INTUITIONISTICFUZZY CONNECTEDNESS WITH RESPECT TO INTUITIONISTIC FUZZY IDEALS

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ABSTRACT:
Study is done on the fundamentals of intuitionistic fuzzy topology qualities. The study and some extension of previous work is done on the intuitionistic fuzzy left ideals, the set of all normal intuitionistic fuzzy left ideals, and the set of all completely normal intuitionistic fuzzy left ideals.

KEYWORDS: Intuitionistic fuzzy Connectedness, intuitionistic fuzzy ideals, intuitionistic fuzzy left ideals, normal intuitionistic fuzzy left ideals
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INTRODUCTION

The notion of a fuzzy set was introduced by L.A.Zadeh [1965] andsince then this concept has been applied to various algebraic structures. In1964, N.Nobusawa introduced the notion of a Γ⍰⍰ring, as more general than a ring. W.E.Barnes [1966] weakened slightly the conditions in the definition of the Γ⍰ring in the sense of Nobusawa.

Nobusawa introduced the concept of aΓ⍰ring and W.E.Barnes generalised this concept. After that, W.E.Barnes [1966], S.Kyuno [1978]and J.Luh [1969] studied the structure of Γ⍰rings and obtained various generalizations analogous to corresponding parts in ring theory. Then in1992,Y.B.JunandC.Y.Lee introduced the notion of fuzzy ideals in Γ⍰rings and studied preliminary properties of fuzzy ideals. They applied the concept of fuzzy sets to the theory of Γ⍰rings.

The notion of fuzzy ideals in Γ⍰rings was introduced by Y.B.Jun andC.Y.Lee [1992] and studied some preliminary properties of fuzzy ideals.They applied the concept of fuzzy sets to the theory of Γ⍰rings. LaterS.M.Hong and Y.B.Jun[1995] defined normalized fuzzy ideals and fuzzy maximal ideals in Γ⍰rings and studied them. In this chapter, the concepts ofnormalized intuitionistic fuzzy ideals in Γ⍰rings and its characteristics arestudied. Also the notion of Γ⍰residue classringis discussed and established a relation between the set of all intuitionistic fuzzy left ideals ofΓ⍰rings and the set of all intuitionistic fuzzy left ideals of Γ⍰residue classring.

Throughout this chapter,letMdenoteaΓ⍰ring,IFLI(M),NIFI(M) and CNIFI(M) denote the set of all intuitionistic fuzzy left ideals, the set of all normal intuitionistic fuzzy left ideals and the set of all completely normal intuition isticfuzzy left ideals of M respectively.

PRELIMINARIES

Definition1

A subset 'U' of a Γ�ring V is a left (resp. right)ideal ofV if U isan additive subgroup ofVsuch that VΓU�U whereV�U��a�b�a�V����b�U�� and U��V��b�a | b�U������a�V���If U is both a left and a right ideal, then U is a two sided ideal or simply anidealofV�

Definition2

Afuzzyset�ina��ringViscalledafuzzyleftideal of Vif it satisfies:
�i���a�b�≥��a����b��
�ii���a�b� ≥ ��b��resp. ��a�b� ≥ ��a�]�for all a�b�V and ����If�isbothafuzzyleftandrightidealofV��then�iscalledafuzzyidealofV�

Definition3

An intuitionistic fuzzy set U =��$_U$�$_U$� in V is called anintuitionisticfuzzyleftidealofa��ringVif
�i��$_U$��a�b���γ_U��a���$_U$��b��and�$_U$��a�b�≥�$_U$��b�
�ii�χ_U��a�b�≤��$_U$��a���$_U$��b��and�$_U$��a�b�≤�$_U$��b�
forall a�b�V and ����

Definition4

Anintuitionisticfuzzyleft(resp.right)idealU���$_U$��$_U$�ofa �� ring A is saidtobenormalif�$_U$������and�$_U$�������

Definition5

Anon-constantU�IFI(V)iscalledmaximalifMisamaximal e lementof the poset (NIFI(V),�).

Definition6

A normalintuitionistic fuzzy left idealU of V is called completely normalifthereexistsa$_0$�VsuchthatU(a$_0$)=(0,1).
ClearlyCNIFI(V)�NIFI(V).

Theorem 7

LetU���$_U$��$_U$�beanintuitionisticfuzzyleftidealofa��ringMand let
�$_M$�a���$_U$��a����������$_M$�a���$_U$��a����

If�$_M$�a���$_M$�a�≤�foralla���then����$_M$��$_M$�isanormalintuitionisticfuzzyleft(resp.right)ideal of V.

Proof. We first observethat�$_M$�������$_M$�����,where
�$_M$�a���$_M$�a������� forevery a�V.
So,M=<�$_M$,�$_M$>isa normalintuitionisticfuzzyset.
To prove that it is an intuitionistic fuzzy left ideal,leta,b� Mand���. Then

$□_M$□a□b□□□$_U$ (a□b□□□□□$_U$□□□□
≥ □□$_U$□□a□□□$_U$□□b□□□□□□$_U$□□□□
□□□$_U$□□a□□□□□$_U$□□□□□□□□$_U$□□y□□□□□$_U$□□□□□
□□ $_M$□a□□□ $_M$□b□□
$□_M$□a□b□□□$_U$□□a□b□□□$_U$□□□□
≤ □□$_U$□□a□□□$_U$□□b□□□□$_U$□□□□
□□□$_U$□□a□□□$_U$□□□□□□□□$_U$□□y□□□$_U$□□□□□□□$_U$□□a□□□$_U$□□b□□
$□_U$□□a□b□□□$_U$□□a□b□□□□□$_U$□□□□
≥ $□_U$□□b□□□□□$_U$□□□□□□$_U$□□b□and
$□_U$□□a□b□□□$_U$□□a□b□□□$_U$□□□□
≤$□_U$□□b□□□$_U$□□□□□□$_U$□□b□□
ThisshowsthatMisanintuitionisticfuzzyleftidealofV. So,M isanormalintuitionisticfuzzyleftidealofV.

Corollary8

IfUis normal,then U=Mwhere□□□□ $_M$ □□$_M$□, +
$□_M$□□$_U$ (a□□□□□$_U$□□□□and$□_M$□□$_U$ (a□□□$_U$□□□□□
NotethatNIFI(V)is aposet underthe setinclusion.

Theorem 9

Anon-constantmaximalelementof(NIFI(V),□)takesonlythevalues(0,1) and (1,0) .

Theorem 10

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AmaximalA□IFLI(M)isnormalandtakesonlytwovalues(0,1)and(1,0).

Proof. LetU□IFLI(V)be maximal.
ThenMisanon-constantmaximalelementof(NIFI,□)and
byTheorem9,thepossiblevaluesofMare(0,1)and(1,0),Thatis,
$□_M$takesonlytwovalues0and1.
Clearly$□_M$(a)=1ifandonlyif$□_U$(a)=$□_U$(0)and$□_M$(a)=0ifand onlyif$□_U$ (a)=$□_U$ (0)□1.
But U□$□_M$(Theorem 7),so,$□_U$ (a)≤$□_M$(a)foralla□M.Thus$□_M$(a)=0 implies $□_U$ (a)= 0.
Hence$□_U$ (0)=1.
ThisprovesthatUisnormal.

Theorem 11

U(1,0)–levelsubsetofamaximalintuitionisticfuzzyleftidealof
Vis a maximal left ideal of V.

Proof. LetRbea(1,0)–levelsubsetofamaximalintuitionisticfuzzyleftidealUofV.
Thatis,R=$V_{<1,0>}$={a□V□□$_U$ (a)=1}.
ItisnotdifficulttoverifythatSisaleftidealofV.R□Vbecause$□_U$takestwovalues.
LetSbealeftidealofVcontainingR.Then$□_R$□$□_S$.
Since$□_U$=$□_R$and$□_U$takesonlytwovalues,$□_S$alsotakesthesetwovalues.But,byassumption,Uisamaximalintuitio
nisticfuzzyleftidealofV,so
$□_R$=$□_U$=$□_S$or$□_S$(a) = 1 for all x□V.

Inthelast caseR= V,whichis notpossible.
So,$_R$=$_U$=$_S$,which implies R=S.
ThismeansthatRisamaximalleftidealofV.

Theorem 12

A non constant maximal element of (NIFI(V),) is also a maximalelementof(CNIFI(V),).

Proof. LetUbeanonconstantmaximalelementof(NIFI(V),).
By Theorem 9, ‘U’ takes only the values (0,1) and (1,0), so U(0) = (1,0)andU(a_0)=(0,1)forsomea$_0$ V.
HenceUCNIFI(V).
Assume that there exists W CNIFI(V) such that U W.ItfollowsthatU WinNIFI(V).
Since U ismaximal in(NIFI(V),) andsince Wis non-constant, thereforeU=W.
Thus U is maximal element of (CNIFILI(V),).Thiscompletestheproof.

Theorem 13

EverymaximalintuitionisticfuzzyleftidealofViscompletely normal.

Proof. LetanintuitionisticfuzzyleftidealUof Vbemaximal.
Then by Theorem 10, it is normal and U = M takes only two values (0,1)and(1,0).
SinceUisnon-constant,itfollowsthatU(0)=(1,0)andU(a_0)=(0,1)forsome a$_0$V.HenceUiscompletely 1484
normal.This completesthe proof.

Theorem 14

Letg:[0,1][0,1]beanincreasingfunctionandletubeaIFSona-ringM.
Theng^{-1}(U)==<$_{g-1(U)}$,$_g$ $-1_{(U)}$>where$_{g-1(U)}$(a)=g($_U$(a)) an $_{(U)}$(a)=
g($_U$(a))isanintuitionisticfuzzyleftidealifandonlyifU is an intuitionistic fuzzy left ideal. More over if g($_U$(0))=1 and g($_U$(0))=0,theng $^{-1}$(A) is normal.

Proof.Letg^{-1}(U)=<$_{g-1(U)}$,$_{g-1(U)}$>beanintuitionisticfuzzyleftideal ofV.
Theng($_U$(ab))=$_{g-1(U)}$(ab)
≥$_{g-1(U)}$(a)$_{g^{-1}(U)}$(b)
= g($_U$(a))g($_U$(b))
=g($_U$(a)$_U$(b)).
Thatis,g($_U$(ab))≥g($_U$(a)$_U$(b)) for alla , b V.Since gisincreasing,$_U$(ab)≥$_U$(a) $_U$(b).
Alsog($_U$(ab))=$_{g-1(U)}$(ab)
≤$_{g-1(U)}$(a)$_{g-1(U)}$(b)
= g($_U$(a))g($_U$(b))=g($_U$(a)$_U$(b)).That is, g($_U$(ab))≤ g($_U$(a)$_U$(b))foralla,b V.
Since gisincreasing,$_U$(ab)≤ $_U$(a)$_U$(b).
Similarly
g($_U$(ab))=$_{g-1(U)}$(ab)≥$_{g-1(U)}$(b)=g($_U$(b)).
Thatis,
g($_U$(ab))≥g($_U$(b))foralla,b V.
Also,
g($_U$(ab))=$_{g-1(U)}$(ab)≤$_{g-1(U)}$(b)=g($_U$(b)).

Thatis,
$g(\square_U(a\square b))\leq g(\square_U(b))$foralla,b□V.
Sincegisincreasing$\square_U(a\square b)\geq\square_U(b)and\square_U(a\square b)\leq\square_U(b)$.

Conversely, let U = <□U,□U> bean intuitionistic fuzzy left ideal of V.

Thenforalla,b□V, we have

$\square_{g^{-1}(U)}(a\square b)=g(\square_U(a\square b))\geq g(\square_u(a)\square\square_U(b))$
$=g(\square_U(a))\square g(\square_U(b))$
$=\square_{g^{-1}(U)}(a)\square\square_{g^{-1}(U)}(b)$and
$\square_{g^{-1}(U)}(a\square b)=g(\square_U(a\square b))\leq g(\square_U(a)\square\square_U(b))$
$=g(\square_U(a))\square g(\square_U(b))$
$=\square_{g^{-1}(U)}(a)\square\square_{g^{-1}(U)}(b)$.
Also$\square_{g^{-1}(U)}(a\square b)=g(\square_U(a\square b))\square g(\square_U(b))=\square_{g^{-1}(U)}(b)$and
$\square_{g^{-1}(U)}(a\square b)=g(\square_U(a\square b))\square g(\square_U(b))=\square_{g^{-1}(U)}(b)$.
Thisprovesthatg $^{-1}$(U)isanintuitionisticfuzzyleftidealifandonlyifUisanintuitionisticfuzzyleftideal.

CONCLUSION

The basics of intuitionistic fuzzy topology properties are studied. The intuitionistic fuzzy leftideals, theset of all normal intuitionistic fuzzy left ideals and the set of all completely normal intuitionistic fuzzy left ideals are studied and some extension of already existing work is done.

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