Intuitionistic Fuzzy Translation and Fuzzy Multiplication in BG-Algebra

R.RASHMA[1], K.R.SOBHA[2]
[1]Research scholar,Reg.No:21113182092001
Sree Ayyappa college for Women,Chunkankadai,Nagercoil-629001,TamilNadu, India.
[2]Assistant Professor, Department of Mathematics, Sree Ayyappa College for Women, Chunkankadai, Nagercoil.
[Affiliated to Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli-627012, Tamil Nadu, India.]
[1]rashmamariagiri@gmail.com
[2]vijayakumar.sobha9@gmail.com

Abstract:

In this paper, we introduced the concept of Intuitionistic fuzzy translation and fuzzy multiplication on Intuitionistic fuzzy ideals in BG-Algebras and discussed some of its properties in detail by using concept of Intuitionistic fuzzy BG- ideal and Intuitionistic fuzzy BG-subalgebra.

Keywords: BG-algebra ,BG-subalgebra, Fuzzy subalgebra , BG-Ideal, Fuzzy BG-Ideal, Intuitionistic fuzzy sub-algebra, Intuitionistic fuzzy Ideal, Intuitionistic Fuzzy $\alpha -$ translation, Intuitionistic Fuzzy $\alpha -$Multiplication.

DOI Number: 10.48047/nq.2022.20.22.NQ10127 **NeuroQuantology 2022; 20(22):1467-1475**

1.Introduction:

Zadeh introduced initiatively the concept of fuzzy sets [12] in 1965, from then on some researchers have been conducted on the generalization of the notion of fuzzy sets. Atanassov presented a generalization of the notion of fuzzy sets, the concept of intuitionistic fuzzy sets [2] in 1986, and some basic and main results on intuitionistic fuzzy sets were discussed .Fuzzy sets give a degree of membership of an element in a given set, while intuitionistic fuzzy sets give both degrees of membership and of nonmembership. Both degrees belong to the interval [0, 1], and their sum should not exceed 1.

C.B. Kim and Kim [4] introduced the notion of BG-algebras, which is a generalization of B-algebras. Ahn and Lee [1] fuzzified BG-algebras. Saeid introduced fuzzy topological BG-algebras. In the same year Zarandi and Saeid [13] presented intuitionistic fuzzy ideals of BG-algebras. Senapati et al. [9] presented the concept and basic properties of intuitionistic fuzzy BG- subalgebras. The concept of fuzzy translations in fuzzy subalgebras and ideals in BCK/BCI-algebras has been discussed respectively . They investigated relations among fuzzy translations, fuzzy extensions and fuzzy multiplications of ideals in BG-Algebra. In this paper, intuitionistic fuzzy translations and fuzzy multiplications of intuitionistic fuzzy ideals in BG-algebras are discussed.

2.Preliminaries

Definition:2.1

A non-empty set X with a constant 0 and a binary operation '*' is called a BG-algebra, if it satisfies the following axioms

(i) $x * x = 0$
(ii) $x * 0 = x$
(iii) $(x * y) * (0 * y) = x \ \forall x, y \in X$

Let S be non-empty subset of a BG-Algebra X ,then S is a BG-sub-algebra of X,if $x * y \epsilon S \ \forall x, y \in S$

Definition:2.2

Let μ be a fuzzy set in BG-algebra.Then μ is called a fuzzy sub-algebra of X if $\mu(x * y) \geq min\{\mu(x), \mu(y)\} \ \forall x, y \in X$

Definition:2.3

Let X be a BG-Algebra and I be a subset of X, then I is called a BG-Ideal of X, if it satisfies following conditions
(i) $0 \in I$
(ii) $x * y \in I$ and $y \epsilon I \Longrightarrow x \epsilon I$
(iii) $x \epsilon I \ and \ y \epsilon I \Longrightarrow x * y \epsilon I, I \times I \subseteq I$

Definition:2.4

A fuzzy set A in X is called fuzzy BG-Ideal of X, if it satisfies the following inequalities,
(i) $\mu(0) \geq \mu(x)$,
(ii) $\mu(x) \geq min\{\mu(x * y), \mu(y)\}$,
(iii) $\mu(x * y) \geq min\{\mu(x), \mu(y)\} \ \forall x, y \in X$

Definition:2.5

An intuitionistic fuzzy set $X = \{(x, \mu_A(x), \gamma_A(x): x \epsilon X\}$ in X is called a intuitionistic fuzzy BG-subalgebra of X if
(i) $\mu_A(x * y) \geq \min\{\mu_A(x), \mu_A(y)\}$,
(ii) $\gamma_A(x * y) \leq \max\{\gamma_A(x), \gamma_A(y)\} \ \forall x, y \in X$.

Definition:2.6

An Intuitionistic fuzzy set A of a BG-algebras X is said to be Intuitionistic fuzzy Ideal (IFI) of X if
(i) $\mu_A(0) \geq \mu_A(x)$,
(ii) $\gamma_A(0) \leq \gamma_A(x)$,
(iii) $\mu_A(x) \geq \min\{\mu_A(x * y), \mu_A(y)\}$,
(iv) $\gamma_A(x) \leq \max\{\gamma_A(x * y), \gamma_A(y)\} \ \forall x, y \in X$.

Definition:2.7

A fuzzy set A in X is called Intuitionistic fuzzy BG-Ideal of X, if it satisfies the following inequalities,
(i) $\mu_A(0) \geq \mu_A(x)$,
(ii) $\gamma_A(0) \leq \gamma_A(x)$),
(iii) $\mu_A(x) \geq min\{\mu_A(x * y), \mu_A(y)\}$,
(iv) $\gamma_A(x) \leq max\{\gamma_A(x * y), \gamma_A(y)\}$,
(v) $\mu_A(x * y) \geq min\{\mu_A(x), \mu_A(y)\}$,
(vi) $\gamma_A(x * y) \leq max\{\gamma_A(x), \gamma_A(y)\} \ \forall x, y \in X$.

3.Intuitionistic Fuzzy Translation and Intuitionistic Fuzzy Multiplication of ideals in BG-Algebra

For the sake of simplicity,we shall use the symbol $A = (\mu_A, \gamma_A)$ for the intuitionistic fuzzy subset $A = \{(x, \mu_A(x), \gamma_A(x)): x \epsilon X\}$.Throughtout this paper,we take $T = inf\{\mu_A(x)/x \in X\}$ for any intuitionistic fuzzy set $A = (\mu_A, \gamma_A)$ of X.

Definition:3.1

Let $X = (\mu_A, \gamma_A)$ be an intuitionistic fuzzy subset of X and let $\alpha \in [0, T]$.An object having the form $A_\alpha^T = ((\mu_A)_\alpha^T, (\gamma_A)_\alpha^T)$ is called a intuitionistic fuzzy $\alpha -$ translation of A if

$(\mu_A)_\alpha^T(x) = \mu_A(x) + \alpha$ and $(\gamma_A)_\alpha^T(x) = \gamma_A(x) - \alpha \ \forall \ x \epsilon X$.

Definition:3.2

Let $X = (\mu_A, \gamma_A)$ be an intuitionistic fuzzy subset of X and let $\gamma \in [0,1]$.An object having the form $A_\alpha^M = ((\mu_A)_\alpha^M, (\gamma_A)_\alpha^M)$ is called a intuitionistic fuzzy $\alpha -$ multiplication of A if

$(\mu_A)_\alpha^M(x) = \mu_A(x).\alpha$ and $(\gamma_A)_\alpha^M(x) = \gamma_A(x).\alpha \ \forall \ x \epsilon X$.

Theorem 3.1

If $X = (\mu_A, \gamma_A)$ is a intuitionistic fuzzy BG-subalgebra and $\alpha \in [0,1]$, then the intuitionistic fuzzy $\alpha-$translation $(\mu_A)_\alpha^T(x)$ and $(\gamma_A)_\alpha^T(x)$ of X is also a intuitionistic fuzzy BG-subalgebra of X.

Proof:

Let $x, y \epsilon X$ and $\alpha \in [0, T]$

To Prove: (i) $(\mu_A)_\alpha^T(x * y) \geq min\{(\mu_A)_\alpha^T(x), (\mu_A)_\alpha^T(y)\}$

(ii) $(\gamma_A)_\alpha^T(x * y) \leq max\{(\gamma_A)_\alpha^T(x), (\gamma_A)_\alpha^T(y)\}$

Now,

(i) $(\mu_A)_\alpha^T(x * y) = \mu_A(x * y) + \alpha$

$\geq min\{\mu_A(x), \mu_A(y)\} + \alpha$ $=$ $min\{\mu_A(x) + \alpha, \mu_A(y) + \alpha\}$

$= min\{(\mu_A)_\alpha^T(x), (\mu_A)_\alpha^T(y)\})$

$(\mu_A)_\alpha^T(x * y) \geq min\{(\mu_A)_\alpha^T(x), (\mu_A)_\alpha^T(y)\} \ \forall \ x, y \in X.$

(ii) $(\gamma_A)_\alpha^T(x * y) = \gamma_A(x * y) - \alpha$

$\leq max\{\gamma_A(x), \gamma_A(y)\} - \alpha$ $=$ $max\{\gamma_A(x) - \alpha, \gamma_A(y) - \alpha\}$

$= max\{(\gamma_A)_\alpha^T(x), (\gamma_A)_\alpha^T(y)\})$

Therefore, $(\gamma_A)_\alpha^T(x * y) \leq max\{(\gamma_A)_\alpha^T(x), (\gamma_A)_\alpha^T(y)\} \forall \ x, y \in X.$

Hence the proof.

Theorem 3.2 1469

Let $X = (\mu_A, \gamma_A)$ be a intuitionistic fuzzy subset of X such that the intuitionistic fuzzy $\alpha-$translation $(\mu_A)_\alpha^T(x)$ and $(\gamma_A)_\alpha^T(x)$ of (μ_A, γ_A) is a intuitionistic fuzzy subalgebra of X, for some $\alpha \in [0, T]$,then (μ_A, γ_A) is a intuitionistic fuzzy subalgebra of X.

Proof:

Assume that $(\mu_A)_\alpha^T(x)$ and $(\gamma_A)_\alpha^T(x)$ is a intuitionistic fuzzy subalgebra of X, for some $\alpha \in [0, T]$

Let $x, y \in X$,we have

$\mu_A(x * y) + \alpha = (\mu_A)_\alpha^T(x * y)$

$\geq min\{(\mu_A)_\alpha^T(x), (\mu_A)_\alpha^T(y)\})$

$= min\{\mu_A(x) + \alpha, \mu_A(y) + \alpha\}$

$= min\{\mu_A(x), \mu_A(y)\} + \alpha$

$\mu_A(x * y) \geq min\{\mu_A(x), \mu_A(y)\}$

Clearly,this can be proved for maximal condition.

Hence (μ_A, γ_A) is an intuitionistic fuzzy subalgebra of X.

Theorem 3.3

For any intuitionistic fuzzy BG-algebra (μ_A, γ_A) of X and $\alpha \in [0, T]$,if the intuitionistic fuzzy $\alpha-$multiplication $(\mu_A)_\alpha^M(x)$ and $(\gamma_A)_\alpha^M(x)$ is a intuitionistic fuzzy BG-sub algebra of X.

Proof:

Let $x, y \epsilon X$ and $\alpha \in [0, T]$

Then

$$(\mu_A)_\alpha^M(x * y) = \alpha.\mu_A(x * y)$$
$$\geq \alpha.min\{\mu_A(x), \mu_A(y)\}$$
$$= min\{\alpha.\mu_A(x), \alpha.\mu_A(y)\}$$
$$(\mu_A)_\alpha^M(x * y) \geq min\{(\mu_A)_\alpha^M(x), (\mu_A)_\alpha^M(y)\})$$

And

$$(\gamma_A)_\alpha^M(x * y) = \alpha.\gamma_A(x * y)$$
$$\leq \alpha.max\{\gamma_A(x), \gamma_A(y)\}$$
$$= max\{\alpha.\gamma_A(x), \alpha.\gamma_A(y)\}$$
$$(\gamma_A)_\alpha^M(x * y) \leq max\{(\gamma_A)_\alpha^M(x), (\gamma_A)_\alpha^M(y)\})$$

Hence $((\mu_A)_\alpha^M(x)$, $(\gamma_A)_\alpha^M(x))$ is a intuitionistic fuzzy BG-subalgebra of X.

Theorem 3.4

For any intuitionistic fuzzy subset (μ_A, γ_A) of X and $\alpha \in [0, T]$,if the intuitionistic fuzzy $\alpha-$ multiplication $(\mu_A)^M_\alpha(x)$ and $(\gamma_A)^M_\alpha(x)$ is a intuitionistic fuzzy BG-algebra of X then so in (μ_A, γ_A).

Proof:

Assume that $(\mu_A)^M_\alpha(x)$ and $(\gamma_A)^M_\alpha(x)$ is a intuitionistic fuzzy subalgebra of X, for some $\alpha \in [0, T]$. Let $x, y \in X$,we have

$$\alpha.\mu_A(x*y) = (\mu_A)^M_\alpha(x*y)$$
$$\geq min\{(\mu_A)^M_\alpha(x), (\mu_A)^M_\alpha(y)\}$$
$$= min\{\alpha.\mu_A(x), \alpha.\mu_A(y)\}$$
$$= \alpha.min\{\mu_A(x), \mu_A(y)\}$$
$$\mu_A(x*y) \geq min\{\mu_A(x), \mu_A(y)\}$$

Clearly, this can be proved for maximal condition.

Theorem:3.5

If the intuitionistic fuzzy $\alpha-$ translation$(\mu_A)^T_\alpha(x)$ and $(\gamma_A)^T_\alpha(x)$ of (μ_A, γ_A) is a Intuitionistic fuzzy BG-ideal, then it satisfies the condition

(i) $(\mu_A)^T_\alpha(y*(x*y)) \geq (\mu_A)^T_\alpha(x)$

(ii) $(\gamma_A)^T_\alpha(y*(x*y) \leq (\gamma_A)^T_\alpha(x)$

Proof:

(i) $(\mu_A)^T_\alpha(y*(x*y)) = \mu_A\big(y*(x*y)\big) + \alpha$

$$\geq min\{\mu_A(x*(y*(x*y)), \mu_A\big(y*(x*y)\big)\} + \alpha$$
$$= min\{\mu_A(x*(y*(x*y)) + \alpha, \mu_A\big(y*(x*y)\big) + \alpha\}$$
$$= min\{\mu_A(x*x) + \alpha, \mu_A(x) + \alpha\}$$
$$= min\{\mu_A(0) + \alpha, \mu_A(x) + \alpha\}$$
$$= min\{(\mu_A)^T_\alpha(0), (\mu_A)^T_\alpha(x)\})$$
$$(\mu_A)^T_\alpha(y*(x*y)) \geq (\mu_A)^T_\alpha(x)$$

(ii) $(\gamma_A)^T_\alpha(y*(x*y)) = \gamma_A\big(y*(x*y)\big) - \alpha$

$$\leq max\{\gamma_A(x*(y*(x*y)), \gamma_A\big(y*(x*y)\big)\} - \alpha$$
$$= max\{\gamma_A(x*(y*(x*y)) - \alpha, \gamma_A\big(y*(x*y)\big) - \alpha\}$$
$$= max\{\gamma_A(x*x) - \alpha, \gamma_A(x) - \alpha\}$$
$$= max\{\gamma_A(0) - \alpha, \gamma_A(x) - \alpha\}$$
$$= max\{(\gamma_A)^T_\alpha(0), (\gamma_A)^T_\alpha(x)\})$$
$$(\gamma_A)^T_\alpha(y*(x*y)) \leq (\gamma_A)^T_\alpha(x)$$

Hence the condition proved.

Theorem:3.6

If (μ_A, γ_A) is a intuitionistic fuzzy BG-Ideal of X,then the intuitionistic fuzzy $\alpha-$ translation $(\mu_A)^T_\alpha(x)$ and $(\gamma_A)^T_\alpha(x)$ of (μ_A, γ_A) is a Intuitionistic fuzzy BG-ideal of X $\forall\, \alpha \in [0, T]$

Proof:

Let (μ_A, γ_A) is a intuitionistic fuzzy BG-Ideal of X,and Let $\alpha \in [0, T]$

Then (i) $(\mu_A)^T_\alpha(0) = \mu_A(0) + \alpha$

$$\geq \mu_A(x) + \alpha$$
$$= (\mu_A)^T_\alpha(x)$$
$$(\mu_A)^T_\alpha(0) \geq (\mu_A)^T_\alpha(x)$$

(ii) $(\mu_A)^T_\alpha(x) = \mu_A(x)$

$$\geq \min\{\mu_A(x*y), \mu_A(y)\} + \alpha$$
$$= min\{\mu_A(x*y) + \alpha, \mu_A(y) + \alpha\}$$
$$= min\{(\mu_A)^T_\alpha(x*y), (\mu_A)^T_\alpha(y)\})$$
$$(\mu_A)^T_\alpha(x) \geq min\{(\mu_A)^T_\alpha(x), (\mu_A)^T_\alpha(y)\}$$

(iii) $(\mu_A)^T_\alpha(x*y) = \mu_A(x*y) + \alpha$

$$\geq min\{\mu_A(x), \mu_A(y)\} + \alpha$$
$$= min\{\mu_A(x) + \alpha, \mu_A(y) + \alpha\}$$
$$= min\{(\mu_A)_\alpha^T(x), (\mu_A)_\alpha^T(y)\})$$
$$(\mu_A)_\alpha^T(x * y) \geq min\{(\mu_A)_\alpha^T(x), (\mu_A)_\alpha^T(y)\}$$

(iv) $(\gamma_A)_\alpha^T(0) = \gamma_A(0) - \alpha$
$$\leq \gamma_A(x) - \alpha$$
$$= (\gamma_A)_\alpha^T(x)$$
$$(\gamma_A)_\alpha^T(0) \leq (\gamma_A)_\alpha^T(x)$$

(v)) $(\gamma_A)_\alpha^T(x) = \gamma_A(x) - \alpha$
$$\geq max\{\gamma_A(x * y) - \alpha, \gamma_A(y) - \alpha\}$$
$$= max\{(\gamma_A)_\alpha^T(x * y), (\gamma_A)_\alpha^T(y)\})$$
$$(\gamma_A)_\alpha^T(x) \leq max\{(\gamma_A)_\alpha^T(x), (\gamma_A)_\alpha^T(y)\}$$

(vi) $(\gamma_A)_\alpha^T(x * y) = \gamma_A(x * y) - \alpha$
$$\geq max\{\gamma_A(x), \gamma_A(y)\} - \alpha$$
$$= max\{\gamma_A(x) - \alpha, \gamma_A(y) - \alpha\}$$
$$= max\{(\gamma_A)_\alpha^T(x), (\gamma_A)_\alpha^T(y)\})$$
$$(\gamma_A)_\alpha^T(x * y) \leq max\{(\gamma_A)_\alpha^T(x), (\gamma_A)_\alpha^T(y)\} \forall x, y \in X.$$

Hence $(\mu_A)_\alpha^T(x)$ and $(\gamma_A)_\alpha^T(x)$ of (μ_A, γ_A) is a Intuitionistic fuzzy BG-ideal of X , $\forall \alpha \in [0, T]$

Theorem:3.7

If (μ_A, γ_A) is a intuitionistic fuzzy subset of X, such that intuitionistic fuzzy $\alpha -$ translation $(\mu_A)_\alpha^T(x)$ and $(\gamma_A)_\alpha^T(x)$ of (μ_A, γ_A) is a Intuitionistic fuzzy BG-ideal of X for some $\alpha \in [0, T]$, then (μ_A, γ_A) is a Intuitionistic fuzzy BG-ideal of X.

Proof:

Assume that $(\mu_A)_\alpha^T(x)$, $(\gamma_A)_\alpha^T(x)$ is a intuitionistic fuzzy BG-Ideal of X, for some $\alpha \in [0, T]$.
Let $x, y \in X$
Then (i) $\mu_A(0) + \alpha = (\mu_A)_\alpha^T(0)$
$$\geq (\mu_A)_\alpha^T(x)$$
$$= \mu_A(x) + \alpha$$
$$\mu_A(0) \geq \mu_A(x)$$

(ii) $\mu_A(x) + \alpha = (\mu_A)_\alpha^T(x)$
$$\geq min\{(\mu_A)_\alpha^T(x * y), (\mu_A)_\alpha^T(y)\})$$
$$\geq \min\{\mu_A(x * y) + \alpha, \mu_A(y) + \alpha\}$$
$$= min\{\mu_A(x * y), \mu_A(y)\} + \alpha$$
$$\mu_A(x) \geq min\{\mu_A(x * y), \mu_A(y)\}$$

(iii) $\mu_A(x * y) + \alpha = (\mu_A)_\alpha^T(x * y)$
$$\geq min\{(\mu_A)_\alpha^T(x), (\mu_A)_\alpha^T(y)\})$$
$$= min\{\mu_A(x) + \alpha \quad , \mu_A(y) + \alpha \quad \}$$
$$= min\{\mu_A(x), \mu_A(y)\} + \alpha$$
$$\mu_A(x * y) \geq min\{\mu_A(x), \mu_A(y)\}$$
Clearly,this can be proved for maximal condition.

Hence (μ_A, γ_A) is a Intuitionistic fuzzy BG-ideal of X.

Theorem:3.8

$Let\ \alpha \in [0,T]$ and let (μ_A, γ_A) is a Intuitionistic fuzzy BG-ideal of X .If X is a BG-algebra, then the intuitionistic fuzzy $\alpha -$ translation $(\mu_A)_\alpha^T(x)$ and $(\gamma_A)_\alpha^T(x)$ of (μ_A, γ_A) is a Intuitionistic fuzzy BG-sub-algebra of X.

Proof:

Let $x, y \in X$.

Now, we have

(i) $$(\mu_A)_\alpha^T(x*y) = \mu_A(x*y) + \alpha$$
$$\geq min\{\mu_A(y*(x*y), \mu_A(y)\} + \alpha$$
$$= min\{\mu_A\big(x*(y*y)\big), \mu_A(y)\} + \alpha$$
$$= min\{\mu_A(x), \mu_A(y)\} + \alpha$$
$$= min\{\mu_A(x) + \alpha, \mu_A(y) + \alpha\}$$
$$(\mu_A)_\alpha^T(x*y) \geq min\{(\mu_A)_\alpha^T(x), (\mu_A)_\alpha^T(y)\}$$

(ii) $$(\gamma_A)_\alpha^T(x*y) = \gamma_A(x*y) - \alpha$$
$$\leq max\{\gamma_A(y*(x*y), \gamma_A(y)\} - \alpha$$
$$= max\{\gamma_A\big(x*(y*y)\big), \gamma_A(y)\} - \alpha$$
$$= max\{\gamma_A(x), \gamma_A(y)\} - \alpha$$
$$= max\{\gamma_A(x) - \alpha, \gamma_A(y) - \alpha\}$$
$$(\gamma_A)_\alpha^T(x*y) \leq max\{(\gamma_A)_\alpha^T(x), (\gamma_A)_\alpha^T(y)\}$$

Hence $(\mu_A)_\alpha^T(x)$ and $(\gamma_A)_\alpha^T(x)$ is a Intuitionistic fuzzy BG-sub-algebra of X.

Theorem:3.9

If the intutionistic fuzzy $\alpha -$ translation $(\mu_A)_\alpha^T$ and $(\gamma_A)_\alpha^T$ of (μ_A, γ_A) is a Intuitionistic fuzzy BG-sub-algebra of X, $\alpha \in [0,T]$ then (μ_A, γ_A) is a Intuitionistic fuzzy BG-sub-algebra of X.

Proof:

Let us assume that $(\mu_A)_\alpha^T$ and $(\gamma_A)_\alpha^T$ of (μ_A, γ_A) is a Intuitionistic fuzzy BG-ideal of X.

Then

To Prove:(i) $\mu_A(x*y) \geq min\{\mu_A(x), \mu_A(y)\}$

(ii) $\gamma_A(x*y) \leq max\{\gamma_A(x), \gamma_A(y)\}$

(i)) $$\mu_A(x*y) + \alpha = (\mu_A)_\alpha^T(x*y)$$
$$\geq min\{(\mu_A)_\alpha^T(y*(x*y)), (\mu_A)_\alpha^T(y)\}$$
$$\geq min\{(\mu_A)_\alpha^T(x*(y*y)), (\mu_A)_\alpha^T(y)\}$$
$$= min\{(\mu_A)_\alpha^T(x), (\mu_A)_\alpha^T(y)\}$$
$$= min\{\mu_A(x) + \alpha\ , \mu_A(y) + \alpha\}$$
$$= min\{\mu_A(x), \mu_A(y)\} + \alpha$$
$$\mu_A(x*y) \geq min\{\mu_A(x), \mu_A(y)\}$$

(ii)) $$\gamma_A(x*y) - \alpha = (\gamma_A)_\alpha^T(x*y)$$
$$\leq max\{(\gamma_A)_\alpha^T(y*(x*y)), (\gamma_A)_\alpha^T(y)\}$$
$$= max\{(\gamma_A)_\alpha^T(x*(y*y)), (\gamma_A)_\alpha^T(y)\}$$
$$= max\{(\gamma_A)_\alpha^T(x), (\gamma_A)_\alpha^T(y)\}$$
$$= max\{\gamma_A(x) + \alpha\ , \gamma_A(y) - \alpha\}$$
$$= max\{\gamma_A(x), \gamma_A(y)\} - \alpha$$
$$\gamma_A(x*y) \leq max\{\gamma_A(x), \gamma_A(y)\}$$

Hence (μ_A, γ_A) is a Intuitionistic fuzzy BG-sub-algebra of X.

Theorem.3.10

Let (μ_A, γ_A) intuitionistic fuzzy subset of X such that the intuitionistic fuzzy $\alpha -$ multiplication $(\mu_A)_\alpha^M(x)$ and $(\gamma_A)_\alpha^M(x)$ of (μ_A, γ_A). is a intuitionistic fuzzy BG-ideal of X, for some $\alpha \in [0,T]$, then (μ_A, γ_A) is a intuitionistic fuzzy BG-ideal of X.

Proof:

Assume that $(\mu_A)_\alpha^M(x)$ and $(\gamma_A)_\alpha^M(x)$ is a intuitionistic fuzzy BG-ideal of X, for some $\alpha \in [0,T]$

Let $x, y \in X$,we have

(i) $\alpha.\mu_A(0) = (\mu_A)_\alpha^M(0)$

$\geq (\mu_A)_\alpha^M(x)$

$=\alpha.\mu_A(x)$

$\mu_A(0) \geq \mu_A(x)$

(ii) $\alpha.\mu_A(x) = (\mu_A)_\alpha^M(x)$

$\geq min\{(\mu_A)_\alpha^M(x*y), (\mu_A)_\alpha^M(y)\}$

$= min\{\alpha\mu_A(x*y)\ , \alpha\mu_A(y)\}$

$= \min \alpha\,\{\mu_A(x*y)\ , \mu_A(y)\}$

So, $\mu_A(x) \geq min\{\mu_A(x*y)\ , \mu_A(y)\}$

(iii) $\alpha.\mu_A(x*y) = (\mu_A)_\alpha^M(x*y)$

$\geq min\{(\mu_A)_\alpha^M(x), (\mu_A)_\alpha^M(y)\}$

$= min\{\alpha\mu_A(x)\ , \alpha\mu_A(y)\}$

$= \alpha.min\{\mu_A(x)\ , \mu_A(y)\}$

$\mu_A(x*y) \geq min\{\mu_A(x)\ , \mu_A(y)\}$

(iv) $\alpha.\gamma_A(0) = (\gamma_A)_\alpha^M(0)$

$\leq (\gamma_A)_\alpha^M(x)$

$=\alpha.\gamma_A(x)$

$\gamma_A(0) \leq \gamma_A(x)$

(v) $\alpha.\gamma_A(x) = (\gamma_A)_\alpha^M(x)$

$\leq max\{(\gamma_A)_\alpha^M(x*y), (\gamma_A)_\alpha^M(y)\}$

$= max\{\alpha\gamma_A(x*y)\ , \alpha\gamma_A(y)\}$

$= \max \alpha\,\{\gamma_A(x*y)\ , \gamma_A(y)\}$

So, $\gamma_A(x) \leq max\{\gamma_A(x*y)\ , \gamma_A(y)\}$

(vi) $\alpha.\gamma_A(x*y) = (\gamma_A)_\alpha^M(x*y)$

$\leq max\{(\gamma_A)_\alpha^M(x), (\gamma_A)_\alpha^M(y)\}$

$= max\{\alpha.\gamma_A(x)\ , \alpha.\gamma_A(y)\}$

$= \alpha.max\{\gamma_A(x)\ , \gamma_A(y)\}$

$\gamma_A(x*y) \leq max\{\gamma_A(x)\ , \gamma_A(y)\}$

Hence (μ_A, γ_A) is a intuitionistic fuzzy BG-ideal of X.

Theorem.3.11

If (□$_□$,□$_□$) intuitionistic fuzzy BG-ideal of □ then the intuitionistic fuzzy □ − multiplication (□$_□$)$^□_□$(□) and (□$_□$)$^□_□$(□) of (□$_□$,□$_□$) is a intuitionistic fuzzy BG-ideal of □,∀ □ ∈ [*0*,□].

Proof:

Let (□$_□$,□$_□$). is a intuitionistic fuzzy BG-ideal of □, and let □ ∈ [*0*,□]

Then (i) $(\mu_A)_\alpha^M(0)=\alpha.\mu_A(0))$

$\geq \alpha.\mu_A(x)$

$= (\mu_A)_\alpha^M(x)$

$(\mu_A)_\alpha^M(0) \geq (\mu_A)_\alpha^M(x)$

(ii) $(\mu_A)_\alpha^M(x)=\alpha.\mu_A(x)$

$\geq \alpha.min\{\mu_A(x*y)\ , \mu_A(y)\}$

$\geq min\{\alpha\,\mu_A(x*y)\ , \alpha\,\mu_A(y)\}$

$= min\{(\mu_A)_\alpha^M(x*y), (\mu_A)_\alpha^M(y)\}$

$(\mu_A)_\alpha^M(x) \geq min\{(\mu_A)_\alpha^M(x*y), (\mu_A)_\alpha^M(y)\}$

(iii)) $(\mu_A)_\alpha^M(x*y)=\alpha.\mu_A(x*y)$

$\geq \alpha.min\{\mu_A(x)\ , \mu_A(y)\}$

$\geq min\{\alpha\,\mu_A(x)\ , \alpha\,\mu_A(y)\}$

$= min\{(\mu_A)_\alpha^M(x), (\mu_A)_\alpha^M(y)\}$

$(\mu_A)_\alpha^M(x*y) \geq min\{(\mu_A)_\alpha^M(x), (\mu_A)_\alpha^M(y)\}$

(iv) $(\gamma_A)_\alpha^M(0)=\alpha.\gamma_A(0))$

$\leq \alpha.\gamma_A(x)$

$= (\gamma_A)^M_\alpha(x)$

$(\gamma_A)^M_\alpha(0) \leq (\gamma_A)^M_\alpha(x)$

(v) $(\gamma_A)^M_\alpha(x)=\alpha\, \gamma_A(x)$

$\leq \alpha. max\{\gamma_A(x*y)\ ,\gamma_A(y)\}$

$\leq\ max\{\ \alpha\, \gamma_A(x*y)\ ,\alpha\, \gamma_A(y)\}$

$= max\{(\gamma_A)^M_\alpha(x*y), (\gamma_A)^M_\alpha(y)\}$

$(\mu_A)^M_\alpha(x) \leq\ max\{(\gamma_A)^M_\alpha(x*y), (\gamma_A)^M_\alpha(y)\}$

(vi)) $(\gamma_A)^M_\alpha(x*y)=\alpha. \gamma_A(x*y)$

$\leq \alpha. max\{\gamma_A(x)\ ,\gamma_A(y)\}$

$\leq\ max\{\ \alpha\, \gamma_A(x)\ ,\alpha\, \gamma_A(y)\}$

$= max\{(\gamma_A)^M_\alpha(x), (\gamma_A)^M_\alpha(y)\}$

$(\gamma_A)^M_\alpha(x*y) \leq\ max\{(\gamma_A)^M_\alpha(x), (\gamma_A)^M_\alpha(y)\}$

Hence, $(\mu_A)^M_\alpha(x)$ and $(\gamma_A)^M_\alpha(x)$ of (μ_A, γ_A). is a intuitionistic fuzzy BG-ideal of X,$\forall\ \alpha \in [0, T]$.

Theorem.3.12

Let $\alpha \in [0, T]$ and let (μ_A, γ_A) intuitionistic fuzzy BG-ideal of a BG-algebra of X.
Then the intuitionistic fuzzy $\alpha -$ multiplication $(\mu_A)^M_\alpha(x)$ and $(\gamma_A)^M_\alpha(x)$ of (μ_A, γ_A) is a intuitionistic fuzzy BG-subalgebra of X.

Proof:

Let $x, y\ \in X$,we have
Now, we have

$(\mu_A)^M_\alpha(x*y)=\alpha. \mu_A(x*y)$

$\geq \alpha. min\{\mu_A(y*(x*y)\ ,\mu_A(y)\}$

$=\ min\{\ \alpha. \mu_A(x*(y*y)\ ,\alpha. \mu_A(y)\}$

$= \alpha. min\{\mu_A(x)\ ,\mu_A(y)\}$

$=\ min\{\alpha. \mu_A(x)\ ,\alpha. \mu_A(y)\}$

$= min\{(\mu_A)^M_\alpha(x), (\mu_A)^M_\alpha(y)\}$

$(\mu_A)^M_\alpha(x*y) \geq\ min\{(\mu_A)^M_\alpha(x), (\mu_A)^M_\alpha(y)\}$

Similiarly, $(\gamma_A)^M_\alpha(x*y) \leq\ max\{(\gamma_A)^M_\alpha(x), (\gamma_A)^M_\alpha(y)\}$
Hence, $(\mu_A)^M_\alpha(x)$ and $(\gamma_A)^M_\alpha(x)$ is a intuitionistic fuzzy BG-subalgebra of X,$\forall\ x, y \epsilon [0,1]$.

Theorem.3.13

If the intutionistic fuzzy $\alpha -$multiplication $(\mu_A)^M_\alpha$ and $(\gamma_A)^M_\alpha$ of (μ_A, γ_A) is a Intuitionistic fuzzy BG-sub-algebra of X. $\alpha \in [0, T]$.then (μ_A, γ_A) is a Intuitionistic fuzzy BG-sub-algebra of X.

Proof:

Let us assume that $(\mu_A)^M_\alpha$ and $(\gamma_A)^M_\alpha$ of (μ_A, γ_A) is a intuitionistic fuzzy BG-ideal of X.
Then , $\alpha. \mu_A(x*y) = (\mu_A)^M_\alpha(x*y)$

$\geq min\{(\mu_A)^M_\alpha(y*(x*y), (\mu_A)^M_\alpha(y)\}$

$= min\{(\mu_A)^M_\alpha(x*(y*y), (\mu_A)^M_\alpha(y)\}$

$= min\{(\mu_A)^M_\alpha(x), (\mu_A)^M_\alpha(y)\}$

$=\ min\{\alpha\, \mu_A(x)\ ,\alpha\, \mu_A(y)\}$

$= \alpha\ .min\{\ \mu_A(x)\ ,\mu_A(y)\}$

$\mu_A(x*y) \geq\ min\{\ \mu_A(x)\ ,\mu_A(y)\}$

Similiarly, $\gamma_A(x*y) \leq\ max\{\ \gamma_A(x)\ ,\gamma_A(y)\}$

Hence (μ_A, γ_A) is a Intuitionistic fuzzy BG-sub-algebra of X.

Theorem 3.14

The intersection and union of any two a Intuitionistic fuzzy translation of fuzzy BG-ideal (μ_A, γ_A) of X are also Intuitionistic fuzzy BG-ideal of X.

Proof:

Let $(\mu_{A_1})^T_{\alpha_1}$and $(\mu_{A_2})^T_{\alpha_2}$be two Intuitionistic fuzzy translation of a Intuitionistic fuzzy BG-ideal of (μ_{A_1}, μ_{A_2}) of X.Where $\alpha_1, \alpha_2 \in [0,1]$

Assume that $\alpha_1 \leq \alpha_2$
Then by Theorem 3.12
$(\mu_{A_1})^T_{\alpha_1}$ and $(\mu_{A_2})^T_{\alpha_2}$ are intuitionistic fuzzy BG-ideal of X.
Now, $((\mu_{A_1})^T_{\alpha_1} \cap (\mu_{A_2})^T_{\alpha_2})(x) = ((\mu_{A_1})^T_{\alpha_1}(x) \cap (\mu_{A_2})^T_{\alpha_2}(x))$

$$= min\{\mu_{A_1}(x) + \alpha_1 , \mu_{A_2}(x) + \alpha_2 \}$$
$$= \mu_{A_1}(x) + \alpha_1$$
$$= (\mu_{A_1})^T_{\alpha_1}(x)$$

And

$$((\mu_{A_1})^T_{\alpha_1} \cup (\mu_{A_2})^T_{\alpha_2})(x) = ((\mu_{A_1})^T_{\alpha_1}(x) \cup (\mu_{A_2})^T_{\alpha_2}(x))$$
$$= max\{\mu_{A_1}(x) + \alpha_1 , \mu_{A_2}(x) + \alpha_2 \}$$
$$= \mu_{A_2}(x) + \alpha_2$$
$$= (\mu_{A_2})^T_{\alpha_2}(x)$$

Hence, $((\mu_{A_1})^T_{\alpha_1} \cap (\mu_{A_2})^T_{\alpha_2})$ and $((\mu_{A_1})^T_{\alpha_1} \cup (\mu_{A_2})^T_{\alpha_2})$ are Intuitionistic fuzzy BG-ideal of X.

Clearly,this can be proved for $((\gamma_{A_1})^T_{\alpha_1} \cap (\gamma_{A_2})^T_{\alpha_2})$ and $((\gamma_{A_1})^T_{\alpha_1} \cup (\gamma_{A_2})^T_{\alpha_2})$ are also a Intuitionistic fuzzy BG-ideal of X.

4. Conclusion:

In this paper, the translation and multiplication of Intuitionistic fuzzy ideals in BG-algebra are introduced and investigated some of their useful properties.The relationship between Intuitionistic fuzzy translation, Intuitionistic fuzzy multiplication of Intuitionistic fuzzy BG-ideal have been discussed.

5.References:

1. S.S. Ahn and H.D. Lee, Fuzzy subalgebras of BG-algebras, Commun. Korean Math. Soc. 19(2) (2004), 243-251.
2. K.T. Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets and Systems, 20 (1986), 87-96.
3. T.Bej and M.Pal,Doubt intutionistic fuzzy ideals in BCK/BCI-algebras, Communicated.
4. C.B. Kim and H.S. Kim, On BG-algebras, Demonstratio Mathematica, 41 (2008), 497-505.
5. Muthuraj R,Sridharan M and Sitharselvam P M, Fuzzyy BG-Ideals in BG-Algebra, International Journal of Computer Application, 2,1 (2010),0975-8887.
6. Dr.A.Prasanna,M.Premkumar and Dr.s.Ismail mohiden “fuzzy translation and fuzzy multiplication on BG-Algebras”, 4th Alterman Conference cum workshop on Computational and Geometric Algebra and workshop on Kahler calculus,July 2019.
7. Prasanna A,Premkumar M and Ismil Mohideen S, fuzzy translation and fuzzy multiplication on B-Algebras, International Journal for Science and Advance Research In Technology, Vol.4,No.4 (2018),2898-2901.
8. Priya and Ramachandran T, fuzzy translation and fuzzy multiplication on PS-Algebras,International Journal of Innovation in Science and Mathematics, Vol.2,No.5,(2014),485-489.
9. T.Senapati,M.Bhowmik and M.Pal, Interval-valued Intuitionistic fuzzy BG-subalgebras,J.Fuzzy Math.20(3) (2012) 707-720.
10. T.Senapati,M.Bhowmik and M.Pal, Intuitionistic fuzzifications of ideals in BG-algebras, Math.aterna 2(9-10) (2012) 761-778.
11. Tapan Senapati, Monoranjan Bhowmik and Madhumangal Pal “Atanasso’s Intuitionistic fuzzy translations of Intuitionistic fuzzy $H-$ ideals in BCK/BCI-algebras”, Notes on Intuitionistic fuzzy Sets Vol. 19, 2013,N0.1, 32-47.
12. Zadeh L A, Fuzzy sets, Information and Control,Vol.8, (1965),338-353.
13. A.Zarandi,A.B.Saeid, On intuionistic fuzzy ideals of BG-algebras,World Academy of Sciences Engineering an Technology 5 (2005) 187-189.