuous on X if f is continuous at every point $a \in X$.

Lemma 4: [20] Let (X,S) be a S-metric space. There exists sequence $\{x_n\}$ and $\{y_n\}$ such that $\lim_{n\to\infty} x_n \longrightarrow x$ and $\lim_{n\to\infty} y_n \longrightarrow y$, then

$$\lim_{n\to\infty} S(x_n, x_n, y_n) \longrightarrow S(x,x,y).$$

Definition 2.9: [20] Let (X,S) be a S-metric space. A mapping T: $X \longrightarrow X$ is called contraction if $S(T_x, T_X, T_Y) \le \alpha . S(x,x,y)$ for all $x, y \in X$ with $\alpha \in [0,1)$.

2.10: Throughout this paper (Ω, Σ) denotes a measurable space, C is non empty subset of H, $\xi : \Omega \to X$ is a measurable selector.

2.10.1 **Measurable function:** A function $f : \Omega \to C$ is said to be measurable if $f^{-1}(B \cap C) \in \Sigma$ foe every Borel subset B of H.

2.10.**2 Random operator**: A function $f : \Omega \times C \to C$ is said to be random operator, if $F(.,X) : \Omega \to C$ is measurable for every X $\in$ C

2.10.3. **Continuous Random operator**: A random operator $F : \Omega \times C \to c$ is said to be continuous if for fixed $t \in \Omega, F(t,.) : C \to C$ *is continuous*

2.10.4. **Random fixed point: A measurable function** $g : \Omega \to C$ is said to be random fixed point of the random operator $F : \Omega \times C \to C$, *if* $F(t, g(t)) = g(t), \forall t \in \Omega$

3. Soft S-metric spaces:

In the following we always suppose (S,A) is a soft S-metric in soft Banach spaces. $\tilde{E}$ with Int$(S,A) \neq \Phi$ and $\precapprox$ is soft partial ordering with respect to (S,A).

Definition 3.1: Let X be a non-empty set and $\tilde{X}$ be absolute soft set. A mapping S: $\mathbb{SE}(\tilde{X})$×$\mathbb{SE}$ $(\tilde{X})$×$\mathbb{SE}$ $(\tilde{X}) \longrightarrow [0,\infty)$is said to be a soft S-metric on $\tilde{X}$ if S satisfies the following axioms holds for all $\xi\tilde{x}, \widetilde{\xi y}, \widetilde{\xi z}, \widetilde{\xi a} \mathbin{\tilde{\in}} \tilde{X}$,

(S1) $\Theta \mathbin{\tilde{\leqq}} S(\xi\,\tilde{x}, \widetilde{\xi y}, \tilde{z})$,

(S2) $S(\xi\,\tilde{x}, \widetilde{\xi y}, \xi\,\tilde{z}) = \Theta$ iff $\widetilde{\xi x} = \widetilde{\xi y} = \xi\,\tilde{z}$

(S3) $S(\widetilde{\xi x}, \widetilde{\xi y}, \widetilde{\xi z}) \mathbin{\tilde{\leqq}} S(\widetilde{\xi x}, \xi\,\tilde{x}, \widetilde{\xi a}) + S(\widetilde{\xi y}, \xi\,\tilde{y}, \widetilde{\xi a}) + S(\widetilde{\xi z}, \widetilde{\xi z}, \widetilde{\xi a})$

Then, the soft set $\tilde{X}$ with a soft S-metric$S$ on $\tilde{X}$ is called a soft S-metric space and is denoted by $(\tilde{X}, S, A)$ or $(\tilde{X}, S)$.Then $(\tilde{X}, S)$ is called soft S-metric space with random variable.

Definition 3.2: Let $(\tilde{X}, S)$ be soft S-metric space. Let $\{\widetilde{\xi x_n}\}$ be a sequence of soft elements in $\tilde{X}$ and $\widetilde{\xi x} \in \tilde{X}$. $\{\widetilde{\xi x_n}\}$ converges to $\tilde{x}$ if $S(\xi\,\widetilde{x_n}, \widetilde{\xi x_n}, \widetilde{\xi x}) \longrightarrow 0$ as $n \longrightarrow \infty$. That is, for each $\tilde{\varepsilon} \mathbin{\tilde{>}} 0$, there exists $n_0 \in \mathbb{N}$, such that for all $n \mathbin{\tilde{\geqq}} n_0$, we have, $S(\xi\,\widetilde{x_n}, \widetilde{\xi x_n}, \widetilde{\xi x}) \mathbin{\tilde{<}} \tilde{\varepsilon}$.We denote this $\lim_{n\to\infty} \widetilde{\xi x_n} \longrightarrow \widetilde{\xi x}$.

Definition 3.3: Let $(\tilde{X}, S)$ be soft S-metric space. Let $\{\xi\,\widetilde{x_n}\}$ be a sequence of soft elements in $\tilde{X}$. A sequence $\{\xi\, x_n\}$ Cauchy sequence if $S(\widetilde{\xi x_n}, \widetilde{\xi x_n}, \widetilde{\xi x_m}) \longrightarrow 0$ as $n, m \longrightarrow \infty$. That is, for each $\tilde{\varepsilon} \mathbin{\tilde{>}} 0$, there exists $n_0 \in \mathbb{N}$ (there is natural number $\mathbb{N}$), such that for all $n, m \mathbin{\tilde{\geqq}} n_0$, we have, $S(\xi\,\widetilde{x_n}, \widetilde{\xi x_n}, \widetilde{\xi x_m}) \mathbin{\tilde{<}} \tilde{\varepsilon}$.

Definition 3.4: Let $(\tilde{X}, S)$ be soft S-metric space. If every Cauchy sequence of soft elements in $\tilde{X}$ is convergent in $\tilde{X}$, then $(\tilde{X}, S)$ is called complete soft S-metric space.

Definition 3.5: Letτ be the soft set of all A$\mathbin{\tilde{\subset}}$ $\tilde{X}$ with $\xi\,\tilde{x} \in A$ and there exists $\xi\, r \mathbin{\tilde{>}} 0$ such that $B_S(\xi\,\tilde{x}, \xi\, r) \mathbin{\tilde{\subset}} A$. Then τ is a topolpgy on $\tilde{X}$ with random variable .

Definition 3.6: Let $(\tilde{X}, S)$ be soft S-metric space. A mapping T:$\tilde{X} \longrightarrow \tilde{X}$ is called contraction with respect to random variable if

$S(T\xi\,\tilde{x}, T\widetilde{\xi x}, T\widetilde{\xi y}) \mathbin{\tilde{\leqq}} \alpha . S(\xi\,\tilde{x}, \widetilde{\xi x}.\widetilde{\xi y})$ for all $\widetilde{\xi x}, \widetilde{\xi y} \in \tilde{X}$ with $\alpha \in [0,1)$.

4. Main Results:

Theorem 4.1:$\tilde{X}$ be a soft set. Let the metric space $(\tilde{X}, S)$ be complete soft S-metric space and. supposing the map Q, T: $\tilde{X} \longrightarrow \tilde{X}$ be two self-mappings.

(Ω, Σ) denotes a measurable space, $\xi : \Omega \to X$ is a measurable selector,.satisfies the following condition :

- $Q(\tilde{X}) \mathbin{\tilde{\subseteq}} T(\tilde{X})$; Q *or* T is continuous;
- $S(Q\xi\,\tilde{x}, Q\xi\,\tilde{y}, Q\widetilde{\xi z}) \le \gamma . S(T\xi\,\tilde{x}, T\xi\,\tilde{y}, Q\widetilde{\xi z})$

$$+\delta\left[\frac{S(Q\xi\,\tilde{x},Q\xi\,\tilde{y},T\widetilde{\xi\,z})+S(T\xi\,\tilde{x},T\xi\,\tilde{y},Q\xi\,\tilde{z})}{1+S(Q\xi\,\tilde{x},Q\xi\,\tilde{y},T\widetilde{\xi\,z}).S(T\xi\,\tilde{x},T\widetilde{\xi\,y},Q\widetilde{\xi\,z})}\right]$$

[4.1.*]

For all $\widetilde{\xi\,x},\widetilde{\xi\,y},\widetilde{\xi\,z}\,\tilde{\in}\,\tilde{X}$ and $\gamma,\delta\geq 0$ with $\gamma+\delta<1/3$. Then $Q\ and\ T$ has unique common random soft fixed point if $Q\ and\ T$ are compatible mapping.

Proof: Lert $\xi\,\widetilde{x_0}$ be arbitrary in $\tilde{X}$, define a sequence $\{\xi\,\widetilde{y_n}\}\ in\ \tilde{X}$ such that

$\xi\,\widetilde{y_n}=Q\xi\,\widetilde{x_n}=\ T\xi\,\widetilde{x_{n+1}}$ for $n=0,1,2,\ldots..$

Now to show that $\{\xi\,\widetilde{y_n}\}$ is Cauchy sequence consider

$$S(\widetilde{y_n},\widetilde{y_n},\widetilde{y_{n+1}})=S(Q\xi\,\widetilde{x_n},Q\xi\,\widetilde{x_n},Q\xi\,\widetilde{x_{n+1}})$$

From [4.1.1]

$$S(Q\xi\,\widetilde{x_n},Q\xi\,\widetilde{x_n},Q\xi\,\widetilde{x_{n+1}})$$

$$\leq\gamma.S(T\xi\,\widetilde{x_n},T\xi\,\widetilde{x_n},Q\xi\,\widetilde{x_{n+1}})+\delta\left[\frac{S(Q\xi\,\widetilde{x_n},Q\xi\,\widetilde{x_n},T\xi\,\widetilde{x_{n+1}})+S(T\xi\,\widetilde{x_n},T\xi\,\widetilde{x_n},Q\xi\,\widetilde{x_{n+1}})}{1+S(Q\widetilde{x_n},Q\widetilde{x_n},T\widetilde{x_{n+1}}).S(T\widetilde{x_n},T\widetilde{x_n},Q\widetilde{x_{n+1}})}\right]$$

$$\leq\gamma.S(\xi\,\widetilde{y_{n-1}},\xi\,\widetilde{y_{n-1}},\xi\,\widetilde{y_{n+1}})\quad+\delta\left[\frac{S(\xi\,\widetilde{y_n},\xi\,\widetilde{y_n},\xi\,\widetilde{y_n})+S(\xi\,\widetilde{y_{n-1}},\xi\,\widetilde{y_{n-1}},\xi\,\widetilde{y_{n+1}})}{1+S(\xi\,\widetilde{y_n},\xi\,\widetilde{y_n},\xi\,\widetilde{y_n}).S(\xi\,\widetilde{y_{n-1}},\xi\,\widetilde{y_{n-1}},\xi\,\widetilde{y_{n+1}})}\right]$$

$$S(\xi\,\widetilde{y_n},\xi\,\widetilde{y_n},\xi\,\widetilde{y_{n+1}})\leq(2\gamma+2\delta)\,S(\xi\,\widetilde{y_{n-1}},\xi\,\widetilde{y_{n-1}},\xi\,\widetilde{y_n})+(\gamma+\delta)\,S(\xi\,\widetilde{y_n},\xi\,\widetilde{y_n},\xi\,\widetilde{y_{n+1}})$$

$$S(\xi\,\widetilde{y_n},\xi\,\widetilde{y_n},\xi\,\widetilde{y_{n+1}})\leq\frac{(2\gamma+2\delta)}{(1-\gamma-\delta)}S(\xi\,\widetilde{y_{n-1}},\xi\,\widetilde{y_{n-1}},\xi\,\widetilde{y_n})$$

$S(\xi\,\widetilde{y_n},\xi\,\widetilde{y_n},\xi\,\widetilde{y_{n+1}})\leq\phi.S(\xi\,\widetilde{y_{n-1}},\xi\,\widetilde{y_{n-1}},\xi\,\widetilde{y_n})$

Where $\phi=\frac{(2\gamma+2\delta)}{(1-\gamma-\delta)}<1$

$S(\widetilde{y_n},\widetilde{y_n},\widetilde{y_{n+1}})\leq\phi^n.S(\widetilde{y_{n-2}},\widetilde{y_{n-2}},\widetilde{y_{n-1}})$ (4.1. **)

It is necessary to show that $\{\xi\,\widetilde{y_n}\}$ is Cauchy sequence , consider $m,n\in N$ with $m>n$ and calculating as usual

$S(\xi\,\widetilde{y_n},\xi\,\widetilde{y_n},\xi\,\widetilde{y_m})\leq 2\,\frac{\phi^n}{1-\phi}S(\xi\,\widetilde{y_0},\xi\,\widetilde{y_0},\xi\,\widetilde{y_1})$

On taking limit $m,n\longrightarrow\infty$, we have

$$\lim_{n,m\to\infty}S(\xi\,\widetilde{y_n},\xi\,\widetilde{y_n},\xi\,\widetilde{y_m})=0$$

$\Rightarrow\{\widetilde{y_n}\}$ is a S-Cauchy sequence in $\tilde{X}$ for random variable Since $(\tilde{X},S)$ is complete soft S-metric space so there must exists $\xi\,\tilde{z}$ in $\tilde{X}$ such that,

$$\lim_{n\to\infty}\xi\,\widetilde{y_n}=\lim_{n\to\infty}Q\xi\,\widetilde{x_n}=\lim_{n\to\infty}T\xi\,\widetilde{x_{n+1}}=\widetilde{\xi\,z}$$

$Q\ and\ T$ is continuous for definiteness so taking T is continuous, therefore

$$\lim_{n\to\infty}T\xi\,\widetilde{y_n}=\lim_{n\to\infty}TQ\xi\,\widetilde{x_n}=\lim_{n\to\infty}TT\xi\,\widetilde{x_{n+1}}=T\xi\,\tilde{z}$$

Further $Q\ and\ T$ are compatible, thus

$$\lim_{n\to\infty}S(QT\xi\,\widetilde{x_n},QT\xi\,\widetilde{x_n},TQ\xi\,\widetilde{x_n})=0$$

Which implies,

$\lim_{n\to\infty}QT\xi\,\widetilde{x_n}=T\xi\,\tilde{z}$ (4.1.***)

$$S(QT\xi\,\widetilde{x_n},QT\xi\,\widetilde{x_n},Q\xi\,\widetilde{x_n})\leq\gamma.S(TT\xi\,\widetilde{x_n},TT\xi\,\widetilde{x_n},Qx\widetilde{\xi}_n)$$

$$+\ \delta\left[\frac{S(QT\xi\,\widetilde{x_n},QT\xi\,\widetilde{x_n},T\xi\,\widetilde{x_n})+S(TT\xi\,\widetilde{x_n},TT\xi\,\widetilde{x_n},Q\xi\,\widetilde{x_n})}{1+S(QT\xi\,\widetilde{x_n},QT\xi\,\widetilde{x_n},T\xi\,\widetilde{x_n}).S(TT\xi\,\widetilde{x_n},TT\xi\,\widetilde{x_n},Q\xi\,\widetilde{x_n})}\right]$$

Taking limit$n\longrightarrow\infty$

$$S(T\widetilde{\xi\,z},T\xi\,\tilde{z},\widetilde{\xi\,z})\leq\ \gamma.S(T\xi\,\tilde{z},T\xi\,\tilde{z},\widetilde{\xi\,z})+\delta\left[\frac{S(T\xi\,\tilde{z},T\xi\,\tilde{z},\widetilde{\xi\,z})+S(T\widetilde{\xi\,z},T\xi\,\tilde{z},\xi\,\tilde{z})}{1+S(T\xi\,\tilde{z},T\xi\,\tilde{z},\widetilde{\xi\,z}).S(T\xi\,\tilde{z},T\xi\,\tilde{z},\widetilde{\xi\,z})}\right]$$

$$\leq(\ \gamma+2\delta)S(T\xi\,\tilde{z},T\xi\,\tilde{z},\widetilde{\xi\,z})$$

The above inequality is possible only if $S(T\xi\,\tilde{z},T\xi\,\tilde{z},\widetilde{\xi\,z})=0$ iff $T\xi\,\tilde{z}=\widetilde{\xi\,z}$.

$$S(Q\xi\,\widetilde{x_n},Q\xi\,\widetilde{x_n},Q\xi\,\tilde{z})\leq\ \gamma.S(T\xi\,\widetilde{x_n},T\xi\,\widetilde{x_n},Q\xi\,\tilde{z})$$

$$+\ \delta\left[\frac{S(Q\xi\,\widetilde{x_n},Q\xi\,\widetilde{x_n},T\xi\,\tilde{z})+S(T\xi\,\widetilde{x_n},T\xi\,\widetilde{x_n},Q\xi\,\tilde{z})}{1+S(Q\xi\,\widetilde{x_n},Q\xi\,\widetilde{x_n},T\widetilde{\xi\,z}).S(T\xi\,\widetilde{x_n},T\xi\,\widetilde{x_n},Q\xi\,\tilde{z})}\right]$$

Taking limit$n\longrightarrow\infty$

$$S(\widetilde{\xi z},\widetilde{\xi z},Q\widetilde{\xi z}) \leq \gamma.S(\widetilde{\xi z},\widetilde{\xi z},Q\xi\tilde{z}) + \delta\left[\frac{S(\xi\tilde{z},\widetilde{\xi z},\xi\tilde{z}) + S(\xi\tilde{z},\widetilde{\xi z},Q\xi\tilde{z})}{1+S(\xi\tilde{z},\xi\tilde{z},\xi\tilde{z}).S(\xi\tilde{z},\xi\tilde{z},Q\widetilde{\xi z})}\right]$$

$S(\widetilde{\xi z},\widetilde{\xi z},Q\widetilde{\xi z}) \leq (\gamma+\delta)S(\xi\tilde{z},\widetilde{\xi z},Q\widetilde{\xi z})$

It is possible only if $S(\xi\tilde{z},\widetilde{\xi z},Q\xi\tilde{z}) = 0$ iff $Q\xi\tilde{z} = \widetilde{\xi z}$. Thus $\xi\tilde{z}$ is common random soft fixed point of $Q\ and\ T$.

Uniqueness: It can be proved easily as usual way.

Theorem 4.2: Let the metric space $(\tilde{X},S)$ be complete soft S-metric space and the map $Q,T:\tilde{X}\longrightarrow\tilde{X}$ be a pair of weakly compatible self-mappings satisfies the following condition:

- $Qand\ T$ Satisfy property (E.A.) ; $T(\tilde{X})$ is closed subspace of $\tilde{X}$;
- $S(Q\xi\tilde{x},Q\xi\tilde{y},Q\widetilde{\xi z}) \leq \gamma.S(T\xi\tilde{x},T\xi\tilde{y},Q\xi\tilde{z}) + \delta\left[\frac{S(Q\xi\tilde{x},Q\xi\tilde{y},T\xi\tilde{z})+S(T\xi\tilde{x},T\xi\tilde{y},Q\xi\tilde{z})}{1+S(Q\xi\tilde{x},Q\xi\tilde{y},T\xi\tilde{z}).S(T\xi\tilde{x},T\xi\tilde{y},Q\xi\tilde{z})}\right]$ [4.2.*]

For all $\widetilde{\xi x},\widetilde{\xi y},\widetilde{\xi z} \tilde{\in} \tilde{X}$ and $\gamma,\delta \geq 0$ with $\gamma+\delta < 1$. Then $Q\ and\ T$ has unique common random soft point.

Proof: $Qand\ T$ Satisfy property (E.A.)

∃a sequence $\{\widetilde{\xi x_n}\}\ in\ \tilde{X}$: $\lim_{n\to\infty} Q\widetilde{\xi x_n} = \lim_{n\to\infty} T\widetilde{\xi x_n} = \xi\tilde{t}$ for some $\widetilde{\xi t} \tilde{\in} \tilde{X}$

Now $T(\tilde{X})$ is closed subspace of $\tilde{X}$, for every convergent sequence in $T(\tilde{X})$ has limit point in $T(\tilde{X})$,⇒$\xi\tilde{t} = T\xi\tilde{p}$ for some $\xi\tilde{p} \tilde{\in} \tilde{X}$.

Using idea of previous theorem, result can be proved easily

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