Quantum Collision broadening with Screening Effects in Plasma

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Abstract:
Stark effect is one of the mechanisms of broadening the spectral line, it occurs when an atom, a molecule, or an ion, which emits light in a gas, is perturbed by its interactions with the other constituents of the plasmonic gas, such as other atoms, molecules, ions, or electrons. Among the most used theories in this field is the so-called classical path approximation which has presented these effects and has attracted much attention in recent years. It is based on the classical consideration, where the dynamics of the perturbers is governed by the laws of classical mechanics. Thus, the classical path approximation assumes that each perturber can be located and followed on a classical trajectory. In this work, in the case of a quantum mechanics, the interaction ion-electron is described by a quantum scattering cross section developed with a new effective potential. This potential of ion-electron interaction, taking into consideration both quantum-mechanical effects of diffraction, and also screening field effects, is presented. The amplitude of the electronic collision operator is calculated for isolated ion lines in plasma and compared with the amplitude which has been derived by Griem (classical Coulomb interaction).

Keywords: collision operator; hydrogen ions; electron scattering cross-sections.
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1- Introduction:

The profile reflects the physical properties of the plasma: it could be Doppler or "Stark effect [1,2,3,4,5,6,7,8]. Stark line broadening has become an important means of measuring temperatures across the full range of densities of plasma gas-forming species. Baranger, taking up the article by Anderson developed the basic quantum formalism of isolated spectral lines taking into account inelastic collisions [9,10,11]. Baranger's work showed that electronic broadening (electron-atom or emitting ion) is expressed in the form of a sum of collision cross sections. Griem et al advanced the theory applied to isolated degenerate spectral lines where the interaction cross sections are purely classical [12,13,14,15,16]. In this work, a quantum scattering cross section is developed, taking into consideration both quantum-mechanical effects of diffraction and screening field effects, with a new effective potential. The amplitude of the electronic collision operator is calculated for isolated ion lines in plasma. Comparison with the results obtained on the basis of Griem's model (classical Coulomb interaction) and conclusions were made.

Next, we addressed the electron's motion around the emitting ion by putting ourselves in a quantum mechanical setting in which the idea of a trajectory is meaningless. Furthermore, we have taken into account the possibility that the colliding electron obtains relativistic, or excessive, velocity. The comparison with the non-relativistic method is carried out with an appropriate range of electron concentrations and temperatures. Strong agreement is generally attained at high temperatures and densities. Because electrons travel quickly and have very low masses, their collision delay is too short compared to the time span used to compute the correlation function [3,9]. As a result, the impact approximation handles them.

2- Electronic collision operator for Deutch Debye potentiale

The effective potential is suggested for semiclassical plasma. This potential contains the quantum and screening effects. Therefore, the Deutsh-Debye interaction between an emitting ion and a perturber electron has the following formula:

$$U_{DD}(r) = \frac{\alpha}{r}\left(1-\exp(-\frac{r}{\lambda_t})\right)\exp(-\frac{r}{\lambda_D})$$

Where

$$\alpha = K_e(Z-1)e^2, \lambda_D = 6.9\sqrt{\frac{T}{N_e}}$$

λ_D is the Debye length.

The de Broglie wavelength can determine by:

$$\lambda_{ie} = \frac{\hbar}{\sqrt{2\pi m_{ie}K_BT}}$$

the electronic collision operator calculated according to the coulomb interaction is given by Griem:

$$\Phi = -\frac{4\pi}{3}\frac{N_e}{\hbar^2}\left(\frac{m}{z-1}\right)^2\iint \upsilon^3 f(\upsilon)d\upsilon\rho d\rho\sin(\frac{\theta}{2})^2$$

The classical differential cross section is defined as:

$$\sigma(\theta) = 2\pi\rho d\rho$$

In the other hand, it is related to the scattering amplitude F(θ), in the Born approximation, by

$$\sigma(\theta) = |F(\theta)|^2 d\omega$$

Where dω=2πsinθdθ is the element of solid angle, then we can write the electronic collision operator:

$$\Phi = -\frac{4\pi}{3}\frac{N_e}{\hbar^2}\left(\frac{m}{z-1}\right)^2\iint \upsilon^3 f(\upsilon)d\upsilon|F(\theta)|^2\sin\theta\sin(\frac{\theta}{2})^2 d\theta$$

f(υ) is velocities distribution of Maxwel equilibrium given by:

$$f(v) = 4\pi(\frac{m}{2\pi k_B T})^{\frac{3}{2}} v^2 \exp(-\frac{mv^2}{2k_B T})$$

The diffusion capacity can be defined as an integral over the distance r between the emitting ion and the perturbing electron.

$$F(\vec{q}) = -\frac{m}{2\pi\hbar^2}\int u(r)\exp(-i\vec{q}\vec{r})dr$$

Such that

$$q = 2k\sin(\frac{\theta}{2}),\ k = \frac{m\upsilon}{\hbar^2}$$

with

$$d\vec{r} = r^2\sin\theta d\theta dr d\varphi$$

The amplitude of the collision operator, calculated according the Deutch-Debye potential, is written as:

$$F(\vec{q}) = -\frac{2m\alpha}{q\hbar^2}\int(\exp(-\frac{r}{\lambda_D}) - \exp(-r\frac{\lambda_t+\lambda_D}{\lambda_t\lambda_D}))\sin(qr)dr$$

Considering the notation: 228

$$L = \frac{\lambda_t\lambda_D}{\lambda_t+\lambda_D}, S = \lambda_D$$

After simplification, we discover that the diffusion capacity formula becomes: when we integrate over r.

$$F(\vec{q}) = -\frac{2m\alpha}{\hbar^2}\left[\frac{S^2}{1+q^2S^2} - \frac{L^2}{1+q^2L^2}\right]$$

We were able to derive quantum collision operator by ignoring the emitter's fine structure since it would be too difficult to determine the expression of the electronic collision operator. The operator matrix elements [5,13] can be used to express the electronic collision breadth of the isolated spectral line inside the impact approximation.

$$\emptyset = -\frac{4\pi}{3}\frac{N_e}{\hbar^2}(\frac{m}{(z-1)})^2\int\int v^3 f(v)dv\rho d\rho\, sin^2\frac{\theta}{2}$$

By replacing F(q) in amplitude expresion Φ,we find:

$$\Phi = -\frac{\pi}{3}\frac{N_e}{\hbar^2}\left(\frac{m}{z-1}\right)^2\left(\frac{2m\alpha}{\hbar^2}\right)^2\int_0^\infty\frac{v^3}{k^4}f(v)dv\int_{q\min}^{q\max}q^3\left[\frac{S^4}{(1+q^2S^2)^2} + \frac{L^4}{(1+q^2L^2)^2} - \frac{2S^2L^2}{(1+q^2S^2)(1+q^2L^2)}\right]dq$$

q_{min}, q_{max} are the maximum and minimum impact parameter, The integration over q allows us to get a final result of amplitude of the electronic collision operator:

Following integration, the electronic collision operator takes on its final form, which is expressed as follows:

$$\Phi = -\frac{4\pi^2}{6}\frac{N_e}{\hbar^2}\left(\frac{m}{z-1}\right)^2\left(\frac{2m\alpha}{\hbar^2}\right)^2(\frac{m}{2\pi k_B T})^{\frac{3}{2}}\int_0^\infty\frac{\hbar^4}{m^4}\upsilon\exp(-\frac{mv^2}{2k_BT})d\upsilon \times\left[\frac{S^4}{M} - \frac{S^4}{H} + \frac{L^4}{N} - \frac{L^4}{Y} + \ln(\frac{MN}{HY}) + \frac{2}{S^2-L^2}\ln(\frac{M^{L^2}Y^{S^2}}{H^{L^2}N^{S^2}})\right]$$

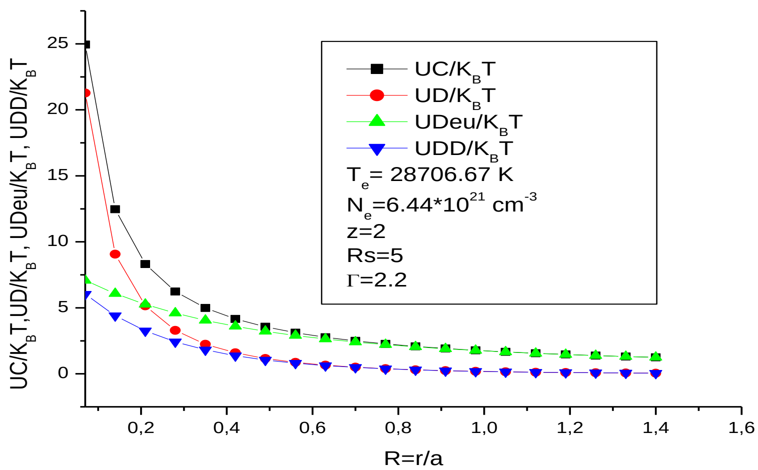
where:

$$N = 1 + q_{\max}^2L^2, Y = 1 + q_{\min}^2L^2, M = 1 + q_{\max}^2S^2, H = 1 + q_{\min}^2S^2$$

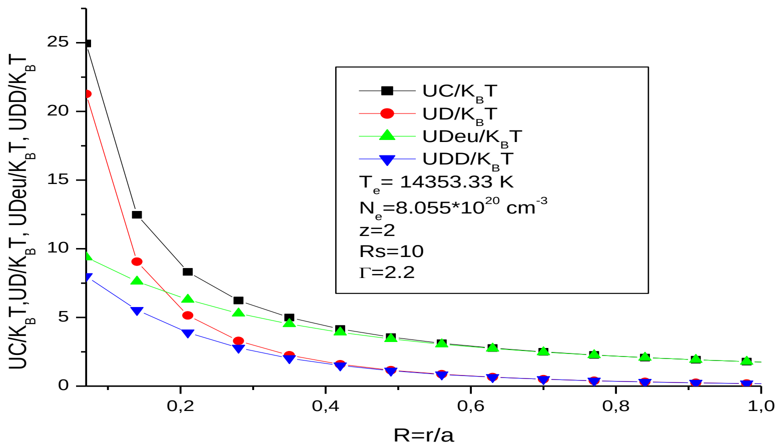
3- Results and discussion

Figure (1), (2), and (3) present the comparison between the Coulomb, Debye, Deutsch, and Deutsch-Debye potentials. It is shown that our potential approaches to the Deutsch at short distance r, and coincides with Debye potential at large distance. Both potentials decrease with growth of the distance r. On the other hand, it is clear that the differences between these potentials increase with the increasing of the coupling parameter Γ at fixed density parameter r_s.

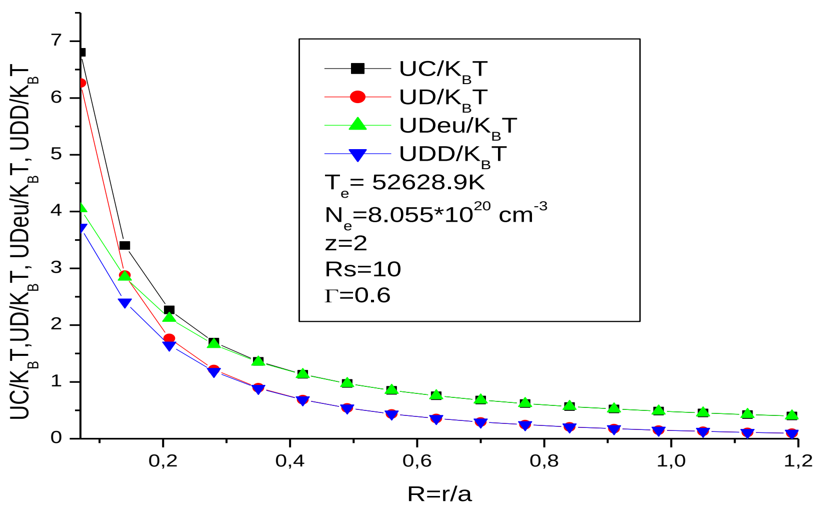
It is well shown that the amplitude of the electronic collision operator, calculated with the Debye-Deutsch potential, has a same behavior with Griem's amplitude (calculated with Coulomb potential) as a function of T (figure 4). Both amplitudes decrease with increasing of the electronic temperature T (figure 4). Under the same condition the discrepancy between our amplitude and Griem's result grows with increasing of the electronic temperature T (figure 5). The figure 6 shows that both amplitudes grow with increasing of the electronic density. We remark that the amplitude calculated according the Deutsch –Debye potential is less than Griem's amplitude (established according the Coulomb interaction) for all values of T and Ne.



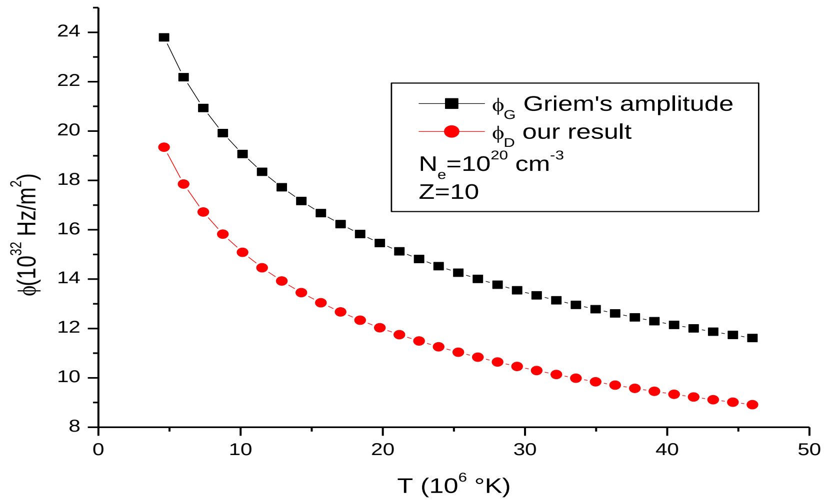
Figures (1): Potential variations of the particle interactions, in a semi-classical plasma, us a function of a distance r between particles: Coulomb potential (UC), Debye-HücKel potential (UD), Deutsch potential (UDeu), Debye-Deutsch potential (UDD).



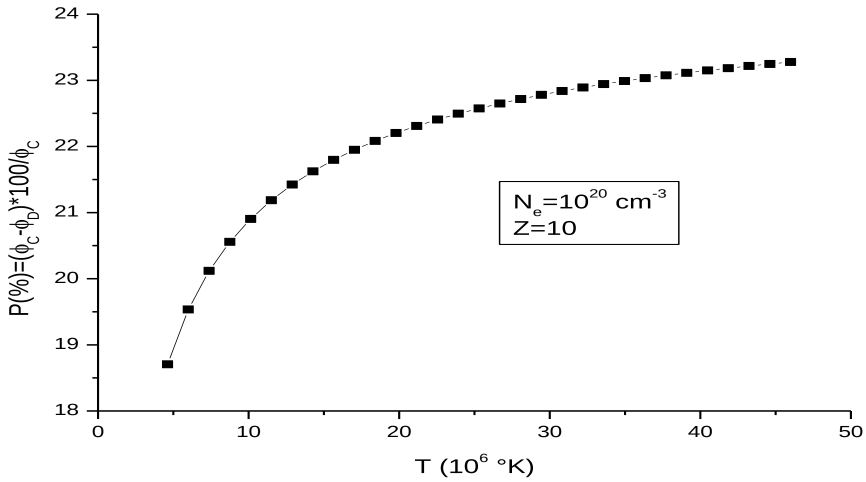
Figures (2): Potential variations of the particle interactions, in a semi-classical plasma, us a function of a distance r between particles: Coulomb potential (UC), Debye-HücKel potential (UD), Deutsch potential (UDeu), Debye-Deutsch potential (UDD).



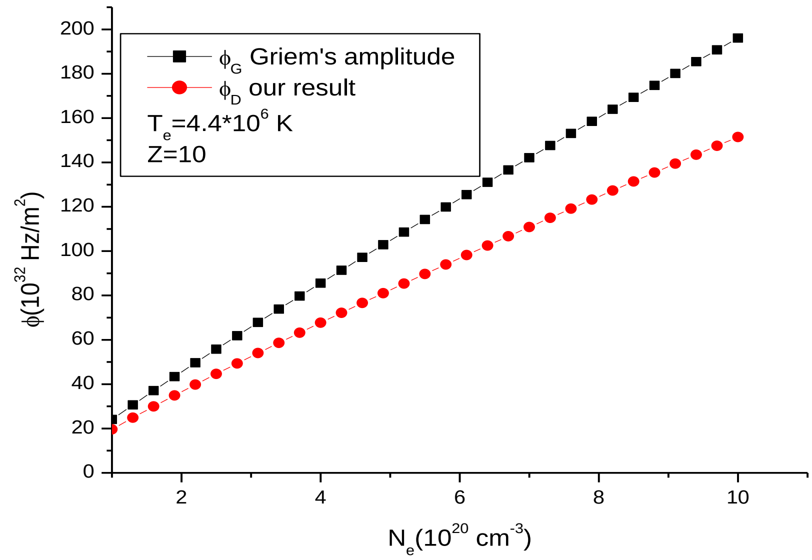
Figures (3): Potential variations of the particle interactions, in a semi-classical plasma, us a function of a distance r between particles: Coulomb potential (UC), Debye-HücKel potential (UD), Deutsch potential (UDeu), Debye-Deutsch potential (UDD).



Figures (4): Behaviors of the electronic collision operator, in asemi-classical plasma, us a function of the electronic temperature T.



Figures (5): Behaviors of the percentage P(%) us a function of the electronic temperature T.



Figures (6): Behaviors of the electronic collision operator, in a semi-classical plasma, us a function of the electronic density N_e.

4-Conclusion

The radiation emitted by the plasma makes it possible to connect, on the one hand, the properties of an isolated emitter with the properties of the plasma which surrounds it. The theoretical study of the electronic collision operator, in the impact approximation, has been the subject of much research. The interactions at short distances between particles are treated within the framework of quantum mechanics, which leads to taking into account the quantum effect on the potential and the cross section of collisions. In addition, the screening effect is taking in the consideration to develop a new electronic collision operator in plasma, The Deutsch-Debye potential presented in this work is more accurate to describing the reel interactions in the plasma. The amplitude of electronic operator derived in this case has a simple and inclusive formula.

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