Multi-Objective Three Bar Truss Design Optimization By Four Valued Refined Neutrosophic Optimization Technique

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Abstract—
Structural design optimization such as three bar truss design optimization by mathematical method is an important notion in civil engineering as well as in mathematics. Traditionally structural optimization is well known concept and in many situations it is treated as single objective form ,where the objective is known as cost function. The extension of this form can be defined as optimization where one or more constraints are simultaneously satisfied next to the minimization of cost function. This does not always hold good in real world problem where multiple and conflicting objects frequently exist. In this context, a methodology known as multi-objective structural optimization is introduced. Again Neutrosophic Theory is the existing mathematical theory that generalizes the notion of Fuzzy Set (FS) theory. It also analyses the connectivity between impreciseness and neutralities. In this present study, an advanced neutrosophic optimization technique namely Four Valued Refined Neutrosophic Optimization(FVRNSO) method is discussed to optimize weight and deflection of the load joint that all satisfy all stress constraints in member of the structure.The design variables are cross section in members.From the set theoretic point of view indeterminacy is further refined as Uncertain(U) and contradiction $C = T \wedge F$ in four valued refined neutrosophic set. Particularly based on truth, uncertain, contradiction and falsity membership functions this multi-objective FVRNSO algorithm has been discussed in this paper.This method and this application show that the method is very much cost effective as compare to prevailing non deterministic method.

***Keywords*:**Neutrosophic Set,Single Valued Neutrosophic Set,Four Valued Refined Neutrosophic Set, Multi-objective Four Valued Refined Neutrosophic Optimization, Multi-objective Three Bar Truss Design Optimization.

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1. Introduction:

While there are undoubtedly some instances that call for extreme precision, not many human problems perform. Even the majority of physical processes are simply understood by relying on erroneous human thinking. Most of the time, this imprecision contains really helpful information. In the scientific world, uncertainty was viewed as an undesirable state that had to be avoided at all costs up to the early 19th century. But physicists gradually came to realize that the issues at the molecular level were not addressed by Newtonian Mechanics and its underlying calculus.At that time, scientists began searching for noble statistical mechanics-based techniques so that statistical averages may take the place of specific examples of microscopic organisms. These true numbers, which described the actions of several microscopic elements, could then be linked to appropriate microscopic parameters in a model. After statistical mechanics development, uncertainties were taken into consideration. The main foundation of statistical mechanics is probability theory, which can handle a variety of uncertainties.These have assisted in reducing the amount of uncertainty and achieving dependable solutions.It is truth that probability theory led the way in characterizing uncertainty in scientific models from the late 19th to the late 20th centuries. Probability theory was originally contested by Black, who also added ambiguity. Dempster introduced the idea of lack of information into his well-known theory of evidence for the first time in the 1960s. In 1965, Zadeh presented his seminal notion in logic, which he called fuzzyset theory. In addition to having an impact on the notion of uncertainty, Zadeh's work cast doubt on the idea that probability theory is the sole way to describe uncertainty. Zadeh questioned probability theory's binary logic and also provided an illustration of possibility theory, a particular case of fuzzy sets. The paradigm changed in the 20th century to address a wider range of uncertainty.In the 1970s, Glenn Shafer expanded on Dempster's research and created a comprehensive theory of evidence. His idea takes into account data from multiple sources. Many researchers also combined the ideas of evidence theory, possibility theory, and probability theory with fuzzy measurements later in the 1980s. In terms of a philosophical theory of knowledge, uncertainty might be thought of as the opposite of information. A specific scientific or engineering topic may have information or data that is insufficient, ambiguous, partial, dubious, contradicting, or deficient in some other way. By learning more and more about the issue at hand, we may get past any uncertainty that may arise.As a result, we are less unsure of its formulation and resolution. Uncertainty is a major factor in poorly worded or complicated problems. We are surrounded by a great deal of uncertainty. The most prevalent ones are possible, unclear, hazy, and vague.Additionally, after doing some experiments, we turn a lot of physical problems into mathematical models.Additionally, there are always uncertainties associated with experimentation, and we require fuzzy notions to deal with them. Fuzzy numbers, which Zadeh (1965) proposed, are a good representation of this kind of inaccurate data.Typically, a fuzzy set is made up of objects with various membership functions or grades. Fuzzy sets were included in the set theoretic concepts of relation, union, intersection, complement, concavity, and convexity, among others. In addition to having several properties, fuzzy set context is used to construct algorithms. Fuzzy logic (FL) was established to describe uncertainty in many applications with the invention of fuzzy sets (Singh et al., 2013).To incorporate the degree of uncertainty in the membership By Attanassov (1997), the Intuitionistic Fuzzy Set (IFS) was first presented.The Neutrosophic Set (NS) theory was established by Smarandache (1999) to address problems in the real world that involve indefinite knowledge and imprecise, ambiguous, and vague parameters. Considering the discourse universe as a linear space Single Valued Neutrosophic Set (SVNS) was introduced by Wang et al. (2010) as a particular instance of NS.It stands for inaccurate, ambiguous, indeterminate, and inconsistent information.More effectively than SVNS, the Double Refined and Triple Refined Neutrosophic sets are employed to handle inconsistent and missing data (Kandasamy 2016 and Smarandache 2018, Zadeh 2018).Refined Neutrosophic Set with four values was used in the Sajida et al. (2020) article.However, all of these optimization method parameters are fixed or presumed to be exact, making them unsuitable for problems encountered in everyday life.The linear programming problem was initially introduced by Zimmermann in 1978, and this work has been viewed as an extension .In their studies and applications, numerous researchers have made use of the concepts of fuzzy goals, fuzzy constraints, and fuzzy decisions.In this context, it is important to keep in mind the numerous researchers who have examined optimization techniques in imprecise environments for both linear and nonlinear programming problems. These researchers include Tanaka and Asai (1984), Chanas (1983), Verdigay (1984), Carlson and Korhonen (1986), Campos (1989), Lavandula (1989), Sakawa and Yano (1989), Carls and Dev (2013), Guuand Wu (2019), Zhou et al. (2020), Ghodousian (2019), Sahinidis (2004), Chakraborty et al. (2013), Wan et.al. (2017), Bharati (2018a, b), among others. Similar to this, several researchers have looked into the optimization process to optimize various nonlinear structural design problem criteria over time. Examples of these criteria include a continuous time nonlinear stochastic system with actuator fault and a robust and dependable static output feedback control for nonlinear systems. Furthermore, deterministic optimization for the design of single-objective welded beams has been a well-known civil engineering technique for decades.Among them it is the mathematical conventional optimization technique developed by David-Fletcher-Powel in 1976.The sequential linear approximation (APPROX) developed by Griffith and Stewart in 1976,penalty function and Simplex Method (SIMPLEX) (1976),the harmony search method (2005), the GA-based method (2000), the improved harmony search algorithm by Madhabi et al. (2007), and the Simple Constrained Particle Swarm Optimization (SIC-PSO) (2008) are some of the methods that have been proposed by Recherdson.Constrained optimization using the PSO algorithm (COPSO) (2007), GA based on a coevolution model (GA1) (2000), and GA using Dominance-based tornament selection (GA2) (2002) are some of the works that have been published .Coevolutionary particle swarm optimization (CPSO) (2007) and evolutionary programming with a cultural algorithm (EP) (2004),a feasibility-based rule for hybrid particle swarm optimization (HPSO) was developed in 2007. Particle swarm optimization and hybrid Nelder Mead Simplex Search technique (NM-PSO) (2009),Optimization of Particle Swarms (PSO) (2016), Evaluate Goldlike (GL) (2016) and Anneling (SA) (2016) were developed. In 2016 it was seen the release of Cuckoo Search (Cuckoo), Firefly Algorithm (FF), Flower Polination (FP), and Ant Lion Optimizer (ALO),Multiverse optimizer (MVO) (2016) and gravitational search algorithm (GSA) (2016).However, there is a lot of uncertainty in engineering design challenges, such as the design of welded beams and various structural design issues.There were very few researchers who dealt with these kinds of issues.In this regard, Das et al. (2015)'s discussion of neutrosophic nonlinear programming with numerical examples can be given.In a similar view, Singh B et al. (2016) and Sarkar et al. (2016) have examined single- and multi-objective truss design optimization in intuitionistic fuzzy environments. In the papers of Sarkar et al. (2017), various optimization techniques, including neutrosophic optimization technique, were examined from the standpoint of structural design and welded beam design.

The aim of this paper is to show the efficiency of the four valued refined neutrosophic optimization technique in finding the optimum weight of three bar truss in imprecise environment and to make a comparison of result obtained from different deterministic method. This paper is organised as follows.In section 2 I have stated mathematical prelimineries of four valued refined neutrosophic set .In section 3 I have discussed multi-objective nonlinear optimization algorithm based on (SVRNS). In section 4 we have studied the detailed description of optimum design of three bar truss and solution using SVRNO technique.In section 5 I have the compared the result obtained with existing method like fuzzy, intuitionistic fuzzy, neutrosophic optimization method lastly we make a conclusion in the section 6.

1. Preliminaries

1.1 Fuzzy Set(2019): A fuzzy set (Zadeh 1965) is a set that can contains elements partially that is the property that an elements belong to the set under consideration be a truth with a partial degree of truth. Given a universe set X , a fuzzy set $\tilde{A}^F$ is an ordered set (Universe element ,truth degree of membership of that element) denoted mathematically as

$$\tilde{A}^F = \left\{x, T_{\tilde{A}^F}(x) : x \in X\right\} \text{ where } T_{\tilde{A}^F}(x) \in [0,1]$$

1.2 Intuitionistic Fuzzy Set: Given a Universe X , an intuitionistic fuzzy set (Attanassov 1986) is a set of triplet $(x, T_{\tilde{A}^F}(x), F_{\tilde{A}^F}(x))$ where $T_{\tilde{A}^F}(x), F_{\tilde{A}^F}(x)$ represent the truth and falsity grade respectively and $0 \le T_{\tilde{A}^F}(x), F_{\tilde{A}^F}(x) \le 1, T_{\tilde{A}^F}(x), F_{\tilde{A}^F}(x) \in [0,1]$.Clearly one can obtain a fuzzy set when $T_{\tilde{A}^F}(x) + F_{\tilde{A}^F}(x) = 1$

1.3 Neutrosophic Set: Neutrosophic Set (Smarandache 1995) is a generalized concept in which each component $x \in X$ to a set $\tilde{A}^N$ has the membership degree $T_{\tilde{A}^F}(x)$ non-membership degree $F_{\tilde{A}^F}(x)$ as well as a degree of indeterminacy $I_{\tilde{A}^F}(x)$ where $T_{\tilde{A}^F}(x), I_{\tilde{A}^F}(x)$ and $F_{\tilde{A}^F}(x)$ are real slandered or nonstandard subsets of $\left]0^-, 1^+\right[$.

1.4 Single Valued Neutrosophic Set (SVNS): In single valued Neutrosophic Set (Smarandache 2010) each $x \in X$ to a set $\widetilde{A}^{SN}$ is characterized by $T_{\widetilde{A}^F}(x), I_{\widetilde{A}^F}(x), F_{\widetilde{A}^F}(x)$ belong to $[0,1]$ And $0 \le T_{\widetilde{A}^F}(x) + I_{\widetilde{A}^F}(x) + F_{\widetilde{A}^F}(x) \le 3$. Thus a single valued Neutrosophic set is expressed as $\widetilde{A}^{SN} = \{x, T_{\widetilde{A}^F}(x), I_{\widetilde{A}^F}(x), F_{\widetilde{A}^F}(x) : x \in X\}$.

1.5 Fuzzy Optimization: Based only on the formulation of fuzzy information in terms of membership functions, fuzzy optimization (G.Freen et al., 2019) is the process of solving the fuzziness model "optimally" using currently available optimization tools and techniques. Tang et al. proposed seven steps to implement fuzzy optimization.These procedures can be used to any type of problem, regardless of whether it has discrete or continuous choice variables, is single- or multi-objective, or is linear or non-linear. Recognizing the issue, as well as the framework's goals, limitations, and relationships between them.creation of fuzzy constraints, fuzzy objectives, and fuzzy coefficients based on the problem's fuzzy facts.The creation of a fuzzy optimization using fuzzy coefficient, fuzzy objective, fuzzy constraints, and mathematical procedures.

1.6 Four Valued Refined Neutrosophic Set(FVRNS)(G. Freen et.al. 2019): Refinement of any of T,I,F involves the extenics (Zadeh 2018).Four valued refined Neutrosophic Set can be defined in a number of ways by splitting indeterminacy in different manners. Here in the present work we only focus in the below mentioned criteria.A four valued refined Neutrosophic set is such a type of Neutrosophic set in which indeterminacy split into two parts as U=Uncertain and C=Contradiction where $C = T \wedge F$.The values of T,I,C and F belong to [0,1] and $0 \le T + U + C + F \le 4$ Thus FVRNS is represented as $\widetilde{A}^{RN} = \{(x, T_{\widetilde{A}^{RN}}(x), I_{\widetilde{A}^{RN}}(x), F_{\widetilde{A}^{RN}}(x)) : x \in X\}$ When X is continuous then $\widetilde{A}^{RN} = \int \{x, T_{\widetilde{A}^{RN}}(x), I_{\widetilde{A}^{RN}}(x), F_{\widetilde{A}^{RN}}(x) / dx : x \in X\}$ And when X is discrete its representation will be $\widetilde{A}^{RN} = \sum_{i=1}^{n} \{x, T_{\widetilde{A}^{RN}}(x), I_{\widetilde{A}^{RN}}(x), F_{\widetilde{A}^{RN}}(x) / x_i : x_i \in X\}$ The complement of four valued refined Neutrosophic Set is denoted by C_r $T_{C_r}(x) = F_{\widetilde{A}^{RN}}(x)$, $U_{C_r}(x) = 1 - U_{\widetilde{A}^{RN}}(x)$ $C_{C_r}(x) = 1 - C_{\widetilde{A}^{RN}}(x)$, $F_{C_r}(x) = T_{\widetilde{A}^{RN}}(x)$ For all $x \in X$ .The definition of FVRNS and the complement guarantees the following results. 584

2. Four Valued Refined Multi-objective Neutrosophic Optimization Technique

Consider a nonlinear multi-objective optimization problem

Minimize $[f_i(x)],\ i = 1,2,\ldots p$ (1)

Such that $[g_i(x)] \le b_j \ \ j = 1,2\ldots.q$ (2)

Where x are decision variables , $f_i(x)$ represents here objective functions, $g_i(x)$ represents constraint functions and p and q represent the number of objective functions and constraint respectively.Now the decision set σ a conjunction of four valued neutrosophic objectives and constraints is defined as

$$\widetilde{D} = \left(\bigcap_{p=1}^{k} \widetilde{O}_k\right) \cap \left(\bigcap_{p=1}^{k} \widetilde{L}_k\right) = \left(x, T_{\widetilde{D}}, U_{\widetilde{D}}, C_{\widetilde{D}}, F_{\widetilde{D}},\right) \quad (3)$$

Where

$$T_{\widetilde{D}}(x) = \min\left(T_{\widetilde{O}_1}(x), .T_{\widetilde{O}_2}(x), \ldots..T_{\widetilde{O}_p}(x); L_{\widetilde{L}_1}(x), T_{\widetilde{L}_2}(x) \ldots..T_{\widetilde{L}_q}(x)\right) \quad (4)$$

$$U_{\widetilde{D}}(x) = \min\left(U_{\widetilde{O}_1}(x), .U_{\widetilde{O}_2}(x), \ldots..U_{\widetilde{O}_p}(x); U_{\widetilde{L}_1}(x), U_{\widetilde{L}_2}(x) \ldots..U_{\widetilde{L}_q}(x)\right) \quad (5)$$

$$C_{\widetilde{D}}(x) = \min\left(C_{\widetilde{O}_1}(x), .C_{\widetilde{O}_2}(x), \ldots..C_{\widetilde{O}_p}(x); C_{\widetilde{L}_1}(x), C_{\widetilde{L}_2}(x) \ldots..C_{\widetilde{L}_q}(x)\right) \quad (6)$$

$$F_{\tilde{D}}(x) = \max\left(F_{\tilde{O}_1}(x), .F_{\tilde{O}_2}(x), ...F_{\tilde{O}_p}(x); F_{\tilde{L}_1}(x), F_{\tilde{L}_2}(x).....F_{\tilde{L}_q}(x) \right) \quad (7)$$

Where $T_{\tilde{D}}(x), U_{\tilde{D}}(x), C_{\tilde{D}}(x), F_{\tilde{D}}(x)$ represent truth,uncertainty,contradictory and falsity membership of four valued refined neutrosophic decision set respectively.Now using the four valued refined neutrosophic optimization the above problem is remodeled as non linear optimization as

$$Max\alpha, Max\gamma, Max\beta, Max\delta \quad (8)$$

such that $T_{\tilde{O}_k}(x) \geq \alpha, U_{\tilde{O}_k}(x) \geq \gamma, F_{\tilde{O}_k}(x) \geq \delta, C_{\tilde{O}_k}(x) \geq \beta$ (9)

$$T_{\tilde{L}_j(x)}(x) \geq \alpha, U_{\tilde{L}_j(x)}(x) \geq \gamma, F_{\tilde{L}_j(x)}(x) \geq \delta, C_{\tilde{L}_j(x)}(x) \geq \beta \quad (10)$$

$$\alpha \geq \beta, \alpha \geq \gamma, \alpha \geq \delta \quad (11)$$

$$\alpha + \beta + \gamma + \delta \leq 4 \quad (12)$$

$$\alpha, \beta, \gamma, \delta \in [0,1] \quad (13)$$

$$g_j(x) \leq b_j, x \geq 0, j = 1,2...q \quad (14)$$

$$x \geq 0 \quad (15)$$

Computational Algorithm:

Step-1: Solve the first objective function as single objective function taken from set of k objectives. The values of decision variables and objective function will be computed subject to the given constraints.

Step-2:Now compute the values of unresolved objectives i.e (k-1) using decision variables from step 1.

Step-3: Continue to the remaining (k-1) objective functions by going through step 1 and step 2

$$\begin{bmatrix} f_1^*(x^1) & f_2(x^1) & .... & f_p(x^1) \\ f_1(x^2) & f_2^*(x^2) & .... & f_p(x^2) \\ .... & .... & .... & .... \\ f_1(x^r) & f_2(x^r) & .... & f_p^*(x^r) \end{bmatrix} \quad (16)$$

Step-4 :Find the lower bound $\tilde{L}_p^{\,T}$ and the upper bound $\tilde{U}_p^{\,T}$ corresponding to each objective $f_k(x)$ .The lower and upper bounds for truth membership of objectives are $\tilde{U}_p^{\,T} = \max\left[f_2(x^r)\right]$ (17)

$\tilde{L}_p^{\,T} = \min\left[f_2(x^r)\right]$ where $r = 1,2..., p$ The upper bound $\tilde{U}_p^{\,F}$ and lower bound $\tilde{L}_p^{\,F}$ for falsity membership of objectives are

$\tilde{U}_p^{\,F} = \tilde{U}_p^{\,T}$ and $\tilde{L}_p^{\,F} = \tilde{L}_p^{\,T} + t(\tilde{U}_p^{\,T} - \tilde{L}_p^{\,T})$. (18)

Upper bound $\tilde{U}_p^{\,F}$ and lower bound $\tilde{L}_p^{\,U}$ for uncertainty membership of objectives are $\tilde{L}_p^{\,U} = \tilde{L}_p^{\,T}$ (19)

$$\tilde{U}_p^{\,U} = \tilde{L}_p^{\,T} + s(\tilde{U}_p^{\,T} - \tilde{L}_p^{\,T}) \quad (20)$$

And upper bound $\tilde{U}_p^{\,F}$ and lower bound $\tilde{L}_p^{\,U}$ for contradictory membership of objectives are

$$\tilde{L}_p^{\,C} = \tilde{L}_p^{\,T} \wedge \tilde{L}_p^{\,F} \quad (21)$$

$$\tilde{U}_p^{\,C} = \tilde{L}_p^{\,T} \wedge \tilde{L}_p^{\,F} + l(\tilde{U}_p^{\,T} \wedge \tilde{U}_p^{\,F} - \tilde{L}_p^{\,T} \wedge \tilde{L}_p^{\,F}) \quad (22)$$

Where $t, s, l \in [0,1]$

Step-5: In this step ,we will define truth,uncertainty,falsity and contradictory membership functions as follows

$$T_p(f_p(x)) = \begin{cases} 1 \text{ if } f_p(x) \le \tilde{L}_p^T \\ \dfrac{\tilde{U}_p^T - f_P(x)}{\tilde{U}_p^T - \tilde{L}_p^T} \text{ if } \tilde{L}_p^T \le f_P(x) \le \tilde{U}_p^T \\ 0 \text{ if } f_P(x) \ge \tilde{U}_p^T \end{cases} \quad (23)$$

$$U_p(f_p(x)) = \begin{cases} 1 \text{ if } f_p(x) \le \tilde{L}_p^U \\ \dfrac{\tilde{U}_p^U - f_P(x)}{\tilde{U}_p^U - \tilde{L}_p^U} \text{ if } \tilde{L}_p^U \le f_P(x) \le \tilde{U}_p^U \\ 0 \text{ if } f_P(x) \ge \tilde{U}_p^U \end{cases} \quad (24)$$

$$F_p(f_p(x)) = \begin{cases} 0 \text{ if } f_p(x) \le \tilde{L}_p^F \\ \dfrac{f_P(x) - \tilde{L}_p^F}{\tilde{U}_p^F - \tilde{L}_p^F} \text{ if } \tilde{L}_p^F \le f_P(x) \le \tilde{U}_p^F \\ 1 \text{ if } f_P(x) \ge \tilde{U}_p^F \end{cases} \quad (25)$$

$$C_p(f_p(x)) = \begin{cases} 1 \text{ if } f_p(x) \le \tilde{L}_p^C \\ \dfrac{\tilde{U}_p^C - f_P(x)}{\tilde{U}_p^C - \tilde{L}_p^C} \text{ if } \tilde{L}_p^C \le f_P(x) \le \tilde{U}_p^C \\ 0 \text{ if } f_P(x) \ge \tilde{U}_p^C \end{cases} \quad (26)$$

Step-6:Now four valued refined neutrosophic optimization method for multi-objective nonlinear programming problem gives a corresponding non-linear problem as

Max $\alpha - \beta + \gamma + \delta$ (27)

Such that $T_p(f_p(x)) \ge \alpha$ (28)

$U_p(f_p(x)) \ge \gamma$ (29)

$F_p(f_p(x)) \le \beta$ (30)

$C_p(f_p(x)) \ge \delta$ (31)

$g_j(x) \le b_j,\ j = 1,2\ldots,q$ (32)

$\alpha + \beta + \gamma + \delta \le 4$ (33)

$\alpha \ge \beta, \alpha \ge \gamma, \alpha \ge \delta$ (34)

$x \ge 0$ (35)

Where $\alpha, \beta, \gamma, \delta \in [0,1]$

4.Optimization of Three Bar Truss Design in Refined Neutrosophic Environment

Sometimes slight change in stress or deflection reduces the weight of the structure and indirectly the cost of processing. At this juncture when decision maker is in doubt to design the stress constraint goal, the decision maker can include the acceptance of boundary,uncertainty and contradictory response margin of constraint goal. This facts seems to take the constraint goal as a refined neutrosophic set instead of neutrosophic set, fuzzy set, intuitionistic set. So the above mentioned crisp model can be solved by FVRNO technique in the following way. Consider the following optimization problem

A well-known planer truss framework of the three bar is described in figure -2 in order to decrease the weight of the structure $W(A_1, A_2)$ and to decrease the vertical bending at leading point $\rho(A_1, A_2)$ of a three bar planer truss under stress $\sigma_i(A_1, A_2)$ constraints on each of the truss elements $i = 1,2,3\ldots.$

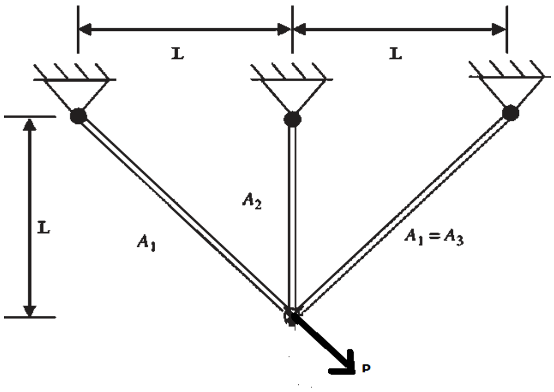

figure -1: Tree Bar Truss Design

In the following way, the MOSOP can be expressed

Minimize $W(A_1, A_2) = \delta L\left(2\sqrt{2}A_1 + A_2\right)$ (36)

Minimize $\rho(A_1, A_2) = \dfrac{PL}{E(A_1 + \sqrt{2}A_2)}$ (37)

Subject to $\sigma_1(A_1, A_2) \equiv \dfrac{P\left(\sqrt{2}A_1 + A_2\right)}{2A_1A_2 + \sqrt{2}A_1^2} \le \left[\sigma_1^T\right]$ (38)

$\sigma_2(A_1, A_2) \equiv \dfrac{P}{A_1 + \sqrt{2}A_2} \le \left[\sigma_2^T\right]$ (39)

$\sigma_3(A_1, A_2) \equiv \dfrac{PA_2}{2A_1A_2 + \sqrt{2}A_1^2} \le \left[\sigma_3^T\right]$ (40)

$A_i \in \left[A_i^{\min}, A_i^{\max}\right], i = 1,2$ where applied load $= P$,material density $= \delta, L =$ length of each bar, $\left[\sigma_i^T\right] =$ maximum tensile stress limit for $i = 1,2.\left[\sigma_3^C\right] =$ maximum compressive stress limit. $E =$ Young's modulus, $A_1 =$ cross section of bar 1 and bar 3 and $A_2 =$ cross section area of bar 2.The input values of MOSOP (1) are as follows P (Applied force) $= 20KN, \delta = 100KN/m^3, L$ (bar length) $= 1m, [\sigma_T]$ (Maximum tensile stress limit for bar 1 and 3) $= 20KN/m^2, [\sigma_C]$ (maximum compressive stress limit for bar 2) $= 15KN/m^2, E$ (Young's modulus) $= 2\times10^8 KN/m^2$,range of bar cross section $= 0.1\times10^{-4}m^2 \le A_1, A_2 \le 5\times10^{-4}m^2$.

Solution : The values obtained in pay off matrix according to step 2 is as follows

$$\begin{array}{c} \quad W(A_1, A_2) \quad \rho(A_1, A_2) \\ \begin{matrix} A_1 \\ A_2 \end{matrix}\begin{bmatrix} 2.638958 & 14.64102 \\ 19.14214 & 1.656854 \end{bmatrix} \end{array} \quad (41)$$

$U_W^T = 19.12412, L_W^T = 2.638958; U_W^C = 2.638958 + t(19.12412 - 2.638958); L_W^C = 2.638958;$

$U_W^F = 19.12412; L_W^F = 2.638958 + r(19.12412 - 2.638958); U_W^U = 2.638958 + S(19.12412 - 2.638958),$

$L_W^U = 2.638958; U_\rho^T = 14.64102; L_\rho^T = 1.656854; U_\rho^C = 1.656854 + t(14.64102 - 1.656854);$

$L_\rho^C = 1.656854; U_\rho^F = 14.64102; L_\rho^F = 1.656854 + r(14.64102 - 1.656854);$

$U_\rho^U = 1.656854 + s(14.64102 - 1.656854); L_\rho^U = 1.656854;$

where $t, s, r \in (0,1); t = 0.96; s = 0.78; r = 0.3$

Now we define the membership function for T, F, U and C as

$$T_W(W(A_1,A_2)) = \begin{cases} 1 \text{ if } W(A_1,A_2) \le 2.638958 \\ \dfrac{19.12412 - W(A_1,A_2)}{19.12412 - 2.638958} \text{ if } 2.638958 \le W(A_1,A_2) \le 19.12412 \\ 0 \text{ if } W(A_1,A_2) \ge 19.12412 \end{cases} \quad (42)$$

$$U_W(W(A_1,A_2)) = \begin{cases} 1 \text{ if } W(A_1,A_2) \le 2.638958 \\ \dfrac{2.638958 - W(A_1,A_2)}{(19.12412 - 2.638958)s} \text{ if } 2.638958 \le W(A_1,A_2) \le 2.638958 + s16.485162 \\ 0 \text{ if } W(A_1,A_2) \ge 2.638958 + s16.485162 \end{cases} \quad (43)$$

$$F_W(W(A_1,A_2)) = \begin{cases} 0 \text{ if } W(A_1,A_2) \le 2.638958 + r16.485162 \\ \dfrac{W(A_1,A_2) - 2.638958}{19.12412 - 2.638958 - r16.485162} \text{ if } 2.638958 + r16.485162 \le W(A_1,A_2) \le 19.12412 \\ 1 \text{ if } W(A_1,A_2) \ge 19.12412 \end{cases} \quad (44)$$

$$C_W(W(A_1,A_2)) = \begin{cases} 1 \text{ if } W(A_1,A_2) \le 2.638958 \\ \dfrac{2.638958 + t16.485162 - W(A_1,A_2)}{t16.485162} \text{ if } 2.638958 \le W(A_1,A_2) \le 2.638958 + t16.485162 \\ 0 \text{ if } W(A_1,A_2) \ge 2.638958 + t16.485162 \end{cases} \quad (45)$$

$$T_\rho(\rho(A_1,A_2)) = \begin{cases} 1 \text{ if } \rho(A_1,A_2) \le 1.656854 \\ \dfrac{1.656854 - \rho(A_1,A_2)}{14.64102 - 1.656854} \text{ if } 1.656854 \le \rho(A_1,A_2) \le 14.64102 \\ 0 \text{ if } \rho(A_1,A_2) \ge 14.64102 \end{cases} \quad (46)$$

$$U_\rho(\rho(A_1,A_2)) = \begin{cases} 1 \text{ if } \rho(A_1,A_2) \le 1.656854 \\ \dfrac{1.656854 - \rho(A_1,A_2)}{s12.984166} \text{ if } 1.656854 \le \rho(A_1,A_2) \le 1.656854 + s12.984166 \\ 0 \text{ if } \rho(A_1,A_2) \ge 1.656854 + s12.984166 \end{cases} \quad (47)$$

$$F_\rho(\rho(A_1,A_2)) = \begin{cases} 0 \text{ if } \rho(A_1,A_2) \le 1.656854 \\ \dfrac{\rho(A_1,A_2) - 1.656854 - r12.984166}{14.64102 - 1.656854 - r12.984166} \text{ if } 1.656854 - r12.984166 \le \rho(A_1,A_2) \le 14.64102 \\ 1 \text{ if } \rho(A_1,A_2) \ge 14.64102 \end{cases} \quad (48)$$

$$C_\rho(\rho(A_1,A_2)) = \begin{cases} 1 \text{ if } \rho(A_1,A_2) \le 1.656854 \\ \dfrac{1.656854 + t12.984166 - \rho(A_1,A_2)}{t12.984166} \text{ if } 1.656854 \le \rho(A_1,A_2) \le 1.656854 + t12.984166 \\ 0 \text{ if } \rho(A_1,A_2) \ge 1.656854 + t12.984144 \end{cases} \quad (49)$$

Now nonlinear programming problem is

Maximize $\alpha - \beta + \gamma + \delta$ (50)

Such that

$\delta L\left(2\sqrt{2}A_1 + A_2\right) + 16.485162\alpha \le 19.12412$ (51)

$\delta L\left(2\sqrt{2}A_1 + A_2\right) + s16.485162\gamma \le 2.638958 + s16.485162$ (52)

$\delta L\left(2\sqrt{2}A_1 + A_2\right) - (16.485162 - r16.485162)\beta \le 2.638958 + r16.485162$ (53)

$\delta L\left(2\sqrt{2}A_1 + A_2\right) - 16.485162\delta \le 2.638958 + t16.485162$ (54)

$$\frac{PL}{E\left(A_1+\sqrt{2}A_2\right)}+12.984166\alpha \le 14.64102 \quad (55)$$

$$\frac{PL}{E\left(A_1+\sqrt{2}A_2\right)}+s12.984166\gamma \le 1.656854+s12.984166 \quad (56)$$

$$\frac{PL}{E\left(A_1+\sqrt{2}A_2\right)}-\left(12.984166-r12.984166\right)\beta \le 1.656854+r12.984166 \quad (57)$$

$$\frac{PL}{E\left(A_1+\sqrt{2}A_2\right)}+12.984166\delta \le 1.656854+t12.984166 \quad (58)$$

$$\frac{P}{\left(A_1+\sqrt{2}A_2\right)} \le 20 \quad (59)$$

$$\frac{PA_2}{\left(2A_1A_2+\sqrt{2}A_1^2\right)} \le 15 \quad (60)$$

$$\frac{P\left(\sqrt{2}A_1+A_2\right)}{\left(2A_1A_2+\sqrt{2}A_1^2\right)} \le 20 \quad (61)$$

$$0 \le \alpha \le 1, 0 \le \beta \le 1, 0 \le \gamma \le 1, 0 \le \delta \le 1 \quad (62)$$

$$\alpha \ge \beta, \alpha \ge \gamma, \alpha \ge \delta \quad (63)$$

$$\alpha+\beta+\gamma+\delta \le 4 \quad (64)$$

Table:1- Input data for refined Neutrosophic model(eq 50-64)

Applied load $P(KN)$	Length $L(in)$	Material Density $\delta(KN/m^3)$	Maximum limit of Tensile Stress $[\sigma_T](KN/m^2)$	Maximum limit of compressive stress $[\sigma_C](KN/m^2)$	Young's Modulus $E(KN/m^2)$	Cross section of bars $A_i^{\min}(10^{-4}m^2)$ $A_i^{\max}(10^{-4}m^2)$
20	1	100	20	15	2×10^8	$0.1 \le A_1 \le 5$ $0.1 \le A_2 \le 5$

Table:2- A comparative result of structural weight and deflection for $s=0.78, t=0.96, r=0.3$

Methods	$A_1\times10^{-4}$	$A_2\times10^{-4}$	$W(A_1,A_2)\times10^2 KN$	$\rho(A_1,A_2)\times10^{-7}m$
Fuzzy Optimization (FO)	0.5995887	3.789761	5.485654	3.356200
Intuitionistic Fuzzy optimization(IFO)	0.5766526	3.694181	5.325201	3.447673
Neutrosophic optimization (NSO)	0.581611	3.462786	5.140011	3.628012
Four valued refined neutrosophic optimization(FVRNO)	0.7886751	0.4082483	2.638958	0.732508

It can be observed that FVRNO is the best method in finding minimum cost as compare to other deterministic method.

5. Conclusion

This comparison analysis demonstrates that enhanced neutrosophic optimization outperforms other optimization methods in terms of performance. Moreover, this study implies that other nonlinear programming issues in a variety of engineering domains may be resolved by expanding the use of neutrosophic optimization. By offering more precise answers to challenging optimization issues, the creation of an efficient neutrosophic optimization method may have substantial practical ramifications in the engineering area. This strategy can get beyond restrictions brought on by ambiguity and imprecision in the data by using an approximation approach. Because of its adaptability, it is a useful tool for academics dealing with difficult optimization problems where ambiguity and imprecision are common. Furthermore, the time and computational efficiency of the suggested method are significantly better than those of other methods mentioned in the literature. Because of this benefit, it is a potentially useful tool for academics dealing with challenging optimization issues where efficiency and speed are important factors.

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