



# Labeling of Some Graphs Using Python Programming

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## Abstract

In this paper we investigate a Labeling on Grotzsch graph and Peterson Graph using programming language. We have derived more than thousands different Graceful and Odd Even Graceful labeling pattern using Python programming for the Grotzsch graph and Peterson Graph.

**Keywords:** Graceful Labeling, Odd Even Graceful Labeling, Grotzsch Graph, Peterson Graph, Python programming

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## 1. Introduction

Most graph labelings trace their origins to labelings presented by Alexander Rosa [8] in his paper in the year 1967. Rosa identified three types of labeling, which he called as the  $\alpha$ ,  $\beta$  and  $\rho$  labelings, out of which the  $\beta$  - labeling later came to be known as the "Graceful labeling". R. B. Gananjothi [11] defined the Odd Even Graceful labeling in 1991 motivated from the original idea of the Graceful labeling.

Graph Labeling has been done manually since last 56 years by researchers. As per the vastness of the Graph structure and the various Graph Labeling techniques, it is required to find some way such that people can easily identify and apply the Graph Labeling techniques as per their requirement. To address and solve this problem, an unconventional, out of the box attempt has been made in this research to simplify complex and tedious labelings of some graphs by taking help of a programming language.

## 2. Preliminaries

This topic aims at providing some basic definitions of this research work.

### Definition 2.1

Graceful labeling: Consider a graph  $G = (V, E)$  with  $|V| = m$  and  $|E| = n$ . If there exists an injective labeling function  $f: V(G) \rightarrow \{0, 1, 2, \dots,$

$n\}$  and an induced bijective function  $f^*: E(G) \rightarrow \{1, 2, 3, \dots, n\}$  defined as  $f^*(uv) = |f(u) - f(v)|$ , then such a labeling function  $f$  is called a Graceful labeling of graph  $G$ . A graph which admits a Graceful labeling is called a Graceful graph.

### Definition 2.2

Odd Even Graceful labeling: Consider a graph  $G = (V, E)$  with  $|V| = m$  and  $|E| = n$ . If there exists an injective labeling function  $f: V(G) \rightarrow \{1, 3, 5, \dots, 2n + 1\}$  and an induced bijective function  $f^*: E(G) \rightarrow \{2, 4, 6, \dots, 2n\}$  defined as  $f^*(uv) = |f(u) - f(v)|$ , then such a labeling function  $f$  is called an Odd Even Graceful labeling of graph  $G$ . A graph which admits an Odd Even Graceful labeling is called an Odd Even Graceful graph.

### Definition 2.3

Grotzsch graph: A Grotzsch graph, denoted by  $G$ , is a triangle free graph with 11 vertices and 20 edges, satisfying the following conditions. It is a graph that contains a Star graph  $K$ , in which, each pendant vertex of  $K$ , is connected with two rim vertices of the cycle  $C$ . The vertex set and the edge set of a  $G$  can respectively be given by  $V(G_z) = \{w, v_1, \dots, v_5, u_1, \dots, u_5\}$  and  $E(G_z) = \{wv_i; 1 \leq i \leq 5\} \cup \{u_5u_1, uu_i; 1 \leq i \leq 4\} \cup \{v_i u_{i+1}, u_i v_{i+1}, v_j u_{j+4}, u_i v_{(j+4)}; 1 \leq i \leq 4, j = 1\}$

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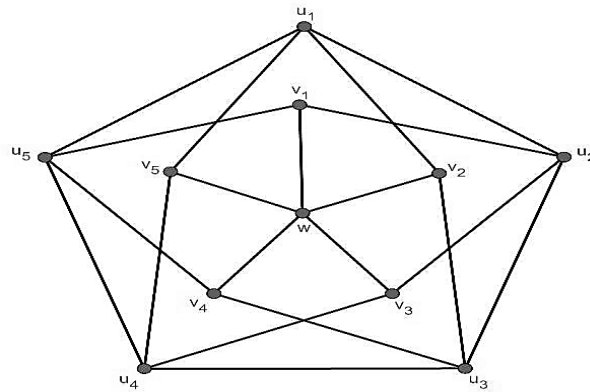
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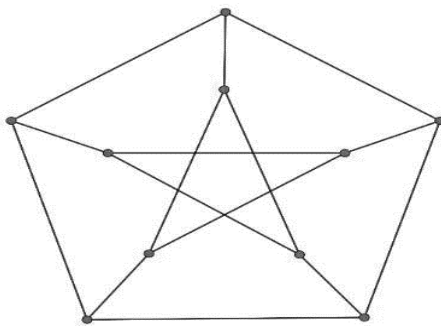


Demonstrated below, is the figure of the Grotzsch graph.



**Definition 2.4**

**Petersen Graph:** The Petersen graph is the complement of the line graph of  $K_5$  and it contains 10 vertices and 15 edges. Demonstrated below, is the figure of the Petersen graph.



**3. Main Results**

More than 1000 labeling patterns for Graceful and Odd Even Graceful labelings of the Grotzsch graph and the Petersen graph were

observed upon forming a coding by usage of Python programming language, some of which have been mentioned along with the following results.

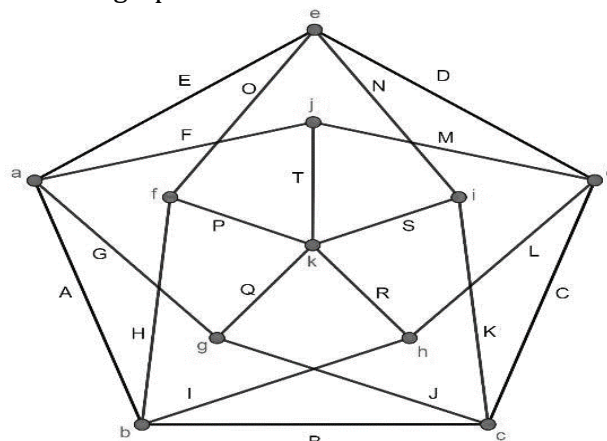
**Theorem 3.1:** The Grotzsch graph admits a Graceful labeling.

Consider the Grotzsch graph  $G$ . Now to generate its Graceful as well as Odd Even Graceful labelings using a programming language we take into account, the following nomenclature for its vertices and edges.

$$V = \{a, b, c, d, e, f, g, h, i, j, k\}$$

$$E = \{A, B, C, D, E, F, G, H, I, J, K, L, M, N, O, P, Q, R, S, T\}$$

The arrangement of these vertices and edges in  $G$  has been demonstrated in the below figure.



The Grotzsch graph  $G$

In order to define a Graceful labeling function, say  $f$ , of  $G$ ; we consider its domain as  $V(G_z)$  and its co-domain as  $\{0, 1, 2, \dots, 20\}$  ( $\because |E| = 20$ ) i.e.,  $f: V(G_z) \rightarrow \{0, 1, 2, \dots, 20\}$

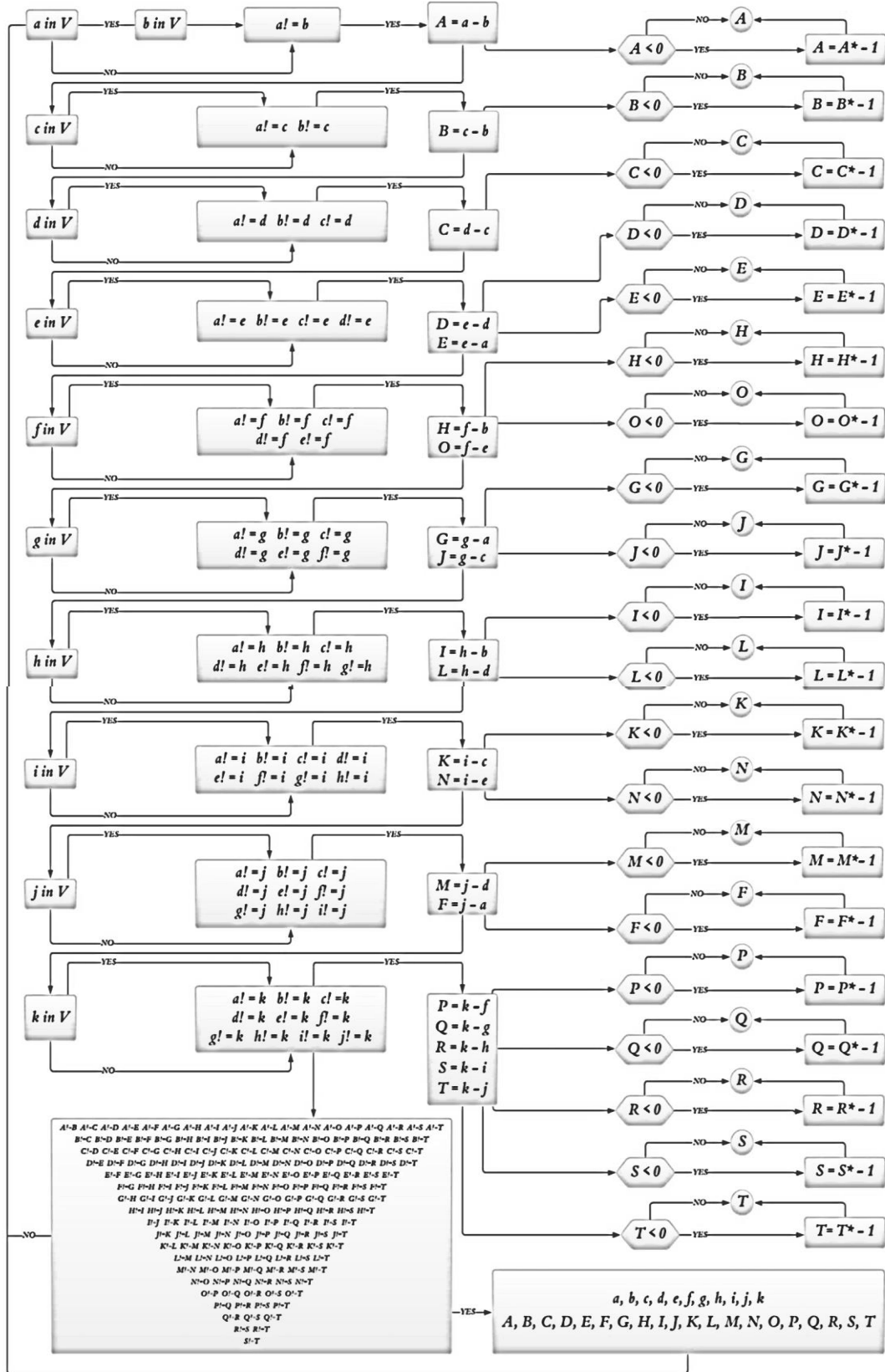
In order to define a Odd Even Graceful labeling function, say  $g$ , of  $G$ ; we consider its domain as  $V(G_z)$  and its co-domain as  $\{1, 3, 5, \dots, 41\}$  ( $\because |E| = 20$ )



i.e.,  $g : V(G_z) \rightarrow \{0, 1, 2, \dots, 20\}$

labelings of the Grotzsch graph, is presented below.

□ The flowchart of coding for extracting patterns of Graceful and Odd Even Graceful



□ A few out of several obtained patterns have been listed in the below tables.



Vertex	Graceful Labeling patterns									
a	0	0	0	0	0	0	0	0	0	0
b	1	2	1	1	1	1	1	1	1	1
c	4	14	3	3	5	7	7	13	14	14
d	2	9	7	10	2	2	9	2	3	5
e	20	20	19	19	20	19	19	20	20	7
f	12	16	16	5	10	12	5	7	17	12
g	10	8	8	15	16	20	15	4	19	20
h	16	18	18	18	9	16	18	17	15	17
i	8	12	12	8	18	10	14	6	16	10
j	19	19	20	20	19	18	20	19	18	19
k	3	1	2	2	4	8	2	9	9	2

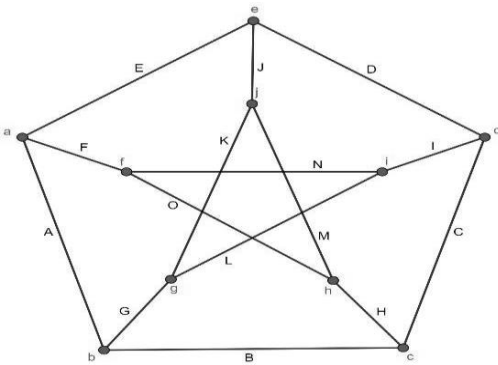
Vertex	Odd Even Graceful Labeling patterns									
a	1	1	1	1	1	1	1	1	1	1
b	7	7	7	7	7	9	9	9	9	9
c	29	29	39	39	39	21	21	3	3	21
d	3	21	21	15	23	7	11	5	5	17
e	35	19	13	41	13	11	41	41	41	11
f	31	31	33	25	33	37	7	37	37	37
g	41	39	41	11	37	41	39	33	33	41
h	21	37	31	19	31	39	31	23	25	39
i	27	33	17	3	17	5	35	25	29	35
j	39	41	37	35	41	25	27	39	39	19
k	11	5	3	33	3	3	3	13	15	3

**Theorem 3.2:** The Petersen graph admits a Graceful labeling.  
 Consider the Petersen graph  $G$ . Now to generate its Graceful as well as Odd Even Graceful labelings using a programming

language we take into account, the following nomenclature for its vertices and edges.  
 $V = \{a, b, c, d, e, f, g, h, i, j\}$   
 $E = \{A, B, C, D, E, F, G, H, I, J, K, L, M, N, O, P, Q, R, S, T\}$



The arrangement of these vertices and edges in  $G$  has been demonstrated in the below figure

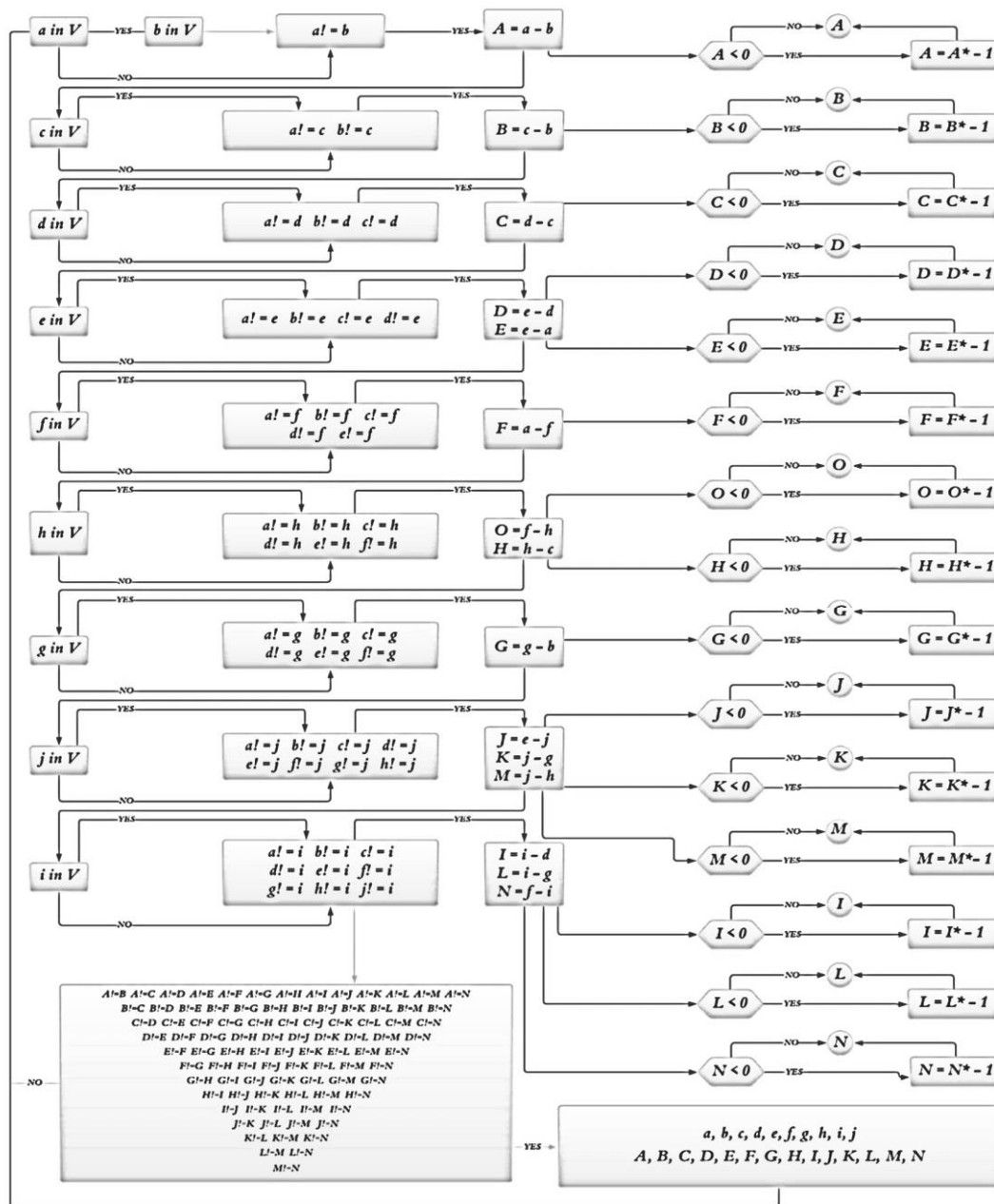


The Petersen graph

In order to define a Graceful labeling function, say  $f$ , of  $G$ ; we consider its domain as  $V(G)$  and its co-domain as  $\{0, 1, 2, \dots, 15\}$  ( $\because |E| = 15$ )  
 i.e.,  $f: V(G) \rightarrow \{0, 1, 2, \dots, 15\}$

In order to define an Odd Even Graceful labeling function, say  $g$ , of  $G$ ; we consider its domain as  $V(G)$  and its co-domain as  $\{1, 3, 5, \dots, 31\}$  ( $\because |E| = 15$ )  
 i.e.,  $g: V(G) \rightarrow \{0, 1, 2, \dots, 31\}$

❑ The Flowchart of coding for extracting patterns of Graceful and Odd Even Graceful labeling of the Petersen graph is presented below.



❑ A few out of several obtained patterns have been listed in the below tables.



Vertex	Graceful Labeling pattern									
a	0	0	0	0	0	0	0	0	0	0
b	11	11	12	12	13	13	14	14	15	15
c	5	5	3	3	2	2	11	11	2	2
d	2	10	5	10	3	7	2	4	11	4
e	15	14	15	14	15	14	15	12	14	14
f	14	15	14	15	14	15	12	15	12	11
g	12	12	13	13	4	4	3	3	5	3
h	10	2	10	5	12	12	4	2	8	5
i	4	3	8	2	10	3	8	5	4	12
j	3	4	2	8	7	6	5	8	3	10

Vertex	Odd Even Graceful Labeling patterns									
a	1	1	1	1	1	1	1	1	1	1
b	3	3	3	3	5	5	5	5	7	7
c	7	7	7	7	17	19	19	21	29	27
d	13	15	19	21	27	13	29	3	3	17
e	29	31	29	31	29	31	9	29	31	25
f	31	29	31	29	31	9	31	31	25	31
g	27	27	27	27	21	27	27	17	15	19
h	21	19	15	13	25	29	13	7	11	5
i	5	9	5	9	7	25	3	25	5	3
j	9	5	9	5	3	3	25	27	27	23

**4. Concluding remarks**

In this paper we have created a Python program to derive Graceful and Odd Even Graceful labeling pattern for Grotzsch graph and Peterson graph.

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