Implementation of Second Order Fuzzy Relational Equation evaluation for the Behaviour of Faculty Teaching Performance by Using Max-Min Composition

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Abstract:
Second order fuzzy relation is used to construct the second order fuzzy relational equations where the developed model uses max-min composition. The application of the model to engineering institute is discussed. The existing feedback system in the engineering institute and its effect on faculty performance involving uncertain parameters is studied. Second Order Binary Fuzzy Relation between faculty performance and student feedback is developed and its impact on result of a particular course is studied. An attempt is made making the decision processes more authentic. The process is exemplified by considering real life example.

Keywords: Second Order Binary Fuzzy Relation, Second order Fuzzy relational equation, Max-min composition, Epsilon-delta triangular fuzzy number, Engineering institute, Faculty performance

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I.INTRODUCTION

The role of faculty in teaching-learning process is vital in order to holistic development of the students. Through teaching-learning process faculty makes student capable to imbibe the required levels of knowledge, skills and application so that student will confidently demonstrate these skills at workplace in particular and society in general. For evaluating the different capabilities of faculty, different feedback systems are used which involve uncertainty. Feedback mechanism system ensures a detailed analysis of the performance of the faculty members with respect to different capabilities as various parameters. The feedback system provides capabilities for selecting a particular course for feedback. It is expected to provide proper feedback to the concerned faculty so that respective faculty can bring improvement on their weaker points. The aim of the present research work is to study the present feedback system in the specific engineering institute and its effect on faculty performance involving uncertain parameters. The system so developed helps in making the decision processes more authentic.

In this paper, we propose new approach to evaluate the faculty performance using Second Order Fuzzy relational equations (SOFRE). Some researchers have showed that the second order fuzzy relation can be obtained by annexing some uncertainty into type-1 fuzzy relation [1-2]. They also developed some applications [3]. No more attention is given to apply the higher order fuzzy relational

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equations to real life situations due to its computational difficulty. The notion of fuzzy relation equation was first proposed and investigated by Sanchez [4]. Many real-life applications can be modelled within fuzzy relational equations framework. A solution of fuzzy relational equation uses different composition operations [5-7]. In various papers researcher tried to solve FREs [8-10]. Many researches have been used fuzzy modeming in their research to improvements [11-27].

In this paper, we use max-min composition of SOFRE. The epsilon-delta triangular fuzzy numbers $r_{\varepsilon,\delta}$ are used for computational task.

The paper is organized in five sections. The second section contains the preliminaries from second order Fuzzy Binary Relation and SOFRE. In the third section the Mathematical Model is developed that uses SOFRE for studying the relation between faculty performance and students feedback. The level of mathematical anxiety among middle school students. Based on the correlation and standard deviation [28]. Moreover, the relation between faculty performance and result of a particular course is obtained. Finally, the fourth section is devoted to our conclusion and future scope of the study.

II. PRELIMINARIES

Zadeh introduced fuzzy set theory in 1965 [29]. Sanchez studied theoretical and application aspects of fuzzy relational equation [30]. The notion of Fuzzy Relational Equations is associated with the concept of composition of fuzzy binary relations and useful in decision making, modelling of non-probabilistic from of uncertainty. Before developing the mathematical model, we will discuss the basic definition of second order fuzzy relations, SOFRE etc.

Definition 2.1[10] Triangular epsilon-delta fuzzy number

If r is a real number then ε-δ fuzzy number $r_{\varepsilon,\delta}$ is the triangular fuzzy number, for some $\varepsilon,\delta \in R, (\varepsilon,\delta > 0)$ is a fuzzy set $r_{\varepsilon,\delta} : R \to [0,1]$ defined by

$$r_{\varepsilon,\delta}(x) = \begin{cases} \dfrac{x-(r-\varepsilon)}{\varepsilon}, & \text{if } r-\varepsilon < x \le r, \\ \dfrac{x-(r+\delta)}{-\delta}, & \text{if } r \le x < r+\delta, \\ 0, & \text{otherwise.} \end{cases}$$

A triangular fuzzy number $A = (l, m, n)$ in above notation is denoted by $A = m_{m-l,n-m}$.

Also, $r_{\varepsilon,\delta} = (r-\varepsilon, r, r+\delta)$.

Definition 2.2 [2] Second Order Fuzzy Relation

A function $\mathrm{R}^2 : X \times Y \to \mathrm{F}(I)$ defined by $\mathrm{R}^2(x,y) = r^{xy}_{\varepsilon,\delta}$ where $r^{xy}_{\varepsilon,\delta} = r_{\varepsilon,\delta}$ is a triangular fuzzy number on $I = [0,1]$ such that $r-\varepsilon \ge 0, r+\delta \le 1$, which indicates the fuzzy relationship between x and y for every $x \in X, y \in Y$, $\mathrm{F}(I)$ is a fuzzy power set of $I = [0,1]$, is called a second order fuzzy relation. It is denoted by $\mathrm{R}^2(X,Y)$. If $r^{xy}_{\varepsilon,\delta} = (r_{\varepsilon,\delta})_{xy} = \tilde{1}$ for some $x \in X$, $y \in Y$ implies that x is completely related to y and if $r^{xy}_{\varepsilon,\delta} = (r_{\varepsilon,\delta})_{xy} = \tilde{0}$ for some $x \in X, y \in Y$ then there is no relation between x and y, where $\tilde{1} = 1_{\varepsilon,0}$ and $\tilde{0} = 0_{0,\delta}$.

Definition 2.3 ([2]) Let $r_{\varepsilon_1,\delta_1}$ and $s_{\varepsilon_2,\delta_2}$ be any two epsilon-delta fuzzy numbers where $r \le s$.

1. If $r-\varepsilon_1 \le s-\varepsilon_2$ and $r+\delta_1 \le s+\delta_2$ then we define $r_{\varepsilon_1,\delta_1} \wedge s_{\varepsilon_2,\delta_2} = r_{\varepsilon_1,\delta_1}$ and $r_{\varepsilon_1,\delta_1} \vee s_{\varepsilon_2,\delta_2} = s_{\varepsilon_2,\delta_2}$.
2. If $s-\varepsilon_2 < r-\varepsilon_1$ and $r+\delta_1 \le s+\delta_2$ then we define $r_{\varepsilon_1,\delta_1} \wedge s_{\varepsilon_2,\delta_2} = r_{\varepsilon_2-|r-s|,\delta_1}$ and $r_{\varepsilon_1,\delta_1} \vee s_{\varepsilon_2,\delta_2} = s_{\varepsilon_1+|r-s|,\delta_2}$.
3. If $r-\varepsilon_1 \le s-\varepsilon_2$ and $s+\delta_2 < r+\delta_1$ then $r_{\varepsilon_1,\delta_1} \wedge s_{\varepsilon_2,\delta_2} = r_{\varepsilon_1,\delta_2+|r-s|}$ and $r_{\varepsilon_1,\delta_1} \vee s_{\varepsilon_2,\delta_2} = s_{\varepsilon_2,\delta_1-|r-s|}$.
4. If $s-\varepsilon_2 \le r-\varepsilon_1$ and $s+\delta_2 < r+\delta_1$ then

$$r_{\varepsilon_1,\delta_1} \wedge s_{\varepsilon_2,\delta_2} = r_{\varepsilon_2-|r-s|,\,\delta_2+|r-s|}$$ and
$$r_{\varepsilon_1,\delta_1} \vee s_{\varepsilon_2,\delta_2} = s_{\varepsilon_1+|r-s|,\,\delta_1-|r-s|}$$

Definition 2.4 Second Order Fuzzy Relational equation (SOFRE)

Let the membership matrices P^2, Q^2 and R^2 be denoted by $P^2 = \left[\left(p_{ij}^2\right)_{\varepsilon_{ij},\delta_{ij}}\right]$,
$Q^2 = \left[\left(q_{jk}^2\right)_{\varepsilon_{jk},\delta_{jk}}\right]$ and $R^2 = \left[\left(r_{ik}^2\right)_{\varepsilon_{ik},\delta_{ik}}\right]$ respectively, where
$\left(p_{ij}^2\right)_{\varepsilon_{ij},\delta_{ij}} = P^2(x_i, y_j), \left(q_{jk}^2\right)_{\varepsilon_{jk},\delta_{jk}} = Q^2(y_j, z_k)$
and $\left(r_{ik}^2\right)_{\varepsilon_{ik},\delta_{ik}} = R^2(x_i, z_k)$ for all $i \in I(=N_n)$, $i \in J(=N_m)$ and $k \in K(=N_s)$.

The three second order fuzzy binary relation constrains each other in such a way that

$$R^2(X,Z) = P^2(X,Y) \circ Q^2(Y,Z) \quad \ldots (1)$$

where $\circ$ denotes the max-min composition, $P^2(X,Y), Q^2(Y,Z), R^2(X,Z)$ are second order fuzzy binary relations defined on the sets $X = \{x_i / i \in I\}; Y = \{y_j / j \in J\}; Z = \{z_k / k \in K\}$ where $I = N_n, J = N_m$ and $K = N_s$.[7, 8]

This means that,

$\left(r_{ik}^2\right)_{\varepsilon_{ik},\delta_{ik}} = \max_{j\in J} \min\left[\left(p_{ij}^2\right)_{\varepsilon_{ij},\delta_{ij}}, \left(q_{jk}^2\right)_{\varepsilon_{jk},\delta_{jk}}\right]$ for all $i \in I,\ k \in K$

$$\left(r_{ik}^2\right)_{\varepsilon_{ik},\delta_{ik}} = \bigvee_{j\in J}\left[\left(p_{ij}^2\right)_{\varepsilon_{ij},\delta_{ij}} \wedge \left(q_{jk}^2\right)_{\varepsilon_{jk},\delta_{jk}}\right] \ldots (2)$$

The matrix equation (1) consists of $n \times s$ simultaneous equations of the form given in (2).

When two of the components in each of the equations are given and one is unknown such equations are referred as second order fuzzy relational equations (SOFRE). SOFRE in matrix notation can be given as

$$\left[\left(r_{ik}^2\right)_{\varepsilon_{ik},\delta_{ik}}\right] = \left[\left(p_{ij}^2\right)_{\varepsilon_{ij},\delta_{ij}}\right] \circ \left[\left(q_{jk}^2\right)_{\varepsilon_{jk},\delta_{jk}}\right]$$

$$\left[\left(r_{ik}^2\right)_{\varepsilon_{ik},\delta_{ik}}\right] = \bigvee_{j\in J}\left\{\left[\left(p_{ij}^2\right)_{\varepsilon_{ij},\delta_{ij}}\right] \wedge \left[\left(q_{jk}^2\right)_{\varepsilon_{jk},\delta_{jk}}\right]\right\}$$

When membership matrices of two second order fuzzy relation P^2, Q^2 are given and membership matrix R^2 is to be determined by max-min composition, the problem is trivial. Clearly, unique solution exists in this case.

On the other side when membership matrices of any one of second order fuzzy relation P^2, Q^2 are unknown. In such case SOFRE does not have any guarantee of solution.

III. MATHEMATICAL MODEL FOR SOFRE

Mathematical Model (MM) is a formulation that expresses essential features of process in mathematical terms. Mathematical model is a solution of real-world problems and appears as a dynamic tool for evaluating the faculty performance based on student's feedback which incorporates specially designed questionnaire (Appendix I). Moreover, the relation between the faculty performance and course result is studied. The proposed model is validated by case study.

The design of the MM

1. Know the problem,
2. Mathematical formulation,
3. Solution of the problem,
4. Solution interpretation,
5. Control of the model [29]

1. Know the problem:

Making the decision related to faculty performance and students' feedback and its effect on the outcome of the course is little authentic if the performance parameters are uncertain.

2. Mathematical formulation:

We propose new approach to evaluate the faculty performance using Second Order Fuzzy relational equations (SOFRE).

Let the crisp set of faculties teaching the course

$X = \{\text{F1, F2, F3, F4, F5, F6}\}$,

Y = {Question wise feedback},

Z = {Result of faculty members for a particular course}.

The faulty feedback on a percentage scale is represented in the form

$$\left\{F_i, \frac{Q_j\text{Feedback}}{\text{Total marks for particular question}} * 100\right\}$$

where $i = 1,2,3,4,5,6$ (fixed) , $j = \{1,2,3,4,5,6,7\}$. In the present case total marks for particular question is *1.43*. TABLE I presents the faculty wise feedback received in percentage.

TABLE I Faculty wise Feedback

Sr. No.	Q.1	Q.2	Q.3	Q.4	Q.5	Q.6	Q.7
F1	Question wise feedback received to each faculty member on percentage scale						
⋮							
F6							

The fuzzy sets representing concepts of a poor, satisfactory, good, excellent is considered. TABLE II presents the quantification of numerical data. The quantification is given by membership function $A: X \to [0,1]$ where $X = [0,100]$ as follows

$$A(x) = \begin{cases} 1, & x \ge 90 \\ \dfrac{x-40}{50}, & 40 \le x < 90 \\ 0, & \text{otherwise} \end{cases}$$

Membership grade for poor P = [0, 0.3],
Membership grade for satisfactory S = (0.3, 0.6],
Membership grade for good G = (0.6, 0.9],
Membership grade for Excellent E = (0.9, 1]

TABLE II Type1 Fuzzy Relation Between Faculty and Feedback

Sr. No.	Q1	Q2	Q3	Q4	Q5	Q6	Q7
F1	Type 1 Fuzzy Relation						
⋮							
F6							

The fuzzy set P induces a fuzzy binary relation (FBR)
$P(x_i, y_j)$, i=1,2,3,4,5,6 and j=1,2,3,4,5,6,7 , $x_i \in X$ and $y_j \in Y$
giving relation between faculty members and feedback received.

Effect of results of a particular course related to questionnaire is given by the matrix,
$[Q] = Q_{ij} \cdot r_i$
and represented in TABLE III where
i = 1,2,3,4,5,6 and j = 1,2,3,4,5,6,7 and k = 1,2,3,4,5,6, $x_i \in X$, $y_j \in Y$ and $z_k \in Z$
wherein r_i is result of faculty members courses which is known (See Appendix 2).
The fuzzy set Q induces a FBR $Q(y_j, z_k)$,i = 1,2,3,4,5,6,7 and k = 1,2,3,4,5,6 giving relation between faculty members and feedback received.
The fuzzy set R induces a FBR $R(x_i, z_k)$,i = 1,2,3,4,5,6 and k = 1,2,3,4,5,6 giving relation between faculty members and effect of questionnaire on result.

TABLE III: Relation Between Questionnaire and Result

Question number	Effect of Questionnaire on Result
Q.1	$Q_{ij} \cdot r_i$
⋮	
Q.7	

3. Solution of the problem

Assume now that three relations constraints each in such a way that $P \circ Q = R$ ......(A) where $_0$ denotes max-min composition.
This means that $\vee(p_{ij} \circ q_{jk}) = r_{ik}$, $i = 1,2,3,4,5,6$ and $k = 1,2,3,4,5,6$ 1565
The matrix P and Q are so far obtained are type-1 fuzzy relation. Conversion of type-1 fuzzy relation into second order fuzzy relation can be done by adding some uncertainty into it.
Semester I (Computational Task)
TABLE IV to XV presents semester wise faculty feedback

TABLE IV: Feedback of Faculty 1 Sem I.

Faculty 1	Questions						
	Q.1	Q.2	Q.3	Q.4	Q.5	Q.6	Q.7
	1.23	1.21	1.25	1.22	1.24	1.25	1.20
Feedback received on percentage scale	86.01	84.62	87.41	85.31	86.71	87.41	83.92
Fuzzified data	0.92	0.89	0.95	0.91	0.93	0.95	0.88
Quantification of Numerical Data	E	G	E	E	E	E	G

Conversion of type-1 fuzzy relation into second order fuzzy relation can be done by adding some uncertainty into it.

Thus
$P^2 = [0.92_{0.9,0.04} \quad 0.89_{0.85,0.08} \quad 0.95_{0.9,0.04} \quad 0.91_{0.9,0.08}$

$0.93_{0.8,0.04} \quad 0.95_{0.9,0.04} \quad 0.88_{0.8,0.1}]$

Effect of result on Questionnaire,

$$Q^2 = \begin{bmatrix} 0.73_{0.71,0.2} \\ 0.71_{0.7,0.2} \\ 0.75_{0.7,0.2} \\ 0.72_{0.7,0.25} \\ 0.74_{0.7,0.1} \\ 0.75_{0.7,0.2} \\ 0.7_{0.6,0.2} \end{bmatrix}$$

Resultant matrix

$$R^2 = \left(r_{1k}^2\right)_{\varepsilon_{ik},\delta_{1k}} = \max_{j\in J} \min\left[\left(p_{1j}^2\right)_{\varepsilon_{1j},\delta_{1j}}, \left(q_{jk}^2\right)_{\varepsilon_{jk},\delta_{jk}}\right]$$

for all $i \in I,\ k \in K$

$$R^2 = r_{11}^2 = \vee\left[\left(0.92_{0.9,0.04} \wedge 0.73_{0.71,0.2}\right), \left(0.89_{0.85,0.08} \wedge 0.71_{0.7,0.2}\right)\right.$$
$$\left(0.95_{0.9,0.04} \wedge 0.75_{0.7,0.2}\right), \left(0.91_{0.9,0.08} \wedge 0.72_{0.7,0.25}\right), \left(0.93_{0.8,0.04} \wedge 0.74_{0.7,0.1}\right)$$
$$\left.\left(0.95_{0.9,0.04} \wedge 0.75_{0.7,0.2}\right), \left(0.88_{0.8,0.1} \wedge 0.7_{0.6,0.2}\right)\right]$$
$$R^2 = r_{11}^2 = \vee\left(0.73_{0.71,0.2}, 0.71_{0.7,0.2}, 0.75_{0.7,0.2}, 0.72_{0.71,0.25}, 0.74_{0.7,0.1}, 0.75_{0.7,0.2}, 0.7_{0.62,0.2}\right)$$
$$= 0.75_{0.67,0.22}$$

TABLE V: Feedback of Faculty 2 Sem I.

Faculty 2	Questions						
	Q.1	Q.2	Q.3	Q.4	Q.5	Q.6	Q.7
	1.01	0.99	1.07	1.03	1.04	1.06	1.04
Feedback received on percentage scale	70.63	69.23	74.83	72.03	72.73	74.13	72.73
Fuzzified data	0.61	0.58	0.70	0.64	0.65	0.68	0.65
Quantification of Numerical Data	G	S	G	G	G	G	G

Thus

$$P^2 = \left[0.61_{0.6,0.3} \quad 0.58_{0.5,0.4} \quad 0.7_{0.6,0.2} \quad 0.64_{0.6,0.3} \quad 0.65_{0.6,0.2} \quad 0.68_{0.6,0.3} \quad 0.65_{0.6,0.2}\right]$$

Effect of result on Questionnaire,

$$Q^2 = \begin{bmatrix} 0.41_{0.4,0.5} \\ 0.39_{0.3,0.6} \\ 0.46_{0.4,0.5} \\ 0.43_{0.4,0.5} \\ 0.44_{0.4,0.5} \\ 0.46_{0.4,0.5} \\ 0.44_{0.4,0.5} \end{bmatrix}$$

Resultant matrix

$$R^2 = r_{22}^2 = \vee\left(0.41_{0.4,0.5}, 0.39_{0.31,0.59}, 0.46_{0.4,0.44}, 0.43_{0.4,0.5}, 0.44_{0.4,0.41}, 0.46_{0.4,0.5}, 0.44_{0.4,0.41}\right)$$
$$= 0.46_{0.38,0.52}$$

TABLE VI: Feedback of Faculty 3 Sem I

Faculty 3	Questions						
	Q.1	Q.2	Q.3	Q.4	Q.5	Q.6	Q.7
	1.1	1.11	1.11	1.1	1.13	1.09	1.09
Feedback received on percentage scale	76.92	77.62	77.62	76.92	79.02	76.22	76.22
Fuzzified data	0.74	0.75	0.75	0.74	0.78	0.72	0.72
Quantification of Numerical Data	G	G	G	G	G	G	G

Thus

$$P^2 = \left[0.74_{0.7,0.2} \quad 0.75_{0.72,0.24} \quad 0.75_{0.72,0.24} \quad 0.74_{0.7,0.2} \quad 0.78_{0.7,0.2} \quad 0.72_{0.7,0.2} \quad 0.72_{0.7,0.2}\right]$$

Effect of result on Questionnaire,

$$Q^2 = \begin{bmatrix} 0.25_{0.2,0.7} \\ 0.26_{0.2,0.74} \\ 0.26_{0.2,0.74} \\ 0.25_{0.2,0.7} \\ 0.27_{0.25,0.7} \\ 0.25_{0.2,0.7} \\ 0.25_{0.2,0.7} \end{bmatrix}$$

Resultant matrix

$$R^2 = r_{33}^2 = \vee\left(0.25_{0.21,0.69}, 0.26_{0.23,0.73}, 0.26_{0.2,0.74}, 0.25_{0.2,0.7}, 0.27_{0.25,0.7}, 0.25_{0.23,0.67}, 0.25_{0.23,0.67}\right)$$
$$= 0.27_{0.23,0.72}$$

TABLE VII: Feedback of Faculty 4 Sem I

	Q.1	Q.2	Q.3	Q.4	Q.5	Q.6	Q.7
	1.26	1.25	1.29	1.27	1.27	1.28	1.27
Feedback received on percentage scale	88.11	87.41	90.21	88.81	88.81	89.51	88.81
Fuzzified data	0.96	0.95	1.00	0.98	0.98	0.99	0.98
Quantification of Numerical Data	E	E	E	E	E	E	E

Thus

$$P^2 = \left[0.96_{0.9,0.2} \quad 0.95_{0.9,0.04} \quad 1_{0.9,0} \quad 0.98_{0.9,0.01} \quad 0.98_{0.9,0.01} \quad 0.99_{0.9,0.01} \quad 0.98_{0.9,0.01}\right]$$

Effect of result on Questionnaire,

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$$Q^2 = \begin{bmatrix} 0.67_{0.6,0.3} \\ 0.66_{0.62,0.32} \\ 0.69_{0.6,0.3} \\ 0.68_{0.62,0.3} \\ 0.68_{0.62,0.3} \\ 0.69_{0.6,0.3} \\ 0.68_{0.62,0.3} \end{bmatrix}$$

Resultant matrix

$$R^2 = r_{44}^2 = \vee(0.67_{0.61,0.31}, 0.66_{0.62,0.32}, 0.69_{0.6,0.3}, 0.68, 0.68_{0.62,0.3}, 0.69_{0.6,0.3}, 0.68_{0.62,0.3}) = 0.69_{0.6,0.3},$$

TABLE VIII: Feedback of Faculty 5 Sem I

Faculty 5	Questions						
	Q.1	Q.2	Q.3	Q.4	Q.5	Q.6	Q.7
	1.26	1.25	1.29	1.27	1.27	1.28	1.27
Feedback received on percentage scale	94.41	93.01	94.41	93.71	93.01	93.71	90.21
Fuzzified data	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Quantification of Numerical Data	E	E	E	E	E	E	E

Thus

$$P^2 = [1_{0.9,0} \quad 1_{0.9,0} \quad 1_{0.9,0} \quad 1_{0.9,0} \quad 1_{0.9,0} \quad 1_{0.9,0} \quad 1_{0.9,0}]$$

Effect of result on Questionnaire,

$$Q^2 = \begin{bmatrix} 0.7_{0.6,0.2} \\ 0.7_{0.6,0.2} \\ 0.7_{0.6,0.2} \\ 0.7_{0.6,0.2} \\ 0.7_{0.6,0.2} \\ 0.7_{0.6,0.2} \\ 0.7_{0.6,0.2} \end{bmatrix}$$

Resultant matrix

$$R^2 = r_{55}^2 = \vee(0.7_{0.6,0.2}, 0.7_{0.6,0.2}, 0.7_{0.6,0.2}, 0.7_{0.6,0.2}, 0.7_{0.6,0.2}, 0.7_{0.6,0.2}, 0.7_{0.6,0.2}) = 0.7_{0.6,0.2}$$

TABLE IX: Feedback of Faculty 6 Sem I

	Q.1	Q.2	Q.3	Q.4	Q.5	Q.6	Q.7
	1.31	1.29	1.32	1.3	1.27	1.3	1.27
Feedback received on percentage scale	91.61	90.21	92.31	90.91	88.81	90.91	88.81
Fuzzified data	1.00	1.00	1.00	1.00	0.98	1.00	0.98
Quantification of Numerical Data	E	E	E	E	E	E	E

Thus

$$P^2 = [1_{0.9,0} \quad 0.99_{0.9,0.01} \quad 1_{0.9,0} \quad 1_{0.9,0} \quad 0.99_{0.9,0.01} \quad 1_{0.9,0} \quad 0.99_{0.9,0.01}]$$

Effect of result on Questionnaire,

$$Q^2 = \begin{bmatrix} 0.42_{0.4,0.5} \\ 0.42_{0.4,0.5} \\ 0.42_{0.4,0.5} \\ 0.42_{0.4,0.5} \\ 0.41_{0.4,0.55} \\ 0.42_{0.4,0.5} \\ 0.41_{0.4,0.55} \end{bmatrix}$$

Resultant matrix

$$R^2 = r_{66}^2 = \vee(0.42_{0.4,0.5}, 0.42_{0.4,0.5}, 0.42_{0.4,0.5}, 0.42_{0.4,0.5}, 0.41_{0.4,0.55}, 0.42_{0.4,0.5}, 0.41_{0.4,0.55}) = 0.42_{0.4,0.54}$$

Thus the decision matrix is,

$$R^2 = \begin{matrix} F1 \\ F2 \\ F3 \\ F4 \\ F5 \\ F6 \end{matrix} \overset{R_{ik}^2}{\begin{bmatrix} 0.75_{0.67,0.22} \\ 0.46_{0.38,0.52} \\ 0.27_{0.23,0.72} \\ 0.69_{0.6,0.3,} \\ 0.7_{0.6,0.2} \\ 0.42_{0.4,0.54} \end{bmatrix}} \text{ or}$$

$$R^2 = \frac{0.75_{0.67,0.22}}{F1} + \frac{0.46_{0.38,0.52}}{F2} + \frac{0.27_{0.23,0.72}}{F3} + \frac{0.69_{0.6,0.3}}{F4} + \frac{0.7_{0.6,0.2}}{F5} + \frac{0.42_{0.4,0.54}}{F6}$$

Semester II (Computational Task)

TABLE X: Feedback of Faculty 1Sem II

Faculty 1	Questions						
	Q.1	Q.2	Q.3	Q.4	Q.5	Q.6	Q.7
	1.28	1.27	1.29	1.26	1.3	1.3	1.25
Feedback received on percentage scale	89.51	88.81	90.21	88.11	90.91	90.91	87.41
Fuzzified data	0.99	0.98	1.00	0.96	1.00	1.00	0.95
Quantification of Numerical Data	E	E	E	E	E	E	E

Thus

$$P^2 = \left[0.99_{0.9,0.01} \quad 0.98_{0.9,0.01} \quad 1_{0.9,0} \quad 0.96_{0.9,0.02} \quad 1_{0.9,0} \quad 1_{0.9,0} \quad 0.95_{0.9,0.01}\right]$$

Effect of result on Questionnaire

$$Q^2 = \begin{bmatrix} 0.86_{0.8,0.1} \\ 0.85_{0.8,0.1} \\ 0.87_{0.8,0.1} \\ 0.72_{0.7,0.25} \\ 0.87_{0.8,0.1} \\ 0.87_{0.8,0.1} \\ 0.82_{0.8,0.16} \end{bmatrix}$$

Resultant matrix

$$R^2 = r_{11}^{\,2} = \vee\left(0.86_{0.8,0.1}, 0.85_{0.8,0.1}, 0.87_{0.8,0.1}, 0.72_{0.71,0.25}, 0.74_{0.7,0.1}, 0.75_{0.7,0.2}, 0.7_{0.62,0.2}\right)$$

$$= 0.87_{0.8,0.11}$$

TABLE XI: Feedback of Faculty 2 Sem II

Faculty 2	Questions						
	Q.1	Q.2	Q.3	Q.4	Q.5	Q.6	Q.7
	1.29	1.28	1.31	1.29	1.28	1.29	1.28
Feedback received on percentage scale	90.21	89.51	91.61	90.21	89.51	90.21	89.51
Fuzzified data	1.00	0.99	1.00	1.00	0.99	1.00	0.99
Quantification of Numerical Data	E	E	E	E	E	E	E

Thus

$$P^2 = \left[1_{0.9,0} \quad 0.99_{0.9,0.01} \quad 1_{0.9,0} \quad 1_{0.9,0} \quad 0.99_{0.9,0.01} \quad 1_{0.9,0} \quad 0.99_{0.9,0.01}\right]$$

Effect of result on Questionnaire

$$Q^2 = \begin{bmatrix} 0.85_{0.8,0.1} \\ 0.84_{0.8,0.1} \\ 0.85_{0.8,0.1} \\ 0.85_{0.8,0.1} \\ 0.84_{0.8,0.1} \\ 0.85_{0.8,0.1} \\ 0.84_{0.8,0.1} \end{bmatrix}$$

Resultant matrix

$$R^2 = r_{22}^{\,2} = \vee\left(0.85_{0.8,0.1}, 0.84_{0.8,0.14}, 0.85_{0.8,0.1}, 0.85_{0.8,0.1} \; 0.84_{0.8,0.14}, 0.85_{0.8,0.1}, 0.84_{0.8,0.14}\right)$$

$$= 0.85_{0.8,0.13}$$

TABLE XII Feedback of Faculty 3 Sem II

Faculty 3	Questions						
	Q.1	Q.2	Q.3	Q.4	Q.5	Q.6	Q.7
	1.21	1.2	1.21	1.2	1.2	1.2	1.21
Feedback received on percentage scale	84.62	83.92	84.62	83.92	83.92	83.92	84.62
Fuzzified data	0.89	0.88	0.89	0.88	0.88	0.88	0.89
Quantification of Numerical Data	G	G	G	G	G	G	G

Thus

$$P^2 = \left[0.89_{0.8,0.1} \quad 0.88_{0.8,0.1} \quad 0.89_{0.8,0.1} \quad 0.88_{0.8,0.1} \quad 0.88_{0.8,0.1} \quad 0.88_{0.8,0.1} \quad 0.89_{0.8,0.1}\right]$$

Effect of result on Questionnaire

$$Q^2 = \begin{bmatrix} 0.54_{0.5,0.4} \\ 0.53_{0.4,0.4} \\ 0.54_{0.5,0.4} \\ 0.53_{0.4,0.4} \\ 0.53_{0.4,0.4} \\ 0.53_{0.4,0.4} \\ 0.54_{0.5,0.4} \end{bmatrix}$$

Resultant matrix

$$R^2 = r_{33}^{\,2} = \vee\left(0.54_{0.5,0.4}, 0.53_{0.45,0.4}, 0.54_{0.5,0.4}, 0.53_{0.45,0.4} \; 0.53_{0.45,0.4}, 0.53_{0.45,0.4}, 0.54_{0.5,0.4}\right)$$

$$= 0.54_{0.46,0.4}$$

TABLE XIII: Feedback of Faculty 4 Sem II

Faculty 4	Questions						
	Q.1	Q.2	Q.3	Q.4	Q.5	Q.6	Q.7
	1.28	1.26	1.28	1.28	1.27	1.26	1.23
Feedback received on percentage scale	89.51	88.11	89.51	89.51	88.81	88.11	86.01
Fuzzified data	0.99	0.96	0.99	0.99	0.98	0.96	0.92
Quantification of Numerical Data	E	E	E	E	E	E	E

Thus

$$P^2 = [0.99_{0.9,0.01} \quad 0.96_{0.9,0.2} \quad 0.99_{0.9,0.01} \quad 0.99_{0.9,0.01} \quad 0.98_{0.9,0.01} \quad 0.96_{0.9,0.2} \quad 0.92_{0.9,0.04}]$$

Effect of result on Questionnaire

$$Q^2 = \begin{bmatrix} 0.79_{0.7,0.2} \\ 0.77_{0.7,0.2} \\ 0.79_{0.7,0.2} \\ 0.79_{0.7,0.2} \\ 0.78_{0.7,0.2} \\ 0.77_{0.7,0.2} \\ 0.73_{0.71,0.2} \end{bmatrix}$$

Resultant matrix

$$R^2 = r_{44}^{\ 2} = \vee(0.79_{0.7,0.2}, 0.77_{0.71,0.2}, 0.79_{0.7,0.2}, 0.79_{0.7,0.2}, 0.78_{0.7,0.2}, 0.77_{0.71,0.2}, 0.73_{0.71,0.2})$$
$$= 0.79_{0.7,0.2}$$

TABLE IIV: Feedback of Faculty 5 Sem II

Faculty 5	Questions						
	Q.1	Q.2	Q.3	Q.4	Q.5	Q.6	Q.7
	1.2	1.29	1.3	1.29	1.27	1.3	1.28
Feedback received on percentage scale	83.92	90.21	90.91	90.21	88.81	90.91	89.51
Fuzzified data	1.00	1.00	1.00	1.00	1.00	1.00	0.99
Quantification of Numerical Data	E	E	E	E	E	E	E

Thus

$$P^2 = [1_{0.9,0} \quad 1_{0.9,0} \quad 1_{0.9,0} \quad 1_{0.9,0} \quad 1_{0.9,0} \quad 1_{0.9,0} \quad 0.99_{0.9,0}]$$

Effect of result on Questionnaire

$$Q^2 = \begin{bmatrix} 0.83_{0.7,0.1} \\ 0.83_{0.7,0.1} \\ 0.83_{0.7,0.1} \\ 0.83_{0.7,0.1} \\ 0.83_{0.7,0.1} \\ 0.83_{0.7,0.1} \\ 0.82_{0.72,0.1} \end{bmatrix}$$

Resultant matrix

$$R^2 = r_{55}^{\ 2} = \vee(0.83_{0.73,0.1}, 0.83_{0.73,0.1}, 0.83_{0.73,0.1}, 0.83_{0.73,0.1}, 0.83_{0.73,0.1}, 0.83_{0.73,0.1}, 0.82_{0.73,0.1})$$
$$= 0.83_{0.73,0.1}$$

TABLE III: Feedback of faculty 6 Sem II

Faculty 6	Questions						
	Q.1	Q.2	Q.3	Q.4	Q.5	Q.6	Q.7
	1.29	1.28	1.28	1.3	1.28	1.3	1.29
Feedback received on percentage scale	90.21	89.51	89.51	90.91	89.51	90.91	90.21
Fuzzified data	1.00	0.99	1.00	1.00	0.99	1.00	1.00
Quantification of Numerical Data	E	E	E	E	E	E	E

Thus

$$P^2 = [1_{0.9,0} \quad 0.99_{0.9,0} \quad 1_{0.9,0} \quad 1_{0.9,0} \quad 0.99_{0.9,0} \quad 1_{0.9,0} \quad 1_{0.9,0}]$$

Effect of result on Questionnaire

$$Q^2 = \begin{bmatrix} 0.62_{0.6,0.3} \\ 0.61_{0.6,0.3} \\ 0.62_{0.6,0.3} \\ 0.62_{0.6,0.3} \\ 0.61_{0.6,0.3} \\ 0.62_{0.6,0.3} \\ 0.62_{0.6,0.3} \end{bmatrix}$$

Resultant matrix

$$R^2 = r_{66}^{\ 2} = \vee(0.62_{0.6,0.3}, 0.61_{0.6,0.3}, 0.62_{0.6,0.3}, 0.62_{0.6,0.3}, 0.61_{0.6,0.3}, 0.62_{0.6,0.3}, 0.62_{0.6,0.3})$$
$$= 0.62_{0.6,0.3}$$

Thus the decision matrix is,

$$R^2 = \begin{matrix} F1 \\ F2 \\ F3 \\ F4 \\ F5 \\ F6 \end{matrix} \overset{R^2_{ik}}{\begin{bmatrix} 0.87_{0.8,0.11} \\ 0.85_{0.8,0.13} \\ 0.54_{0.46,0.4} \\ 0.79_{0.7,0.2} \\ 0.83_{0.73,0.1} \\ 0.62_{0.6,0.3} \end{bmatrix}}$$

Decision Matrix is

$$R^2 = \frac{0.87_{0.8,0.11}}{F1} + \frac{0.85_{0.8,0.13}}{F2} + \frac{0.54_{0.46,0.4}}{F3} + \frac{0.79_{0.7,0.2}}{F4} + \frac{0.83_{0.73,0.1}}{F5} + \frac{0.62_{0.6,0.3}}{F6}$$

4. Solution interpretation,

Fuzzy decision,

$$\begin{matrix} \\ F1 \\ F2 \\ F3 \\ F4 \\ F5 \\ F6 \end{matrix} \begin{matrix} \text{Sem I } (R^2_{ik}) & \text{Sem II } (R^2_{ik}) \\ 0.75_{0.67,0.22} & 0.87_{0.8,0.11} \\ 0.46_{0.38,0.52} & 0.85_{0.8,0.13} \\ 0.27_{0.23,0.72} & 0.54_{0.46,0.4} \\ 0.69_{0.6,0.3,} & 0.79_{0.7,0.2} \\ 0.7_{0.6,0.2} & 0.83_{0.73,0.1} \\ 0.42_{0.4,0.54} & 0.62_{0.6,0.3} \end{matrix}$$

Defuzzification

The height method is used for defuzzifying the fuzzy decision. This method uses the maximum membership grade as rank one. Therefore, the faculty F1 showed best performance as compared to rest faculty. The comparison made for semester I and II revealed that there is improvement in performance of the faculty.

5. Control of the model

Actual reality and the solution obtained by proposed Mathematical model are identical, so it is acceptable.

IV CONCLUSION AND FUTURE SCOPE

In the present research work, authors have presented mathematical model for evaluation of faculty teaching performance using SOFRE. The Second Order Fuzzy Relational Equations are used to analyses the feedback system and its impact on course result. Existing feedback system in the engineering institute is considered and its effect on facultys' performance involving uncertain parameters is studied. By correctly processing data from the student feedback system about the faculty performance of specified course and using the data to the fuzzy system, author succeed to realise the excellent result. The results provide the comment for further improvement in the performance of the faculty.

The proposed model can be used for modelling uncertainty in other application fields. As membership functions are hand optimised there is still room for improvement by using intelligent means of optimisation such as genetic algorithm or neural network. Still, the proposed model proved to be accurate as well as easy-to-execute.

APPENDICES

Appendix 1.

Questionnaire:

1. Are the course contents made clear to you?
2. Do you feel that the complex topics were made simple during teaching?
3. Do you get opportunities for raising doubts within and outside classroom?
4. Is enough confidence and interest getting created in the course?
5. Are all course components and evaluations challenging?
6. Are you given enough opportunities for learning by doing?
7. Are you able to connect the content learnt (theory) to outside practices (application)?

Appendix 2.

Result of the faculty members per semester

Table XVI *Result of faculty*

Faculty	Semester I	Semester II
F1	79.41	86.76
F2	66.67	79.31
F3	33.96	60.47
F4	69.49	79.66
F5	70	83.33
F6	41.82	61.82

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