Analysis of Priority Bi-Series Bulk Queue Network Model linked with common server and parallel server

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Abstract

This paper is an analysis of priority Bi-series queue Network Model having one bi-series queuing subsystems and one parallel queuing subsystems both centrally connected to a server. Infinite number of Low priority and high priority customers in groups arrive at each of the bi-series subsystems. In this model arrival and departure follow Poisson law at each server. Lawof calculus and generating function techniques and results were used to calculate queue length, steady -state equation, average customer waiting time, and variance. A numerical presentation of this model is defined.

Keywords: -Priority, bi-series channel, queue length, variance, parallel server

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1 Introduction

Priority in queuing theory is still hot topic. In which, customers are labelled and served according to priority discipline. High priority customers get an advantage as compared to low priority customers. Priority queuing models have wide range of application in real world. Sometimes priority in customers is created artificially to maximize profit of firm or maximum utilization of the service facility. You can observe priority in online stores, hospitals, administration, polling booths, Educational Institutions etc. Researchers on serial and biserial queue systems with batch arrivals have done the majority of work. In our research, we propose a non-preemptive priority Queue discipline. [1] derived the steady state solution. [2]introduced the concept of bi-seriesin queuing theory. In a stochastic environment, [3] investigated the problem of an intermediate queue before heterogeneous servers. Use of the priority notion on the intermediate queue model ina fuzzy environment was investigated by [4].The Steady state behavior of queue containing two sub subsystems with biserial servers connected to a common server studied by [5].This paper is an extension of work [6] in the sense that we developed a priority queues in bi-series with Poisson arrivals and Erlang phase service distribution.

2 Description of the Model

A queue network model with low and high priority customers consisting of two bi-series system S_1 and S_2 connected to a single server S_3. System S_i consists of two subsystems S_{i1} and $S_{i2}(i = 1, 2)$. Low and high priority customers arrive at S_{1j} with rates λ_{jL} and λ_{jH} respectively. The service time in S_{i1} is Erlang. The Erlang distribution is a two-parameter family 'K' shape parameter and $'\mu'$ scale parameter of continuous probability distributions. This distribution use in stochastic environment

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for mathematical study of waiting in lines. Service rates $\mu_{jL}K_j$ and $\mu_{jH}K_j$ for the low and high priority customers (j = 1, 2). After getting the service at S_{11} customer either go to server $S_{12}$for service with probability p_{12} or go to interconnecting server S_3 with probability p_{13} where the service rate is $\mu_3 K$ for all customers and $p_{12}+ p_{13}=1$

After getting service from server S_{12}, customers either may go to S_{11} with probability p_{21} forreceiving service or go to server S_3 with probability p_{23} for service with condition that $p_{21}+p_{23}=1$

When customer being served at S_3 service channel, then he moves to subsystem S_{21} or S_{22} with probability p_{34},p_{35} respectively. In subsystem S_{21} the service rate is $\mu_4 K_4$ and in subsystemS_{22} the service rate is $\mu_5 K_5$ for all customers. After service in system S_2 the customer leaves thesystem.

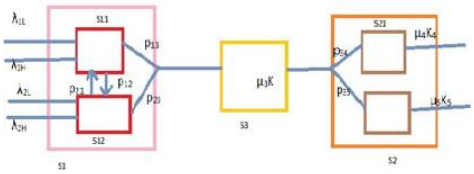

Figure 1: model

3 Notations

λ_{1L}: Mean arrival rate of lower priority customers at S_{11}

λ_{1H}: Mean arrival rate of high priority customers at S_{11}

λ_{2L}: Mean arrival rate of low priority customers at S_{12}

λ_{2H}: Mean arrival rate of high priority customers at $S_{12}\mu_{1L}K_1$: The service rate for low priority customers atS_{11} $\mu_{1H}K_1$: The service rate for high priority customers at$S_{11}\mu_{2L}K_2$: The service rate for low priority customers atS_{12} $\mu_{2H}K_2$: The service rate for high priority customers atS_{12}

$\mu_3 K$: The rate of service at server S_3

$\mu_4 K_4$: The rate of service at server S_{21}

$\mu_5 K_5$: The rate of service at server S_{22}

p_{12}: moving probability from S_{11} to S_{12}

p_{13}: moving probability from S_{11} to S_3

p_{21}: moving probability from S_{12} to S_{11}

p_{23}: moving probability from S_{12} to S_3

p_{34}: moving probability from S_3 to S_{21}

p_{35}: moving probability from S_3 to S_{22}

η_{1L}: customers in low priority queueS_{11}

η_{1H}: customers in high priority queue at S_{11}

η_{2L}: customers in low priority queue at S_{12}

η_{2H}: customers in high priority queue at S_{12}

η_3: customers in queue at S_3

η_4: customers in queue at S_{21}

η_5: customers in queue at S_{22}

4 Practical Application of the model

Students require an admit card to appear in exams during the examination period. There is no due slip in the process of obtaining an admit card. Students go through several phases toclear their dues, such as library, vehicle parking, and so on. In this system, students can go to either service first and then to the other, and priority is given to classes whose examination datesare nearing. After signing the slip from the library and the vehicle parking authority, students proceed to the principal's office for final signature, then to a different server where roll numbers are distributed, collect roll numbers, and finally exit the system, i.e. return home. This model is well suited to this situation.

5 Formation of Steady-State Equations

Define probability function $P_{\eta 1L,\eta 1H,\eta 2L,\eta 2H,\eta 3,\eta 4,\eta 5}(t)$ and (η_{1L}, η_{1H}, η_{2L}, η_{2H}, η_3, η_4, η_5 calling units of customers in queues (Q_{1L}, Q_{1H}), ((Q_{2L}, Q_{2H}), Q_3, Q_4, Q_5 in front of subsystems S_{11}, S_{12}, S_3, S_{21}, S_{22}respectively where η_{1L}, $\eta_{1H} > K_1$, η_{2L}, $\eta_{2H} > K_2$, $\eta_3 > K$, $\eta_4 > K_4$, $\eta_5 > K_5$

In Steady state, Differential Difference equation is

$$(\lambda 1L+\lambda 1H +\lambda 2L+\lambda 2H+\mu 1HK1+\mu 2HK2+\mu 3K +\mu 4K4+\mu 5K5)P_{\eta 1L,\eta 1H,\eta 2L,\eta 2H,\eta 3,\eta 4,\eta 5}$$
$$=\lambda 1LP_{\eta 1L-K1,\eta 1H,\eta 2L,\eta 2H,\eta 3,\eta 4,\eta 5} +\lambda 1HP_{\eta 1L,\eta 1L-K1,\eta 2L,\eta 2H,\eta 3,\eta 4,\eta 5} +\lambda 2LP_{\eta 1L,\eta 1H,\eta 2L-K2,\eta 2H,\eta 3,\eta 4,\eta 5} + \lambda 2HP_{\eta 1L,\eta 1H,\eta 2L,\eta 2H-K2,\eta 3,\eta 4,\eta 5} +\mu 1HK1p12P_{\eta 1L,\eta 1H+1,\eta 2L,\eta 2H-K2,\eta 3,\eta 4,\eta 5} +\mu 1HK1p13P_{\eta 1L,\eta 1H+1,\eta 2L,\eta 2H,\eta 3-K,\eta 4,} + \mu 2HK2p21P_{\eta 1L,\eta 1H-K1,\eta 2L,\eta 2H+1,\eta 3,\eta 4,\eta 5} +\mu 2HK2p23P_{\eta 1L,\eta 1H,\eta 2L,\eta 2H+1,\eta 3-K,\eta 4,\eta 5} +\mu 3Kp34P_{\eta 1L,\eta 1H,\eta 2L,\eta 2H,\eta 3+1,} +\mu 3Kp35P_{\eta 1L,\eta 1H,\eta 2L,\eta 2H,\eta 3+1,\eta 4,\eta 5-K5} +\mu 4K4P_{\eta 1L,\eta 1H,\eta 2L,\eta 2H}$$

$,\eta_3,\eta_{4+1},\eta_5 \quad +\mu 5K5P_{\eta_{1L},\eta_{1H}} \quad ,\eta_{2L},\eta_{2H}$
$,\eta_3,\eta_4,\eta_{5+1}$

(A_1)

6 Steady-State Solution

To solve the steady state equations (A_1) to (A_{128}), introduce the generating functionand its partial generating functions as follows:-

$G(Y_1, Y_2, Y_3, Y_4, Y_5, Y_6, Y_7) =$

$$\sum_{\eta 1L}^{\infty} Y_1^{\eta 1L} \sum_{\eta 2H}^{\infty} Y_2^{\eta 2H} \sum_{\eta 2L}^{\infty} Y_3^{\eta 2L} \sum_{\eta 2H}^{\infty} Y_4^{\eta 2H} \sum_{\eta 3}^{\infty} Y_5^{\eta 3} \sum_{\eta 4}^{\infty} Y_6^{\eta 4} \sum_{\eta 5}^{\infty} Y_7^{\eta 5} P_{\eta 1L,\eta 1H,\eta 2L,\eta 2H,\eta 3,\eta 4,\eta 5} \quad (1)$$

$$G_{\eta 1H,\eta 2L,\eta 2H,\eta 3,\eta 4,\eta 5}(Y_1) = \sum_{\eta 1L}^{\infty} Y_1^{\eta 1L} P_{\eta 1L,\eta 1H,\eta 2L,\eta 2H,\eta 3,\eta 4,\eta 5} \quad (2)$$

$$G_{\eta 2L,\eta 2H,\eta 3,\eta 4,\eta 5}(Y_1, Y_2) = \sum_{\eta 1H}^{\infty} Y_2^{\eta 1H} G_{\eta 1H,\eta 2L,\eta 2H,\eta 3,\eta 4,\eta 5}(Y_1) \quad (3)$$

$$G_{\eta 2H,\eta 3,\eta 4,\eta 5}(Y_1, Y_2, Y_3) = \sum_{\eta 2L}^{\infty} Y_3^{\eta 2L} G_{\eta 2L,\eta 2H,\eta 3,\eta 4,\eta 5}(Y_1, Y_2) \quad (4)$$

$$G_{\eta 3,\eta 4,\eta 5}(Y_1, Y_2, Y_3, Y_4) = \sum_{\eta 2H}^{\infty} Y_4^{\eta 2H} G_{\eta 2H,\eta 3,\eta 4,\eta 5}(Y_1, Y_2, Y_3) \quad (5)$$

$$G_{\eta 4,\eta 5}(Y_1, Y_2, Y_3, Y_4, Y_5) = \sum_{\eta 3}^{\infty} Y_5^{\eta 3} G_{\eta 3,\eta 4,\eta 5}(Y_1, Y_2, Y_3, Y_4) \quad (6)$$

$$G_{\eta 5}(Y_1, Y_2, Y_3, Y_4, Y_5, Y_6) = \sum_{\eta 4}^{\infty} Y_6^{\eta 4} G_{\eta 4,\eta 5}(Y_1, Y_2, Y_3, Y_4, Y_5) \quad (7)$$

$$G(Y_1, Y_2, Y_3, Y_4, Y_5, Y_6, Y_7) = \sum_{\eta 5}^{\infty} Y_7^{\eta 5} G_{\eta 5}(Y_1, Y_2, Y_3, Y_4, Y_5, Y_6) \quad (8)$$

$|Y_1|=|Y_2|=|Y_3|=|Y_4|=|Y_5|=|Y_6|=|Y_7|=1$

On solving, above equations by using partial generating function technique, the probability distribution function is in the form

$$G(Y1, Y2, Y3, Y4, Y5, Y6, Y7) = \frac{\begin{array}{l}\left[\mu 1HK1\left(1-\frac{p12}{Y2}Y_4^{K2}-\frac{p13}{Y}Y_5^{k}\right)-\mu 1LK1\left(1-\frac{p12}{Y1}Y_3^{K2}-\frac{p13}{Y1}Y_5^{k}\right)\right]G1+\\ [\mu 2HK2(1-\frac{p21}{Y4}Y_2^{K1}-\frac{p23}{Y4}Y_5^{K})-\mu 2LK2(1-\frac{p21}{Y3}Y_1^{K1}-\frac{p23}{Y3}Y_5^{K})]G2+\\ \mu 1LK1(1-\frac{p12}{Y1}Y_3^{K2}-\frac{p13}{Y1}Y_5^{K})]G6+\mu 3K[1-\frac{p34}{Y5}Y_6^{K4}-\frac{p35}{Y5}Y_7^{K5}]G3\\ +\mu 2LK2(1-\frac{p21}{Y3}Y_1^{K1}-\frac{p23}{Y3}Y_5^{K})]G7+\mu 4K4(1-\frac{1}{Y6})G4+\mu 5K5(1-\frac{1}{Y7})\,G5\end{array}}{\begin{array}{l}\lambda 1L(1-Y_1^{K1})+\lambda 1H(1-Y_2^{K1})+\lambda 2L(1-Y_3^{K2})\\ +\lambda 2H(1-Y_4^{K2})+\mu 1HK1(1-\frac{p12}{Y2}Y_4^{K2}-\frac{p13}{Y}Y_5^{k})+\mu 3K[1-\frac{p34}{Y5}Y_6^{K4}-\frac{p35}{Y5}Y_7^{K5}]\\ \mu 2HK2(1-\frac{p21}{Y4}Y_2^{K1}-\frac{p23}{Y4}Y_5^{K})+\mu 4K4(1-\frac{1}{Y6})+\mu 5K5(1-\frac{1}{Y7})\end{array}}$$

(A)

On differentiating(A)according L'Hospital rule, w.r.t to one-by-one variable and all the variables approaches to 1 as limit of differentiation. the total probability equal to 1

we get the results

$-\lambda 1L = -\mu 1LG1 + \mu 1LG6 + \mu 2LK2p21G2 - \mu 2Lk2p21G7$ (9)

$-\lambda 1H + \mu 1H - \mu 2HK2p21 = \mu 1HG1 - \mu 2HK2p21G2$ (10)

$-\lambda 2L = -\mu 2LG2 + \mu 2LG7 + \mu 1LK1p12G1 - \mu 1LK1p12G6$ (11)

$-\lambda 2H + \mu 2H - \mu 1HK1p12 = -\mu 1HK1p12G1 - \mu 2HG2$ (12)

$-\mu 1HK1p13 - \mu 2HK2p23 + \mu 3 = [-\mu 1HK1p13 + \mu 1LK1p13]G1 + [-\mu 2HK2p23 + \mu 2LK2p23]G2 + \mu 3G3 - \mu 1LK1p13G6 - \mu 2LK2p23G7$ (13)

$-\mu 3Kp34 + \mu 4 = -\mu 3Kp34G3 + \mu 4G4$ (14)

$-\mu 3KG35 + \mu 5 = -\mu 3Kp35G3 + \mu 5G5$ (15)

Solving equation (9) to (15), we get

$$G2 = 1 - \frac{\lambda 2H + \lambda 1HK1p12}{\mu 2H[1-K1p12K2p21]} \quad (16)$$

$$G1 = 1 - \frac{\lambda 1H + \lambda 2HK2p21}{\mu 1H[1-K1p12K2p21]} \quad (17)$$

$$G7 = 1 - \frac{\mu 2H(\lambda 2L + \lambda 1LK1p12) + \mu 2L(\lambda 2H + \lambda 1HK1p12)}{\mu 2H\mu 2L[1-K1p12K2p21]} \quad (18)$$

$$G6 = 1 - \frac{\mu 1H(\lambda 1L + \lambda 2LK2p21) + \mu 1L(\lambda 1H + \lambda 2HK2p21)}{\mu 1H\mu 1L[1-K1p12K2p21]} \quad (19)$$

$$G3 = 1 - \frac{K2p23[(\lambda 2L + \lambda 1LK1p12) + (\lambda 2H + \lambda 1HK1p12)] + K1p13[(\lambda 1L + \lambda 2LK2p21) + (\lambda 1H + \lambda 2HK2p21)]}{\mu 3[1-K1p12K2p21]} \quad (20)$$

$$G4 = 1 - \frac{Kp34[K2p23[(\lambda 2L + \lambda 1LK1p12) + (\lambda 2H + \lambda 1HK1p12)] + K1p13[(\lambda 1L + \lambda 2LK2p21) + (\lambda 1H + \lambda 2HK2p21)]]}{\mu 4[1-K1p12K2p21]} \quad (21)$$

$$G5 = 1 - \frac{Kp35[K2p23[(\lambda 2L + \lambda 1LK1p12) + (\lambda 2H + \lambda 1HK1p12)] + K1p13[(\lambda 1L + \lambda 2LK2p21) + (\lambda 1H + \lambda 2HK2p21)]]}{\mu 5[1-K1p12K2p21]} \quad (22)$$

Also solution is

$$P_{\eta 1L,\eta 1H,\eta 2L,\eta 2H,\eta 3,\eta 4,\eta 5} = w_1^{\eta 1L} w_2^{\eta 1H} w_3^{\eta 2L} w_4^{\eta 2H} w_5^{\eta 3} w_6^{\eta 4} w_7^{\eta 5} (1-w_1)(1-w_2)(1-w_3)(1-w_4)(1-w_5)(1-w_6)(1-w_7)$$

Where

$$w_1 = \frac{\lambda 1H + \lambda 2HK2p21}{\mu 1H[1-K1p12K2p21]} \quad (23)$$

$$w_2 = \frac{\lambda 2H + \lambda 1HK1p12}{\mu 2H[1-K1p12K2p21]} \quad (24)$$

$$w_3 = \frac{K2p23[(\lambda 2L + \lambda 1LK1p12) + (\lambda 2H + \lambda 1HK1p12)] + K1p13[(\lambda 1L + \lambda 2LK2p21) + (\lambda 1H + \lambda 2HK2p21)]}{\mu 3[1-K1p12K2p21]} \quad (25)$$

$$w_4 = \frac{Kp34[K2p23[(\lambda 2L + \lambda 1LK1p12) + (\lambda 2H + \lambda 1HK1p12)] + K1p13[(\lambda 1L + \lambda 2LK2p21) + (\lambda 1H + \lambda 2HK2p21)]]}{\mu 4[1-K1p12K2p21]} \quad (26)$$

$$w_5 = \frac{Kp35[K2p23[(\lambda 2L + \lambda 1LK1p12) + (\lambda 2H + \lambda 1HK1p12)] + K1p13[(\lambda 1L + \lambda 2LK2p21) + (\lambda 1H + \lambda 2HK2p21)]]}{\mu 5[1-K1p12K2p21]} \quad (27)$$

$$w_6 = \frac{\mu 1H(\lambda 1L + \lambda 2LK2p21) + \mu 1L(\lambda 1H + \lambda 2HK2p21)}{\mu 1H\mu 1L[1-K1p12K2p21]} \quad (28)$$

$$w_7 = \frac{\mu 2H(\lambda 2L + \lambda 1LK1p12) + \mu 2L(\lambda 2H + \lambda 1HK1p12)}{\mu 2H\mu 2L[1-K1p12K2p21]} \quad (29)$$

solution exist in steady state condition if $w_1, w_2, w_3, w_4, w_5, w_6, w_7 < 1$

Marginal Queue Length

$$L_1 = \frac{w_6}{1-w_6}$$
$$L_2 = \frac{w_1}{1-w_1}$$
$$L_3 = \frac{w_7}{1-w_7}$$
$$L_4 = \frac{w_7}{1-w_7}$$
$$L_5 = \frac{w_3}{1-w_3}$$
$$L_6 = \frac{w_4}{1-w_4}$$
$$L_7 = \frac{w_5}{1-w_5}$$

Average Queue Length

$$L_q = L_1 + L_2 + L_3 + L_4 + L_5 + L_6 + L_7 = \frac{w_6}{1-w_6} + \frac{w_1}{1-w_1} + \frac{w_7}{1-w_7} + \frac{w_7}{1-w_7} + \frac{w_3}{1-w_3} + \frac{w_5}{1-w_5}$$

$w_1, w_2, w_3, w_4, w_5, w_6, w_7 < 1$Average Waiting time

$E(W)=\frac{average\ queue\ length}{arrival\ rate}=\frac{Lq}{\lambda}$

$\lambda = \lambda_{1L} + \lambda_{1H} + \lambda_{2L} + \lambda_{2H}$

Variance

$Var(n_{1L} + n_{1H} + n_{2L} + n_{2H} + n_3 + n_4 + n_5)$
$=\frac{w1}{(1-w1)^2} + \frac{w2}{(1-w2)^2} + \frac{w3}{(1-w3)^2} + \frac{w4}{(1-w4)^2} + \frac{w5}{(1-w5)^2} + \frac{w6}{(1-w6)^2} + \frac{w7}{(1-w7)^2}$

Numerical Calculation

Table 1

arrival rate	service rate	batch size	probabilities
λ_{1L}=0.1	μ_{1L}=10	K_1=2	p_{12}=0.2
λ_{2L}=0.2	μ_{2L}=12	K_2=3	p_{21}=0.4
λ_{1H}=2	μ_{1H}=15	K=2	p_{13}=0.8
λ_{2H}=1	μ_{2H}=20		p_{23}=0.6
	μ_3=19		p_{34}=0.6
	μ_4=24		p_{35}=0.4
	μ_5=25		

Marginal Queue Length

$L_1 = .9069$

L_2 =.6954
L_3=.2682
L_4=.2091
L_5=17.1488
L_6=8.7656
L_7=1.3501

Mean Queue Length

$L_q = L_1 + L_2 + L_3 + L_4 + L_5 + L_6 + L_7$

$L_q = 29.3441$

Average Waiting Time $E(W) = \frac{Lq}{\lambda} = \frac{29.3441}{} = 8.8921$

$\lambda = \lambda_{1L} + \lambda_{1H} + \lambda_{2L} + \lambda_{2H} = 3.3$

Variance

$Var(\eta_{1L} + \eta_{1H} + \eta_{2L} + \eta_{2H} + \eta_3 + \eta_4 + \eta_5)$
$=\frac{w1}{(1-w1)^2} + \frac{w2}{(1-w2)^2} + \frac{w3}{(1-w3)^2} + \frac{w4}{(1-w4)^2} + \frac{w5}{(1-w5)^2} + \frac{w6}{(1-w6)^2} + \frac{w7}{(1-w7)^2}$

=1.7300+1.1794+.3401+.2529+311.2308+85.6488+3.1740
=403.556

arrival rate	service rate	batch size	probabilities
λ_{1L}=2	μ_{1L}=14	K_1=2	p_{12}=0.1
λ_{2L}=2	μ_{2L}=15	K_2=3	p_{21}=0.2
λ_{1H}=3	μ_{1H}=18	K=2	p_{13}=0.9
λ_{2H}=4	μ_{2H}=20		p_{23}=0.8
	μ_3=50		p_{34}=0.4
	μ_4=40		p_{35}=0.6
	μ_5=49		

By using table 2, we get

Marginal Queue Length

$L_1 = 1.5041$
L_2 =.5172
L_3=.7956

L_4=.3537
L_5=2.7537
L_6=2.7537
L_7=8.8328

Average Queue Length

$L_q = L_1 + L_2 + L_3 + L_4 + L_5 + L_6 + L_7$
=1.5041+.5172+.7956+.3537+2.7537+2.7537+8.8328
L_q = 17.5108

Average Waiting Time $E(W) = \frac{17.5108}{11}$
= 1.5918

$\lambda = \lambda_{1L} + \lambda_{1H} + \lambda_{2L} + \lambda_{2H} = 11$

Variance

$Var(\eta_{1L} + \eta_{1H} + \eta_{2L} + \eta_{2H} + \eta_3 + \eta_4 + \eta_5)$
=.7847+.4789+10.3469+10.3469+87.2135+3.7678+1.4288
=114.36759

7 conclusion

A priority bi-series queue network design with Erlang phase type service distribution is investigated in this research. From numerical calculation, it is clear that if we increase the service rate at server S_3 and subsystems S_{21} & S_{22} then mean queue length decreases. By changing the parameters optimum utilization of server is possible. If the concept of priority is not considered and service in batches not given the model coincide with the work of [6].

Table 2

Compliance with ethical standards

Conflict of interest We declare that there is no conflict of interest regarding the manuscriptof this paper

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