Application of Weaker Form of Nano Closed Structures by the Use of Ideals and Graph

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Abstract

The intent of this paper is to introduce a new kind of set called $n\Im$-wg - closed sets, $n\Im$-wg - closed sets are weaker than wg-closed sets and stronger than Nwg-closed sets. We study few characterization and basic properties of $n\Im$-wg - closed sets. Also, we established and investigated new type of functions called $n\Im$-wg -continuous function, $n\Im$-wg - irresolute functions and $n\Im$-wghomeomorphism. We prove the isomorphism between simple digraphs through the nano continuity between them.

Key Words:$n\Im$-wg-closed sets, $n\Im$-wg -irresolute,$n\Im$-wg-homeomorphism, digraph.

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Introduction

Kuratowski[12] introduced the ideal topological spaces and the local functions in ideal topological spaces. In 1945, Vaidhyanathaswany[30] have investigated the topic of ideals in topological spaces. Kuratowski closure operator cl^* for the topology $\tau^*(I,\tau)$, called the $\star$ -topology, finer than τ is defined by $cl^*(A) = A \cup A^*(I,\tau)$[5]. When there is no chance for confusion, we will simply write A^* for $A^*(I,\tau)$and $\tau*$ for$\tau^*(I,\tau)$. If I is an ideal on X, then (X,τ,I) is called an ideal space. Jankovic and Hamlett [10] continue this study to get the properties of ideal topological spaces 1990. Abd El-Monsef [1] investigated the notion of I-open sets introduced by Jankovic et al. [11].

Many authors have introduced several open sets and generalized open sets in ideal topological spaces such as $preI$-open sets [24], $semiI$-open sets [4], α-I-open sets, αg-I-open sets and gp-I-open sets [21]. In 2013, Lellis Thivagar and Carmel Richard [14] established the field of nano topological spaces. In 2012 Robert et. Al [26] established the class of $semi^*$ -open sets and $semi^*$ -closed sets in Topological Space. The notion of $nanosemi^*$-open sets and $nanosemi^*$-closed sets in terms of nano generalised closure and nano generalised interior was introduced by Paulraj Gnanachandra [3] in 2015. In 2020 [24], additional properties of $nanosemi*$-open sets were studied. The space known as nano ideal topological spaces was introduced by M. Parimala et al. [23]. In 2018[13], this author established the notion of $nanoI$-open sets and studied some of its properties, also introduced nIg - open sets and nIg- closed sets in Nano Ideal Topological Spaces.

Continuity of functions is one of the fundamental concepts of topology. Lellis Thivagar and Richard [15] introduced nano continuous functions. They also introduced nano-open maps, nano closed maps and nano homeomorphisms and their representations in terms of nano closure and nano interior. Lellis Thivagar et al. [16] defined the concept of nano topological space via a direct simple graph.

Nagaveni introduced a class of sets called weakly generalized closed sets in general topology in 1999[29].Nagaveni et al. [22] introduced the concept of weakly generalized closed sets in Nano Topological spaces and studied some of its properties.

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In this paper we introduce the nano-$\mathfrak{I}$ weakly generalized closed sets in nano ideal topological spaces. The characterization and some of its properties are analysed. Throughout this paper,$(\mathcal{U}, \mathcal{N}, \mathfrak{I})$ $and(\mathcal{V}, \mathcal{N}', \mathfrak{K})$be a nano ideal topological space.

Preliminaries

We take back the following definitions, which will be utilized in this sequel.

Definition 1.1[25] Let U be a certain set called the universe set and let R be an equivalence relation on U. The pair (U, R) is called an approximation space. Elements belonging to the same equivalence class are said to be indiscernible with one another. Let $X \subseteq U$

i. The lower approximation of X with respect to R is the set of all object, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is $L_R(X) = \cup_{x \in U}\{R_x: R_x \subseteq X\}$, where R_x denotes to the equivalence class determined by x.

ii. The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is $U_R(X) = \cup_{x \in U}\{R_x: R_x \cap X \neq \emptyset\}$, where $R_x$denotes to the equivalence class determined by x.

iii. The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not X with respect to R and it is denoted by $B_R(X)$. That is $B_R(X) = U_R(X) - L_R(X)$, where R_x denotes the equivalence class determined by x.

Definition 1.2 [14,15] Let U be a finite universe, $X \subseteq$ U, R be an equivalence relation on U. The collection $\tau_R(X) = \{U, \phi, U_R(X), L_R(X),$ BRX satisfies the following axioms.

i. $U \in \tau_R(X)$ and $\phi \in \tau_R(X)$.

ii. The union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$.

iii. The intersection of the elements of any finite sub collection of $\tau_R(X)$ is $\text{int}\tau_R(X)$.

The collection $\tau_R(X)$ forms a topology on U called as the Nano topology on U with respect to R. The pair $(U, \tau_R(X))$is called as the Nano topological space. The elements of $\tau_R(X)$ are called as Nano open sets. The complement of Nano open sets is named as Nano closed sets. The collection $B = \{U, L_R(X),$ BRX is a basis for the Nano topology.

Definition 1.3[12] An ideal I on a set X is a nonempty collection of subsets of X which satisfies the conditions

i. $A \in I$ and $B \subseteq A$ $implies$ $B \in I$,

ii. $A \in I$ and $B \in I$ $implies$ $A \cup B \in I$

iii. The concept of a set operator $()^{\alpha}: P(X) \rightarrow P(X)$ was introduced by Nasef in 1992, which is called a local function of I with respect to τ.

Definition1.4[8] A subset $\mathcal{A}$ of space $(\mathcal{U}, \mathcal{N}, \mathfrak{I})$is said to be

i. nano*α-I-open (briely* $\alpha - n\mathfrak{I}$ *open) if* $A \subseteq n\text{-}int(n\text{-}cl^*(n\text{-}int(A)))$,

ii. nano *semi-I-open (briefly semi-* $n\mathfrak{I}$ *-open) if* $A \subseteq n\text{-}cl^*(n\text{-}int(A))$,

iii. nano *pre-I-open (briefly pre-* $n\mathfrak{I}$ *-open) if* $A \subseteq n\text{-}int(n\text{-}cl^*(A))$,

iv. nano*β -I-open (briely β-* $n\mathfrak{I}$ *-open) if* $A \subseteq n\text{-}cl^*(n\text{-}int(n\text{-}cl^*(A)))$.

Definition 1.6 A subset A of nano topological space $(U, \tau_R(X))$ is called nano wg-closed [7] if $N\ Cl(Nint(A)) \subseteq V$, whenever $A \subseteq V$ and V is nano open.

Definition 1.7 [16] Let $G(V, E)$ be a directed graph and $u, v \in V(G)$, then

i. u is in vertex of v if $\overrightarrow{uv} \in E(G)$.

ii. u is out vertex of v if $\overrightarrow{vu} \in E(G)$

iii. The neighborhood of v is denoted by $N(v) = \{v\} \cup \{u \in V(G): \overrightarrow{vu} \in E(G)\}$

Definition 1.8 [16] Let$G(V, E)$ be a graph and H be a subgraph of G, $N(v)$ be the neighbourhood of V. then

i. The Lower approximation $L: P(V(G)) \rightarrow P(V, G)$ is $L_N(V(H)) = \cup_{v \in V(G)}\{v: N(v) \subseteq$ *VH;*

ii. The upper approximation $U: P(V(G)) \rightarrow P(V, G)$ is $U_N(V(H)) = \cup_{v \in V(G)}\{N(v): V \in$ *VH;*

iii. The boundary is $B_N(V(H)) = (U_N V(H)) - L_N(V(H))$.

Let G be a graph, N(v) be a neighborhood of v in V and H be a subgraph of G. $\tau_N(V(H)) = \{V(G), \emptyset, L_N(V(H)), U_N(V(H)), B_N(V(H))\}$ forms a topology on V(G) called the nano topology on V(G) with respect to V(H). V(G), $\tau_N(V(H))$ is a nano topological space induced by a graph G.

Definition 1.9 [15] Let $(U, \tau_R(X))$ and $(V, \tau_R(X))$ be a nano topological space. Then a mapping $f: (U, \tau_R(X)) \rightarrow (V, \tau_R(X))$ is a nano continuous on U if the inverse image of every nano-open set in V is nano open in U.

Definition 1.10 [15] A function $f: (U, \tau_R(X)) \rightarrow (V, \tau_R(X))$ is a nano-open map if the image of every nano-open set in U is nano open in V. The mapping f is said to be a nano closed map if the image of every nano closed set in U is nano closed in V.

Definition 1.11 [15] A function $f:(\mathrm{U},\ \tau_{\mathrm{R}}(\mathrm{X}))\rightarrow(\mathrm{V},\ \tau_{\mathrm{R}}(\mathrm{X}))$ is said to be a nano homeomorphism if f is bijective, nano continuous and nano open.

Definition 1.12[6] Two directed graphs G and H are isomorphic if there is an isomorphism f between their underlying graphs that preserves the direction of each edge. That is e is directed from u to v if and only if f(e) is directed from f(u) to f(v).

Definition 1.13 [6] Two directed graph C and D are isomorphic if D can be obtained by relabelling the vertices of C, that is, if there is a bijection between the vertices of C and those of D, such that the arcs joining each pair of vertices in D.

Nano Ideal Weakly Generalized Closed Sets

In this section, we will study some properties of $n\mathfrak{I}$-wg – closed (open)set in ideal topological space.

Definition 3.1

A subset $\mathcal{A}$ of a nano ideal topological space $(\mathcal{U},\mathcal{N},\mathfrak{I})$ is said to be nano ideal weakly generalized closed ($n\mathfrak{I}$-wg-closed) set if $cl_n^*(\mathcal{N}int(\mathcal{A}))\subseteq\mathcal{V}$ where $\mathcal{A}\subseteq\mathcal{V}$ and $\mathcal{V}$is Nano open. A subset $\mathcal{A}$ of a nano ideal topological space $(\mathcal{U},\mathcal{N},\mathfrak{I})$ is said to be nano ideal weakly generalized open ($n\mathfrak{I}$-wg -open) if $\mathcal{U}-\mathcal{A}$ is $n\mathfrak{I}$-wg-closed.

$(\mathcal{U},\mathcal{N},\mathfrak{I})$ be a nano ideal topological space and $\mathcal{A}\subseteq\mathcal{U}$. The nano$\mathfrak{I}$-$wg$ –interior of $\mathcal{A}$ is defined as the union of all nano $n\mathfrak{I}$-wg –open subsets of $\mathcal{A}$ and it is denoted by $n\mathfrak{I}$-$wg-int(A)$. The nano $\mathfrak{I}$-wg –closure of $\mathcal{A}$ is defined as the intersection of all $n\mathfrak{I}$-wg –closed sets containing $\mathcal{A}$ and it is denoted by $n\mathfrak{I}$-$wg-cl(A)$. For simplicity $nano$-$cl^*(\mathcal{A})$ is denoted by $cl_n^*(\mathcal{A})$

Theorem 3.2 If a subset $\mathcal{A}$ of a nano ideal topological space $(\mathcal{U},\mathcal{N},\mathfrak{I})$ is $n\mathfrak{I}$-g-closed, then it is $n\mathfrak{I}$-wg-closed in $(\mathcal{U},\mathcal{N},\mathfrak{I})$ but not conversely.

Proof: suppose $\mathcal{A}$ is$n\mathfrak{I}$-g-closed in $(\mathcal{U},\mathcal{N},\mathfrak{I})$, let $\mathcal{B}$ be an open set containing $\mathcal{A}$ in$(\mathcal{U},\mathcal{N},\mathfrak{I})$.Then $\mathcal{B}$ contains cl_n^*, now $\mathcal{B}\supseteq cl_n^*(\mathcal{A})\supseteq cl_n^*(Nint(\mathcal{A}))$. Thus $\mathcal{A}$ is $n\mathfrak{I}$-wg-closed in $(\mathcal{U},\mathcal{N},\mathfrak{I})$

Remarks 3.3 The converse of the above theorem need not be true as shown in the following example.

Example 3.4 Let $\mathcal{U}=\{\hbar_1,\hbar_2,\hbar_3,\hbar_4\}\mathcal{U}/\mathcal{R}=\{\{\hbar_1\},\{\hbar_3\},\{\hbar_2,\hbar_4\}\},\mathcal{X}=\{\hbar_1,\hbar_2\}\subseteq\mathcal{U},\mathcal{N}=\{\mathcal{U},\emptyset,\{\hbar_1\},\{\hbar_1,\hbar_2,\hbar_4\},\{\hbar_2,\hbar_4\}\}$ and ideal $\mathfrak{I}=\{\varphi,\{\hbar_1\}\}$. Then the set $\mathcal{A}=\{\hbar_2\}$ is $n\mathfrak{I}$-wg-closed but not $n\mathfrak{I}$-g-closed.

Theorem3.5 If a subset $\mathcal{A}$ of a nano ideal topological space is both nano open and $n\mathfrak{I}$-wg-closed, then it is closed.

Proof

Suppose a subset $\mathcal{A}$ is both $n\mathfrak{I}$ -open and $n\mathfrak{I}$-wg-closed in $(\mathcal{U},\mathcal{N},\mathfrak{I})$. Now $\mathcal{A}\supseteq\mathcal{A}_n^*(NInt(\mathcal{A}))\supseteq\mathcal{A}_n^*$. That is $\mathcal{A}\supseteq\mathcal{A}_n^*$. Since $\mathcal{A}_n^*\supseteq\mathcal{A}$, we have $A=\mathcal{A}_n^*$. Thus $\mathcal{A}$ is nano closed in $(\mathcal{U},\mathcal{N},\mathfrak{I})$.

Theorem 3.6 If a subset $\mathcal{A}$ of a nano ideal topological space is both $n\mathfrak{I}$-wg-closed and semi-$n\mathfrak{I}$-open, then it is $n\mathfrak{I}$-g- closed.

Proof

Suppose a subset $\mathcal{A}$ is both $n\mathfrak{I}wg$-closed and semi-nI-open in $(\mathcal{U},\mathcal{N},\mathfrak{I})$. Let $\mathcal{B}$ be an open set containing $\mathcal{A}$. As $\mathcal{A}$ is $n\mathfrak{I}$-wg-closed, $\mathcal{B}\supseteq\mathcal{A}_n^*(Nint(\mathcal{A}))$. Now $\mathcal{B}\supseteq\mathcal{A}_n^*$. Since $\mathcal{A}$ is semi-$n\mathfrak{I}$ -open. Thus $\mathcal{A}$ is gnI- closed in $(\mathcal{U},\mathcal{N},\mathfrak{I})$..

Theorem 3.7 A subset $\mathcal{A}n\mathfrak{I}$-wg-closed if and only if $\mathcal{A}_n^*(Nint(\mathcal{A}))-\mathcal{A}$ contains no nonempty nano closed sets.

Proof

Suppose that $\mathcal{F}$ isa non-empty closed subset of$\mathcal{A}_n^*(Nint(\mathcal{A}))$.Now $\mathcal{F}\subseteq\mathcal{A}_n^*(Nint(\mathcal{A}))-\mathcal{A}$. Implies $\mathcal{F}\subseteq cl_n^*(NInt(\mathcal{A}))\cap\mathcal{A}^c$, since $(Nint(\mathcal{A}))-\mathcal{A}=\mathcal{A}_n^*(NInt(\mathcal{A}))\cap\mathcal{A}^c$. Thus $\mathcal{F}\subseteq\mathcal{A}_n^*(Nint(\mathcal{A}))$. Now $\mathcal{F}\subseteq\mathcal{A}^c$, implies $\mathcal{A}\subseteq\mathcal{F}$. Here $\mathcal{F}^c$ is open and $\mathcal{A}$ $isn\mathfrak{I}$-wg-closed, we have $\mathcal{A}_n^*(Nint(\mathcal{A}))\subseteq\mathcal{F}^c$. Thus $\mathcal{F}\subseteq\left(cl_n^*(Nint(\mathcal{A}))\right)^c$. Hence $\mathcal{F}\subseteq\mathcal{A}_n^*(NInt(\mathcal{A}))\cap\left(\mathcal{A}_n^*(Nint(\mathcal{A}))\right)^c=\emptyset$. That is $\mathcal{F}=\emptyset$ implies $(Nint(\mathcal{A}))-\mathcal{A}$ contains no nonempty nano closed sets.

Conversely, let $\mathcal{A}\subseteq\mathcal{B}$, $\mathcal{B}$ is nano open. Suppose that $\mathcal{A}_n^*(Nint(\mathcal{A}))\nsubseteq\mathcal{B}$, then $\mathcal{A}_n^*(Nint(\mathcal{A}))\cap\mathcal{B}^c$ is a non-empty closed set of $\mathcal{A}_n^*(Nint(\mathcal{A}))\cap\mathcal{A}^c$, which is a contradiction. Therefore $\mathcal{A}_n^*(NInt(\mathcal{A}))\subseteq\mathcal{B}$ and hence $\mathcal{A}$ is $n\mathfrak{I}wg$-closed.

Theorem 3.8 If $\mathcal{A}$ is $n\mathfrak{I}wg$-closed and$\mathcal{A}\subseteq\mathcal{B}\subseteq cl_n^*(Nint(\mathcal{A}))$, then $\mathcal{B}$ is $n\mathfrak{I}wg$-closed set.

Proof Given that $\mathcal{B}\subseteq cl_n^*(Nint(\mathcal{A}))$, then $cl_n^*(\mathcal{N}int(\mathcal{B}))\subseteq cl_n^*(Nint(\mathcal{A}))$. $cl_n^*(Nint(\mathcal{B}))-\mathcal{B}\subseteq cl_n^*(\mathcal{N}int(\mathcal{A}))-\mathcal{A}$, since $\mathcal{A}\subseteq\mathcal{B}$. As $\mathcal{A}$ is $n\mathfrak{I}wg$-closed, theorem 3.7 $cl_n^*(Nint(\mathcal{A}))-\mathcal{A}$ contains no nonempty closed set, $cl_n^*(Nint(\mathcal{B}))-\mathcal{B}$ contains no empty closed set. Again, by the above theorem,$\mathcal{B}$ is $n\mathfrak{I}wg$-closed.

Theorem 3.9 if a subset $\mathcal{A}$ of a nano ideal topological space$(\mathcal{U},\mathcal{N},\mathfrak{I})$ is nowhere dense then it is $n\mathfrak{I}wg$-closed but not conversely.

Proof Suppose $\mathcal{A}$ is nowhere dense then $Nint(cl_n^*(\mathcal{A})) = \emptyset$. It is clear that $\mathcal{A} \subseteq cl_n^*(\mathcal{A})$ and $Nint(\mathcal{A}) \subseteq \mathcal{N}int(cl_n^*(\mathcal{A}))$. As $\mathcal{A}$ is nowhere dense, $int(\mathcal{A}N) = \emptyset$, which implies $cl_n^*(Nint(\mathcal{A})) = \emptyset$. Thus $\mathcal{A}$ is $n\mathfrak{I}wg$-closed in $(\mathcal{U},\mathcal{N},\mathfrak{I})$.

Remark 3.10 The converse of the above theorem need not be true as shown in the following example.

Example 3.11 Let $\mathcal{U} = \{k_1,k_2,k_3,k_4,k_5\}$,$U/\mathcal{R} = \{\{k_1\},\{k_2,k_3\},\{k_4\}\{k_5\}\}$, $\mathcal{X} = \{k_2,k_4\} \subseteq \mathcal{U}$,$\mathcal{N} = \{\mathcal{U},\emptyset,\{k_4\},\{k_2,k_3\},\{k_2,k_3,k_4\}\}$ and ideal $\mathfrak{I} = \{\varphi,\{k_1\}\}$. Then the set $\mathcal{A}$ =$\{k_2\}$ is $n\mathfrak{I}$-wg-closed but not nowhere dense.

Remark 3.12 If a subset $\mathcal{A}$ and $\mathcal{B}$ of a nano ideal topological space are$n\mathfrak{I}wg$-closed then their union need not be $n\mathfrak{I}wg$-closed.

Example3.13 Let $\mathcal{U} = \{h_1,h_2,h_3,h_4\}$,$\mathcal{X} = \{h_1,h_2\} \subseteq \mathcal{U}$,$\mathcal{N} = \{\mathcal{U},\emptyset,\{h_1\},\{h_1,h_2,h_4\},\{h_2,h_4\}\}$ and ideal $\mathfrak{I} = \{\varphi,\{h_1\}\}$. Let $\mathcal{A}$ =$\{h_2\}$and $\mathcal{B} = h_4$ Then the sets $\mathcal{A}$and $\mathcal{B}$ are $n\mathfrak{I}$-wg-closed but their union $\mathcal{A} \cup \mathcal{B} = \{h_2,h_4\}$ is not $n\mathfrak{I}$-wg-closed.

Remark 3.14 If a subset $\mathcal{A}$ and $\mathcal{B}$ are$n\mathfrak{I}wg$-closed then their intersection need not be $n\mathfrak{I}wg$-closed.

Example3.15 Let $\mathcal{U} = \{q_1,q_2,q_3,q_4\}$,$\mathcal{X} = \{q_1,q_2\} \subseteq \mathcal{U}$,$\mathcal{N} = \{\mathcal{U},\emptyset,\{q_1\}\{q_2,q_4\},\{q_1,q_2,q_4\}\}$ and ideal $\mathfrak{I} = \{\varphi,\{q_2\}\}$. Let $\mathcal{A}$ =$\{q_1,q_2\}$and $\mathcal{B} = \{q_1,q_3\}$ Then the sets $\mathcal{A}$and $\mathcal{B}$ are $n\mathfrak{I}$-wg-closed but their intersection $\mathcal{A} \cap \mathcal{B} = \{q_1\}$ is not $n\mathfrak{I}$-wg-closed.

Theorem 3.16

If $\mathcal{A}$is a closed subset of a nano ideal topological space $(\mathcal{U},\mathcal{N},\mathfrak{I})$, then $\mathcal{A}$is $n\mathfrak{I}wg$-closed.

Proof

Let$\mathcal{A}$and $\mathcal{B}$ are the open set in $(\mathcal{U},\mathcal{N},\mathfrak{I})$, such that $\mathcal{A} \subseteq \mathcal{B}$. Since $\mathcal{A}$is closed, $cl_n^*(\mathcal{A}) = \mathcal{A}$. Then $Nint(\mathcal{A}) \subseteq cl_n^*(\mathcal{A}) = \mathcal{A} \subseteq \mathcal{B}$. Hence $\mathcal{A}$ is $n\mathfrak{I}wg$-closed set.

Remark 3.17

The converse of the above theorem need not be true as seen from the following example.

Example 3.18: Let$\mathcal{U} = \{\ell_1,\ell_2,\ell_3,\ell_4,\ell_5\}$,$U/\mathcal{R} = \{\{\ell_1,\ell_2\},\{\ell_3,\ell_4\},\{\ell_5\}\}$, $\mathcal{X} = \{\ell_1,\ell_3,\ell_4\} \subseteq \mathcal{U}$,$\mathcal{N} = \{\mathcal{U},\emptyset,\{\ell_1,\ell_2\},\{\ell_3,\ell_4\},\{\ell_1,\ell_2,\ell_3,\ell_4\}\}$ and ideal $\mathfrak{I} = \varphi,\{\ell_3\},\{\ell_1,\ell_3\},\{\ell_2,\ell_3\}\{\ell_3,\ell_4\},\{\ell_3,\ell_5\}$. Then the set $\mathcal{A}$ =$\{\ell_2\}$ is $n\mathfrak{I}$-wg-closed but not nano closed.

Theorem 3.19

Every pre- n$\mathfrak{I}$ closed sets in a nano ideal topological spaceis $n\mathfrak{I}wg$-closed set.

Proof:

Assume that $\mathcal{A}$ be a pre- n$\mathfrak{I}$ closed set in a nano ideal topological space. Let $\mathcal{B}$ be a Nano open subset in $(\mathcal{U},\mathcal{N},\mathfrak{I})$,such that $\mathcal{A} \subseteq \mathcal{B}$.By assumption$cl_n^*(Nint(\mathcal{A})) \subseteq \mathcal{A}$.Then $Nint(\mathcal{A}) \subseteq \mathcal{A} \subseteq \mathcal{B}$. Therefore$cl_n^*(Nint(\mathcal{A})) \subseteq \mathcal{B}$. Hence $\mathcal{A}$ is $n\mathfrak{I}wg$-closed set.

The converse of the above theorem need not be true as shown in in the following example.

Example 3.20: Let $\mathcal{U} = \{\ell_1,\ell_2,\ell_3,\ell_4,\ell_5\}$,$U/\mathcal{R} = \{\{\ell_1,\ell_3\},\{\ell_3\},\{\ell_4\},\{\ell_5\}\}$, $\mathcal{X} = \{\ell_1,\ell_2\} \subseteq \mathcal{U}$,$\mathcal{N} = \{\mathcal{U},\emptyset,\{\ell_2\},\{\ell_1,\ell_3\},\{\ell_1,\ell_2,\ell_3\}\}$ and ideal $\mathfrak{I} = \{\varphi,\{\ell_2\}\}$. Then the set $\mathcal{A}$ =$\{\ell_4\}$ is $n\mathfrak{I}$-wg-closed but not pre- n$\mathfrak{I}$ closed.

Remark 3.21

Following example shows that semi-$n\mathfrak{I}$-closed sets and $n\mathfrak{I}$-wg-closed sets are independent.

Example 3.22:= $\{g_1,g_2,g_3,g_4,g_5\}$,$\mathcal{U}/\mathcal{R} = \{\{g_1,g_3\},\{g_2\},\{g_4\},\{g_5\}\}$, $\mathcal{X} = \{g_3,g_5\} \subseteq \mathcal{U}$ $\mathcal{N} = \{\mathcal{U},\emptyset,\{g_5\},\{g_1,g_3\},\{g_1,g_3,g_5\}\}$ and ideal $\mathfrak{I} = \{\varphi,\{g_5\}\}$. Then the set $\mathcal{A}$ =$\{g_3\}$ is $n\mathfrak{I}$-wg-closed but not semi-$n\mathfrak{I}$-closed.

Remark 3.23

Following example shows that β-$n\mathfrak{I}$-closed sets and $n\mathfrak{I}$-wg-closed sets are independent.

Example 3.24: Let $\mathcal{U} = \{\ell_1,\ell_2,\ell_3,\ell_4,\ell_5\}$,$U/\mathcal{R} = \{\{\ell_1,\ell_3\},\{\ell_3\},\{\ell_4\},\{\ell_5\}\}$, $\mathcal{X} = \{\ell_1,\ell_2\} \subseteq \mathcal{U}$,$\mathcal{N} = \{\mathcal{U},\emptyset,\{\ell_2\},\{\ell_1,\ell_3\},\{\ell_1,\ell_2,\ell_3\}\}$ and ideal $\mathfrak{I} = \{\varphi,\{\ell_2\}\}$. Then the set $\mathcal{A}$ =$\{\ell_4\}$ is $n\mathfrak{I}$-wg-closed but not β-$n\mathfrak{I}$-closed in $(\mathcal{U},\mathcal{N},\mathfrak{I})$.

Theorem 3.25

Every α-$n\mathfrak{I}$- closed set in a nano ideal topological spaceis $n\mathfrak{I}$-wg-closed set. The converse need not be true.

Proof: Let $\mathcal{B}$be a Nano open subset in $(\mathcal{U},\mathcal{N},\mathfrak{I})$such that $\mathcal{A} \subseteq \mathcal{B}$. Since $\mathcal{A}$ is $\alpha n\mathfrak{I}$ -closed set $cl_n^*(Nint(cl_n^*(\mathcal{A}))) \subseteq \mathcal{A}$. Then $cl_n^*(Nint(\mathcal{A})) \subseteq cl_n^*(Nint(cl_n^*(\mathcal{A}))) \subseteq \mathcal{A} \subseteq \mathcal{B}$.$cl_n^*(Nint(\mathcal{A})) \subseteq \mathcal{B}$. Hence $\mathcal{A}$ is $n\mathfrak{I}wg$-closed set.

Remark 3.26

The converse of above theorem need not be true as seen from the following example.

Example 3.27 Let $\mathcal{U} = \{\hbar_1, \hbar_2, \hbar_3, \hbar_4\}$,$\mathcal{X} = \{\hbar_1, \hbar_2\} \subseteq \mathcal{U}, \mathcal{N} = \{\mathcal{U}, \emptyset, \{\hbar_1\}, \{\hbar_1, \hbar_2, \hbar_4\}, \{\hbar_2, \hbar_4\}\}$ and ideal $\mathfrak{I} = \{\varphi, \{\hbar_1\}\}$. Then the set $\mathcal{A}$ =$\{\hbar_2\}$ is $n\mathfrak{I}$-wg-closed but not α-$n\mathfrak{I}$- closed.

Some Different Types of Functions Via $n\mathfrak{I}$-wg-Closed Sets

Definition 4.1 The map $\varphi: (\mathcal{U}, \mathcal{N}, \mathfrak{I}) \to (\mathcal{V}, \mathcal{N}', \mathfrak{K})$ is called $n\mathfrak{I}$-wg -continuous on $\mathcal{U}$ if the inverse image of every Nano closed(open) set in $\mathcal{V}$ is $n\mathfrak{I}$-wg closed(open) in $\mathcal{U}$.

Theorem 4.2: Let $\varphi: (\mathcal{U}, \mathcal{N}, \mathfrak{I}) \to (\mathcal{V}, \mathcal{N}', \mathfrak{K})$ be a function then φ is $n\mathfrak{I}$-wg-continuous on $\mathcal{U}$ if the inverse image of every Nano open set in $\mathcal{V}$ is $n\mathfrak{I}$-wg closed in $\mathcal{U}$.

Proof: Let $\varphi: (\mathcal{U}, \mathcal{N}, \mathfrak{I}) \to (\mathcal{V}, \mathcal{N}', \mathfrak{K})$ be a $n\mathfrak{I}$-wg -continuous function and $\mathcal{A}$ be a Nano open in $\mathcal{V}$, Then $\mathcal{V} - \mathcal{A}$ is Nano closed. Since φ is $n\mathfrak{I}$-wg -continuous $\mathcal{V} - \mathcal{A}$ is $n\mathfrak{I}$-wg -closed. $\varphi^{-1}(\mathcal{V} - \mathcal{A}) = \mathcal{U} - \varphi^{-1}(\mathcal{A})$ is $n\mathfrak{I}$-wg-closed on $\mathcal{U}$. Therefore $\varphi^{-1}(\mathcal{A})$ is $n\mathfrak{I}$-wg-open.

Conversely let the inverse image of every Nano open set in $\mathcal{V}$ is $n\mathfrak{I}$-wg closed in $\mathcal{U}$. Let $\mathcal{B}$ be nano closed in $\mathcal{V}$. Then $\mathcal{V} - \mathcal{B}$ nano open in $\mathcal{V}$. Then $\varphi^{-1}(\mathcal{V} - \mathcal{B})$ is Nano open in $\mathcal{V}$. $\varphi^{-1}(\mathcal{V} - \mathcal{B}) = \mathcal{U} - \varphi^{-1}(\mathcal{B})$ is Nano open in $\mathcal{U}$. Therefore $\varphi^{-1}(\mathcal{B})$ is $n\mathfrak{I}$-wg-closed in $\mathcal{U}$. Thus, the inverse image of every nano closed set in $\mathcal{V}$ is $n\mathfrak{I}$-wg-closed in $\mathcal{U}$. Therefore, φ is $n\mathfrak{I}$-wg continuous on $\mathcal{U}$.

Proposition 4.3 A function $\varphi: (\mathcal{U}, \mathcal{N}, \mathfrak{I}) \to (\mathcal{V}, \mathcal{N}', \mathfrak{K})$is $n\mathfrak{I}$-wg -continuous function if and only if one of the following is satisfied.

i. $\varphi^{-1}(\mathcal{B})\in n\mathfrak{I}$-$wgC(\mathcal{U})$ for all $\mathcal{B} \in \mathcal{N}'$
ii. The inverse image of every member of the basis B of $\mathcal{N}'$ is $n\mathfrak{I}$-wg-open in $\mathcal{U}$.
iii. $n\mathfrak{I}$-wg-$cl(\varphi^{-1}(\mathcal{B})) \subseteq \varphi^{-1}(cl_n^* (\mathcal{B}))$, for all $\mathcal{B} \subseteq \mathcal{V}$.
iv. $\varphi^{-1}(Nint (\mathcal{B})) \subseteq n\mathfrak{I}$-$wg\ int\ (\varphi^{-1}(\mathcal{B}))$, for all $\mathcal{B} \subseteq \mathcal{V}$.

Proof:(i) Necessity: let φ be nano $n\mathfrak{I}$-wg-continuous and $\mathcal{B} \in \mathcal{N}'$. That is$\mathcal{V} - \mathcal{B} \in \mathcal{N}'^{C}$. Since φ is nano $n\mathfrak{I}$-wg-continuous, $\varphi^{-1}(\mathcal{V} - \mathcal{B}) \in n\mathfrak{I}$-$wgO(U)$. That is, $(\mathcal{U} - \varphi^{-1}(\mathcal{B})) \in n\mathfrak{I}$-$wgO(\mathcal{U})$.Therefore, $\varphi^{-1}(\mathcal{B}) \in n\mathfrak{I}$-$wgC(\mathcal{U})$. Thus, the inverse image of every nano closed set in $\mathcal{V}$ is $n\mathfrak{I}$-wg-closed in $\mathcal{U}$, if φ is $n\mathfrak{I}$-wg-continuous on $\mathcal{U}$. Sufficiency: let $\varphi^{-1}(\mathcal{B}) \in n\mathfrak{I}$-$wgC(\mathcal{U})$, for all $\mathcal{B} \in \mathcal{N}'$. Let $\mathcal{B} \in \mathcal{N}'$, then is$\mathcal{V} - \mathcal{B} \in \mathcal{N}'^{C}$and $\varphi^{-1}(\mathcal{V} - \mathcal{B}) \in n\mathfrak{I}$-$wgC(\mathcal{U})$. That is, $(\mathcal{U} - \varphi^{-1}(\mathcal{B})) \in n\mathfrak{I}$-$wgC(\mathcal{U})$and therefore $\varphi^{-1}(\mathcal{B}) \in n\mathfrak{I}$-$wgO(\mathcal{U})$. Thus, the inverse image of every nano-open set in $\mathcal{V}$ is $n\mathfrak{I}$-wg-open in $\mathcal{U}$. That is, φ is nano $n\mathfrak{I}$-wg-continuous on $\mathcal{U}$.

(ii) Necessity: let φ be nano $n\mathfrak{I}$-wg-continuous on $\mathcal{U}$. Let $\mathcal{B} \in \mathcal{B}'$. Then $\mathcal{B} \in \mathcal{N}'$. Since φ is$n\mathfrak{I}$-wg-continuous, $\varphi^{-1}(\mathcal{B})n\mathfrak{I}$-$wg\alpha O$ $(\mathcal{U})$. That is, the inverse image of every memberof B$\mathcal{B}'$ is $n\mathfrak{I}$-wg-open set in $\mathcal{U}$. Sufficiency: Let the inverse image of every member of $\mathcal{B}'$ be$n\mathfrak{I}$-wg-open set in $\mathcal{U}$. Let G be a nano-open set in V. Then $= \cup \{\mathcal{B}: \mathcal{B} \in \mathcal{B}_1\}$, were$\mathcal{B}_1 \in \mathcal{B}'$. Then $\varphi^{-1}(G) = \varphi^{-1}$ $(\cup \{\mathcal{B} : \mathcal{B} \in \mathcal{B}_1\}) = \cup \{\varphi^{-1}(\mathcal{B}) : \mathcal{B} \in \mathcal{B}_1\}$, where each$\varphi^{-1}(\mathcal{B}) \in n\mathfrak{I}$-$wgO(\mathcal{U})$and hence their union, which is$\varphi^{-1}(G)$ is $n\mathfrak{I}$-wg-open in $\mathcal{U}$. Therefore φ is $n\mathfrak{I}$-wg-continuous on $\mathcal{U}$.

(iii) Necessity: if f is nano $n\mathfrak{I}$-wg-continuous and $\mathcal{B} \subseteq \mathcal{V}, cl_n^*(\mathcal{B}) \in \mathcal{N}'$and from (i) $\varphi^{-1}(cl_n^*(\mathcal{B})) \in n\mathfrak{I}$-$wgC(\mathcal{U})$. Therefore, $n\mathfrak{I}$-wg-cl $(\varphi^{-1}(cl_n^*(\mathcal{B}))] = \varphi^{-1}(cl_n^*(\mathcal{B}))$. Since $\mathcal{B} \subseteq cl_n^*(\mathcal{B})$, $\varphi^{-1}(\mathcal{B}) \subseteq \varphi^{-1}(cl_n^*(\mathcal{B}))$. Therefore, $n\mathfrak{I}$-wg-cl $(\varphi^{-1}(\mathcal{B})) \subseteq n\mathfrak{I}$-$wg - cl(\varphi^{-1}(\varphi^{-1}(\mathcal{B})))$ $=\varphi^{-1}(cl_n^*(\mathcal{B}))$. That is, $n\mathfrak{I}$-$wg - cl(\varphi^{-1}(\mathcal{B})) \subseteq \varphi^{-1}(cl_n^*(\mathcal{B}))$. Sufficiency: let $n\mathfrak{I}$-$wg - cl(\varphi^{-1}(\mathcal{B})) \subseteq \varphi^{-1}(cl_n^*(\mathcal{B}))$ for every $\mathcal{B} \subseteq \mathcal{V}$. Let $\mathcal{B} \in \mathcal{N}'$, then $cl_n^*(\mathcal{B}) = \mathcal{B}$. By assumption, $n\mathfrak{I}$-$wg - cl(\varphi^{-1}(\mathcal{B})) \subseteq \varphi^{-1}(cl_n^*(\mathcal{B})) = \varphi^{-1}(\mathcal{B})$. Thus, $n\mathfrak{I}$-$wg - cl(\varphi^{-1}(\mathcal{B})) \subseteq \varphi^{-1}(\mathcal{B})$. But$\varphi^{-1}(\mathcal{B}) \subseteq n\mathfrak{I}$-$wg - cl(\varphi^{-1}(\mathcal{B}))$. Therefore, $n\mathfrak{I}$-$wg - cl(\varphi^{-1}(\mathcal{B})) = \varphi^{-1}(\mathcal{B})$. That is,$\varphi^{-1}(\mathcal{B})$ is $n\mathfrak{I}$-wg-closed in $\mathcal{U}$ for every nano closed set $\mathcal{B}$ in $\mathcal{V}$. Therefore, φ is $n\mathfrak{I}$-wgcontinuous on $\mathcal{U}$. 1915

(iv) Necessity: Let φ be $n\mathfrak{I}$-wg-continuous and $\mathcal{B} \subseteq \mathcal{V}$. Then $Nint(\mathcal{B}) \in \mathcal{N}'$. Therefore, $\varphi^{-1}(Nint(\mathcal{B})) \in n\mathfrak{I}$-$wgO(\mathcal{U})$. That is, $\varphi^{-1}(Nint(\mathcal{B})) = n\mathfrak{I}$-$wg - int(\varphi^{-1}(Nint(\mathcal{B}))$. Also,$Nint(\mathcal{B}) \subseteq \mathcal{B}$ implies that $n\mathfrak{I}$-$wg - int(\varphi^{-1}(Nint(\mathcal{B}) \subseteq n\mathfrak{I}$-$wg - int(\varphi^{-1}(\mathcal{B}))$. Therefore $\varphi^{-1}(Nint(\mathcal{B})) = n\mathfrak{I}$-$wg - int(\varphi^{-1}(Nint(\mathcal{B}))) \subseteq n\mathfrak{I}$-$wg$-$int(\varphi^{-1}(\mathcal{B}))$. That is,$\varphi^{-1}$ $[Nint\ (\mathcal{B})] \subseteq n\mathfrak{I}$-$wg\ int\ (\varphi^{-1}(\mathcal{B}))$. Sufficiency: φ^{-1} $[Nint\ (\mathcal{B})] \subseteq n\mathfrak{I}$-$wg\ int\ (\varphi^{-1}(\mathcal{B}))\varphi^{-1}(Nint(\mathcal{B}))\subseteq n\mathfrak{I}$-$wg - int(\varphi^{-1}(\mathcal{B}))$for every subset $\mathcal{B}$ of $\mathcal{V}$. If $\mathcal{B} \in \mathcal{N}', \mathcal{B} = NInt(\mathcal{B})$. Also, $\varphi^{-1}(\mathcal{B}) = \varphi^{-1}(Nint(\mathcal{B})), but \varphi^{-1}(Nint(\mathcal{B})) \subseteq$ $n\mathfrak{I}$-$wg\ int\ (\varphi^{-1}(\mathcal{B}))$. That is, $\varphi^{-1}(\mathcal{B}) = \varphi^{-1}(Nint(\mathcal{B})) \subseteq n\mathfrak{I}$-$wg\ int\ (\varphi^{-1}(\mathcal{B}))$. Therefore,$\varphi^{-1}(\mathcal{B}) = n\mathfrak{I}$-$wg\ int\ (\varphi^{-1}(\mathcal{B}))$. Thus, $\varphi^{-1}(\mathcal{B})$ is $n\mathfrak{I}$-wg-open in $\mathcal{U}$ for every nano-open set $\mathcal{B}$ in $\mathcal{V}$. Therefore, φ is $n\mathfrak{I}$-wg-continuous.

Theorem 4.4: Every nano continuous function is $n\mathfrak{I}$-wg-continuous function.

Proof: Let $\varphi:(\mathcal{U},\mathcal{N},\mathfrak{I}) \to (\mathcal{V},\mathcal{N}',\mathfrak{K})$be a Nano continuous function. Let $\mathcal{A}$ be a Nano closed set in the nano ideal topological space$(\mathcal{V},\mathcal{N}',\mathfrak{K})$. Then the inverse image of $\mathcal{A}$ under the map φ is Nano closed in the nano ideal topological space$(\mathcal{U},\mathcal{N},\mathfrak{I})$. Since every Nano closed set is $n\mathfrak{I}$-wg-closed set. Hence φ is $n\mathfrak{I}$-wg-continuous.

Theorem 4.5: Every nano$\mathfrak{I}\alpha$-continuous function is $n\mathfrak{I}$-wg- continuous function.

Proof: The proof of the theorem follows from the fact that every α-$n\mathfrak{I}$- closed set in a nano ideal topological spaceis $n\mathfrak{I}$-wg-closed set.

Definition 4.6 The map $\varphi:(\mathcal{U},\mathcal{N},\mathfrak{I}) \to (\mathcal{V},\mathcal{N}',\mathfrak{K})$is $n\mathfrak{I}$-wg- irresolute on $\mathcal{U}$ if the inverse image of $n\mathfrak{I}$-wg-closed set in $\mathcal{V}$ is $n\mathfrak{I}$-wg-closed set in $\mathcal{U}$.

Theorem 4.7: Composition of two $n\mathfrak{I}$-wg-irresolut functions is again a $n\mathfrak{I}$-wg- irresolute function

Proof: Let $\varphi:(\mathcal{U},\mathcal{N},\mathfrak{I}) \to (\mathcal{V},\mathcal{N}',\mathfrak{K})$and $\psi:(\mathcal{V},\mathcal{N}',\mathfrak{K}) \to (\mathcal{W},\mathcal{N}'',\mathfrak{L})$ are any two $n\mathfrak{I}$-wg-irresolute functions. Let $\mathcal{A}$ be a $n\mathfrak{I}$-wg-closed set in $\mathcal{W}$. Then $\psi^{-1}(\mathcal{A})$ is $n\mathfrak{I}$-wg-closed set in $\mathcal{V}$. Since g is $n\mathfrak{I}$-wg-irresolute function. $\varphi^{-1}(\psi^{-1}(\mathcal{A}))$ is $n\mathfrak{I}$-wg-closed set in $\mathcal{U}$. Since φ is $n\mathfrak{I}$-wg-irresolute function. Hence the composition $\psi \circ \varphi:(\mathcal{U},\mathcal{N},\mathfrak{I}) \to (\mathcal{W},\mathcal{N}'',\mathfrak{L})$ is $n\mathfrak{I}$-wg-irresolute function.

Proposition 4.8 A function $\varphi:(\mathcal{U},\mathcal{N},\mathfrak{I}) \to (\mathcal{V},\mathcal{N}',\mathfrak{K})$ is nano $n\mathfrak{I}$-wg-irresolute continuous function if and only if one of the following is satisfied.

i. $\varphi^{-1}(\mathcal{B}) \in n\mathfrak{I}$-$wgC(\mathcal{U})$ for all $\mathcal{B} \in n\mathfrak{I}$-$wgC(\mathcal{V})$
ii. The inverse image of every member of the basis B of $n\mathfrak{I}$-wg-open set of $\mathcal{V}$ $is n\mathfrak{I}$-wg-open set in $\mathcal{U}$.
iii. $n\mathfrak{I}$-wg-$cl(\varphi^{-1}(\mathcal{B})) \subseteq \varphi^{-1}(n\mathfrak{I}$-$wg$-$cl(\mathcal{B}))$, for all $\mathcal{B} \subseteq \mathcal{V}$.
iv. φ^{-1} $(n\mathfrak{I}$-wg -int $(\mathcal{B})) \subseteq n\mathfrak{I}$-$wg$ int $(\varphi^{-1}(\mathcal{B}))$, for all $\mathcal{B} \subseteq \mathcal{V}$.

Proof

i. Necessity: let φ be nano $n\mathfrak{I}$-wg-irresolutecontinuous and $\mathcal{B} \in n\mathfrak{I}$-$wgC(\mathcal{V})$. That is, $(\mathcal{V} - \mathcal{B}) \in n\mathfrak{I}$-$wgO(V)$. Since φ is nano $n\mathfrak{I}$-wg -irresolute continuous,$\varphi^{-1}(\mathcal{V} - \mathcal{B}) \in n\mathfrak{I}$-$wgO(\mathcal{V})$.That is, $(\mathcal{U}$ - $\varphi^{-1}(\mathcal{B}) \in n\mathfrak{I}$-$wgC(\mathcal{U})$, and therefore $\varphi^{-1}(\mathcal{B}) \in n\mathfrak{I}$-$wgC(\mathcal{U})$. Thus, $\varphi^{-1}(\mathcal{B}) \in n\mathfrak{I}$-$wgC(\mathcal{U})$, for all $\mathcal{B} \in n\mathfrak{I}$-$wgC(\mathcal{V})$, if φ is nano $n\mathfrak{I}$-wg-irresolute continuous on $\mathcal{U}$.Sufficiency: let $\varphi^{-1}(\mathcal{B}) \in n\mathfrak{I}$-$wgC(\mathcal{U})$, for all $\mathcal{B} \in n\mathfrak{I}$-$wgC(\mathcal{V})$. Let $\varphi^{-1}(\mathcal{B}) \in n\mathfrak{I}$-$wgC(\mathcal{U})$. Then $(\mathcal{V} - \mathcal{B}) \in n\mathfrak{I}$-$wgC(\mathcal{V})$. Then, $\varphi^{-1}(\mathcal{V} - \mathcal{B}) \in n\mathfrak{I}$-$wgO(\mathcal{V})$, that is, $(\mathcal{U}$ - $\varphi^{-1}(\mathcal{B}) \in n\mathfrak{I}$-$wgC(\mathcal{U})$. Therefore$\varphi^{-1}(\mathcal{B}) \in n\mathfrak{I}$-$wgO(\mathcal{U})$ for all $\mathcal{B} \in n\mathfrak{I}$-$wgO(\mathcal{V})$. That is,$\varphi$ is $n\mathfrak{I}$-wg-irresolute continuous on $\mathcal{U}$.
ii. Necessity: Let φ be $n\mathfrak{I}$-wg-irresolute continuous on $\mathcal{U}$. Let $\mathcal{B} \in \mathcal{B}'$. Then $\mathcal{B} \in n\mathfrak{I}$-$wg$O $(\mathcal{U})$. Since φ is $n\mathfrak{I}$-wg-irresolute continuous, $\varphi^{-1}(\mathcal{B}) \in n\mathfrak{I}$-$wgO(U)$. That is, theinverse image of every member of $\mathcal{B}'$ is $n\mathfrak{I}$-$wgO(\mathcal{U})$. Sufficiency: Let the inverse image ofevery member of $\mathcal{B}'$ be $n\mathfrak{I}$-wg-open set in $\mathcal{U}$. Let $G \in n\mathfrak{I}$-$wgO(\mathcal{U})$. Then $= \cup\{\mathcal{B}: \mathcal{B} \in \mathcal{B}_1$, where $\mathcal{B}_1 \in \mathcal{B}'$. Then $\varphi^{-1}(G) = \varphi^{-1}$ $(\cup\{\mathcal{B} : \mathcal{B} \in \mathcal{B}_1\}) = \cup\{\varphi^{-1}(\mathcal{B}) : \mathcal{B} \in \mathcal{B}_1\}$, where each $\varphi^{-1}(\mathcal{B}) \in n\mathfrak{I}$-$wgO(\mathcal{U})$ and hence their union, which is $\varphi^{-1}(G)$. Thus φ is nano $n\mathfrak{I}$-wg -irresolute continuous on $\mathcal{U}$.
iii. Necessity: If $\mathcal{U}$ is $n\mathfrak{I}$-wg -irresolute continuous and $\mathcal{B} \subseteq \mathcal{V}$,$n\mathfrak{I}$-wg-$cl(B) \in n\mathfrak{I}$-$wgC(\mathcal{V})$and from (i) φ^{-1} $(n\mathfrak{I}$-wg-$cl(B)) \in n\mathfrak{I}$-wgC$(\mathcal{U})$. Therefore, $n\mathfrak{I}$-wg-$cl(\varphi^{-1}(n\mathfrak{I}$-$wg$-$cl(B))=$ $\varphi^{-1}(n\mathfrak{I}$-$wg$-$cl(\mathcal{B}))$. Since $\mathcal{B} \subseteq n\mathfrak{I}$-$wg$-$cl(\mathcal{B})$, $\varphi^{-1}(\mathcal{B}) \subseteq \varphi^{-1}$ $(n\mathfrak{I}$-wg-$cl(\mathcal{B}))$. Therefore, $n\mathfrak{I}$-wg-$cl(\varphi^{-1}(\mathcal{B})) \subseteq n\mathfrak{I}$-$wg$-$cl$ $(\varphi^{-1}$ $(n\mathfrak{I}$-wg-$cl(\mathcal{B})) = \varphi^{-1}(n\mathfrak{I}$-$wg$-$cl(\mathcal{B}))$. That is, $n\mathfrak{I}$-wg-cl $(\varphi^{-1}(\mathcal{B})) \subseteq \varphi^{-1}(n\mathfrak{I}$-$wg$-$cl(\mathcal{B}))$.
iv. Sufficiency: Let $n\mathfrak{I}$-wg-$cl(\varphi^{-1}(\mathcal{B})) \subseteq$ $\varphi^{-1}(n\mathfrak{I}$-$wg$-$cl(B))$ for every $\mathcal{B} \subseteq \mathcal{V}$. Let $\mathcal{B} \in n\mathfrak{I}$-$wgC(\mathcal{V})$.Then $n\mathfrak{I}$-wg-$cl(\mathcal{B}) = \mathcal{B}$. By assumption, $n\mathfrak{I}$-wg-$cl(\varphi^{-1}(\mathcal{B})) \subseteq \varphi^{-1}(n\mathfrak{I} - wg - cl(\mathcal{B}))$ $= \varphi^{-1}(\mathcal{B})$.Thus,$n\mathfrak{I}$-$wg$-$cl(\varphi^{-1}(\mathcal{B}))$ $\subseteq \varphi^{-1}(\mathcal{B})$. But$\varphi^{-1}(\mathcal{B}) \subseteq n\mathfrak{I}$-$wg$-$cl(\varphi^{-1}(\mathcal{B}))$. Therefore, $n\mathfrak{I}$-wg-$cl(\varphi^{-1}(\mathcal{B})) =$ $\varphi^{-1}(\mathcal{B})$. That is, $\varphi^{-1}(\mathcal{B})$is $n\mathfrak{I}$-wg-closed in $\mathcal{U}$ for every $n\mathfrak{I}$-wg-closed set $\mathcal{B}$ in $\mathcal{V}$.Therefore, φ is $n\mathfrak{I}$-wg-irresolute continuous on $\mathcal{U}$. 1916
v. Necessity: Letφbe $n\mathfrak{I}$-wg-irresolute and $\mathcal{B} \subseteq \mathcal{V}$. Then $n\mathfrak{I}$-wg-$int(\mathcal{B}) \in n\mathfrak{I}$-$wgO(\mathcal{U})$. Therefore, $(\varphi^{-1}(n\mathfrak{I}$-$wg$-$int(\mathcal{B})) \in$ $n\mathfrak{I}$-$wgO(\mathcal{U})$. That is,$n\mathfrak{I}$-wg-$int(\varphi^{-1}(n\mathfrak{I}$-$wg$-$int(B)) = \varphi^{-1}(n\mathfrak{I}$-$wg$-$int(\mathcal{B}))$. Also, $n\mathfrak{I}$-wg-$int(\mathcal{B}) \subseteq \mathcal{B}$ implies that $n\mathfrak{I}$-wg-$int(\varphi^{-1}(n\mathfrak{I}$-$wg$-$int(B)) \subseteq n\mathfrak{I}$-$wg$-$int(\varphi^{-1}((B))$. Therefore $\varphi^{-1}(n\mathfrak{I}$-$wg$-$int(B) = n\mathfrak{I}$-$wg$-$int(\varphi^{-1}(n\mathfrak{I}$-$wg$-$int(B)) \subseteq n\mathfrak{I}$-$wg$-$int(\varphi^{-1}((B))$.That is,$\varphi^{-1}(n\mathfrak{I}$-$wg$-$int(B) \subseteq n\mathfrak{I}$-$wg$-$int(\varphi^{-1}((B))$. Sufficiency: Consider $\varphi^{-1}(n\mathfrak{I}$-$wg$-$int(\mathcal{B}) \subseteq n\mathfrak{I}$-$wg$-$int(\varphi^{-1}((B))$ for every subset $\mathcal{B}$ of $\mathcal{V}$. If $\mathcal{B} \in n\mathfrak{I}$-$wgO(\mathcal{V}), \mathcal{B} =$ $n\mathfrak{I}$-wg-$int((\mathcal{B}))$Also,$\varphi^{-1}(\mathcal{B}) = \varphi^{-1}(n\mathfrak{I}$-$wg$-$int(\mathcal{B})$ but, $\varphi^{-1}(n\mathfrak{I}$-$wg$-$int(B) \subseteq n\mathfrak{I}$-$wg$-$int(\varphi^{-1}((B))$. That is, $\varphi^{-1}(\mathcal{B}) = n\mathfrak{I}$-$wg$-$int(\varphi^{-1}((B))$. Thus,$\varphi^{-1}(\mathcal{B})$ is$n\mathfrak{I}$-wg -open in

$\mathcal{U}$ for every $n\mathfrak{I}$-wg -open set $\mathcal{B}$ in $\mathcal{V}$. Therefore, φ is $n\mathfrak{I}$-wg-irresolute continuous.

Theorem 4.9: If $\varphi: (\mathcal{U}, \mathcal{N}, \mathfrak{I}) \to (\mathcal{V}, \mathcal{N}', \mathfrak{K})$is $n\mathfrak{I}$-wg-irresolute function then φ is $n\mathfrak{I}$-wg-continuous function.

Proof: Let $\mathcal{A}$ be a Nano-closed set in $\mathcal{V}$ then $\mathcal{A}$ is $n\mathfrak{I}$-wg-closed set. Since every Nano closed set is $n\mathfrak{I}$-wg-closed set. Then $\varphi^{-1}(\mathcal{A})$ is $n\mathfrak{I}$-wg-closed set in $\mathcal{U}$, since φ is $n\mathfrak{I}$-wg-irresolute function. Hence φ is $n\mathfrak{I}$-wg-continuous function.

Definition 4.10 The map $\varphi: (\mathcal{U}, \mathcal{N}, \mathfrak{I}) \to (\mathcal{V}, \mathcal{N}', \mathfrak{K})$ be a function. φ is called $n\mathfrak{I}$-wg open [closed] map, if $\varphi(\mathcal{A}) \in n\mathfrak{I}$-$wgO(\mathcal{V})$, for all $\mathcal{A} \in \mathcal{N}$.[$\varphi(\mathcal{A}) \in n\mathfrak{I}$-$wgC(\mathcal{V})$, for all$\mathcal{A} \in \mathcal{N}'$)**].**

Theorem 4.11 Every nano closed map is $n\mathfrak{I}$-wg-closed map.

Proof

Let $\varphi: (\mathcal{U}, \mathcal{N}, \mathfrak{I}) \to (\mathcal{V}, \mathcal{N}', \mathfrak{K})$be a nano closed map. Let $\mathcal{A}$ be a nano closed set in $(\mathcal{U}, \mathcal{N}, \mathfrak{I})$.Then the image of $\mathcal{A}$ under the map φ is nano closed $(\mathcal{V}, \mathcal{N}', \mathfrak{K})$. Since every nano closed set is $n\mathfrak{I}$-wg-closed. $\varphi(\mathcal{A})$is $n\mathfrak{I}$-wg - closed. Hence φ is $n\mathfrak{I}$-wg – closed.

Theorem 4.12 Let $\varphi: (\mathcal{U}, \mathcal{N}, \mathfrak{I}) \to (\mathcal{V}, \mathcal{N}', \mathfrak{K})$ be function, if and only if for each subset $\mathcal{A}$ of $\mathcal{N}$ and for each nano open set $\mathcal{B}$ containing $\varphi^{-1}(\mathcal{A})$there is a $n\mathfrak{I}$-wg - open set $\mathcal{B}$of $\mathcal{V}$such that $\mathcal{B} \subseteq \mathcal{V}, \varphi^{-1}(\mathcal{V}) \subset \mathcal{U}$. $\mathcal{B}$ is an open set of $\mathcal{N}$such that $\varphi^{-1}(\mathcal{V}) \subset \mathcal{U}$. Then $\mathcal{B} = \mathcal{V} - f(\mathcal{U} - \mathcal{A})$ is a $n\mathfrak{I}$-wg-open set containing $\mathcal{B}$ such that $\varphi^{-1}(\mathcal{B}) \subset \mathcal{A}$.

Proof

Suppose $\varphi^{-1}$is $n\mathfrak{I}$-wg - closed. Let B be a subset of $\mathcal{V}$ and $\mathcal{A}$ is an open set of $\mathcal{U}$such that $\varphi^{-1}(\mathcal{V}) \subset \mathcal{U}$. Then $\mathcal{B} = \mathcal{V} - \varphi(\mathcal{U} - \mathcal{A})$ is a $n\mathfrak{I}$-wg - open set containing $\mathcal{B}$ such that $\varphi^{-1}(\mathcal{B}) \subset \mathcal{A}$.

Conversely, suppose that $\mathcal{C}$ is closed subset of $\mathcal{N}$. Then $\varphi^{-1}(\mathcal{V}) - \varphi(\mathcal{C}) \subset \mathcal{U} - \mathcal{C}$ and $\mathcal{U} - \mathcal{C}$is nano open. By hypothesis there is $n\mathfrak{I}$-wg- open set $\mathcal{B}$of $\mathcal{V}$such that $\mathcal{V} - \varphi(\mathcal{C}) \subset \mathcal{B}$, $\varphi^{-1}(\mathcal{B}) \subset \mathcal{U} - \mathcal{C}$. Therefore $\mathcal{C} \subset \mathcal{U} - \varphi^{-1}(\mathcal{B})$. Hence$\mathcal{V} - \mathcal{B} \subset \varphi(\mathcal{C}) \subseteq \varphi(\mathcal{U} - \varphi^{-1}(\mathcal{B})) \subset \mathcal{V} - \mathcal{B}$ which implies that $= \mathcal{V} - \mathcal{B}$. Since$\mathcal{V} - \mathcal{B} n\mathfrak{I}$-$wg$- closed set, $\varphi(\mathcal{C})$is $n\mathfrak{I}$-wg-closed. Hence φ is $n\mathfrak{I}$-wg - closed.

Definition 4.13 Let $\varphi: (\mathcal{U}, \mathcal{N}, \mathfrak{I}) \to (\mathcal{V}, \mathcal{N}', \mathfrak{K})$is bijective function, is said to be to be $n\mathfrak{I}$-wg homeomorphism if φ and φ^{-1} are both $n\mathfrak{I}$-wg continuous functions.

Remark 4.14. Let $\varphi: (\mathcal{U}, \mathcal{N}, \mathfrak{I}) \to (\mathcal{V}, \mathcal{N}', \mathfrak{K})$ be a bijective function. φ is said to be $n\mathfrak{I}$-wg-homeomorphism function if φ is both $n\mathfrak{I}$-wg-continuous and $n\mathfrak{I}$-wg-open function.

Theorem 4.15 Every nano homeomorphism is $n\mathfrak{I}$-wg- homeomorphism.

Proof

Let $\varphi: (\mathcal{U}, \mathcal{N}, \mathfrak{I}) \to (\mathcal{V}, \mathcal{N}', \mathfrak{K})$be a nano homeomorphism. By the definition $\varphi$is one to one and onto, nano continuous and nano open map. Since every nano continuous is $n\mathfrak{I}$-wg- continuous and every nano open map is $n\mathfrak{I}$-wg- open map. φ is $n\mathfrak{I}$-wg homeomorphism.

Remark 4.16 The converse of the above theorem need not be true

Example 4.17 Consider the nano ideal topological spaces $(\mathcal{U}, \mathcal{N}, \mathfrak{I})$ and $(\mathcal{V}, \mathcal{N}^{'}, \mathfrak{K})$. Such that Let $\mathcal{U} = \{\hbar_1, \hbar_2, \hbar_3, \hbar_4\}$, $\mathcal{U}/\mathcal{R} = \{\{\hbar_1\}, \{\hbar_3\}, \{\hbar_2, \hbar_4\}\}, \mathcal{X} = \{\hbar_1, \hbar_2\} \subseteq \mathcal{U}, \mathcal{N} = \{\mathcal{U}, \emptyset, \{\hbar_1\}, \{\hbar_1, \hbar_2, \hbar_4\}, \{\hbar_2, \hbar_4\}\}$ and ideal $\mathfrak{I} = \varphi, \{\hbar_1\}$, and let $\mathcal{V} = \{k_1, k_2, k_3, k_4,\}, \mathcal{V}/\mathcal{R}' = \{\{k_1\}, \{k_2, k_3\}, \{k_4\}\}, \mathcal{Y} = \{k_1, k_3\} \subseteq \mathcal{V}, \mathcal{N} = \{\mathcal{U}, \emptyset, \{k_1\}, \{k_2, k_3\}, \{k_1, k_2, k_3\}\}$ and ideal $\mathfrak{I} = \varphi, \{k_1\}$. Define a function $\varphi: (\mathcal{U}, \mathcal{N}, \mathfrak{I}) \to (\mathcal{V}, \mathcal{N}', \mathfrak{K})$, such that $\varphi(\hbar_1) = k_2, \varphi(\hbar_2) = k_3, \varphi(\hbar_3) = k_4,\ \varphi(\hbar_4) = k_1$. The function$\varphi$ is $n\mathfrak{I}$-wg- homeomorphism. But not nano homeomorphism.

Application

In this section we find two directed graphs are whether isomorphic or not by using nano continuous, nano homeomorphism and $n\mathfrak{I}$-wg-homeomorphism. That is, we are formalizing the structural equivalence for the graphs and their corresponding ideal nano topologies generated by them. Nano homeomorphism between two nano topological spaces is said to be topologically equivalent. Here we are formalizing the structural equivalence for the graphs and their corresponding nano topologies and nano ideal topologies generated by them.

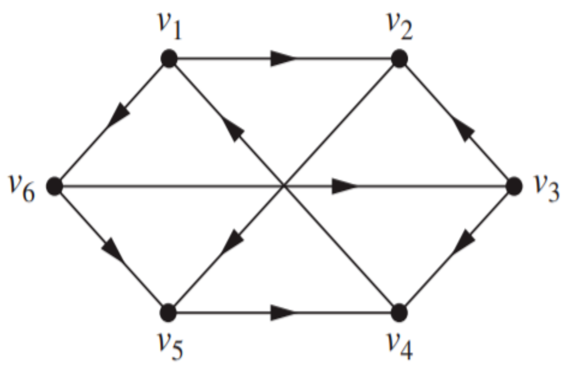

Figure 1. G

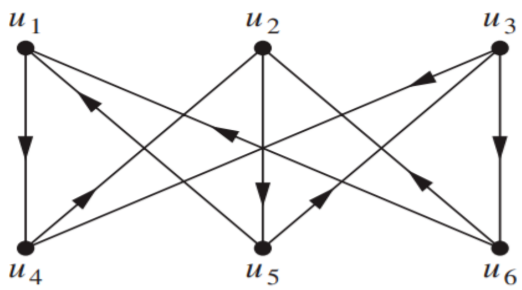

Figure 2. H

Figure 1 shows the simple directed graphs $\mathbb{G}$, where $V(\mathbb{G}) = \{\mathcal{V}_1, \mathcal{V}_2, \mathcal{V}_3, \mathcal{V}_4, \mathcal{V}_5, \mathcal{V}_6\}$ and from the Figure 1 $N(\mathcal{V}_1) = \{\mathcal{V}_1, \mathcal{V}_2, \mathcal{V}_6\}, N(\mathcal{V}_2) = \{\mathcal{V}_2, \mathcal{V}_5\}, N(\mathcal{V}_3) = \{\mathcal{V}_2, \mathcal{V}_3, \mathcal{V}_4\}, N(\mathcal{V}_4) = \{\mathcal{V}_1, \mathcal{V}_4\}$, $N(\mathcal{V}_5) = \{\mathcal{V}_4, \mathcal{V}_5\}$ and $N(\mathcal{V}_6) = \{\mathcal{V}_3, \mathcal{V}_5, \mathcal{V}_6\}$. Let $\mathcal{X} = \{\mathcal{V}_2\}$, then $\mathcal{L}(\mathcal{X}) = \{\mathcal{V}_2\}, \mathcal{U}(\mathcal{X}) = \{\mathcal{V}_2, \mathcal{V}_4, \mathcal{V}_5\}$ and $\mathcal{B}(\mathcal{X}) = \{\mathcal{V}_4, \mathcal{V}_5\}, \mathcal{N} = \{V(G), \emptyset, \{\mathcal{V}_2\}, \{\mathcal{V}_2, \mathcal{V}_4, \mathcal{V}_5\}, \{\mathcal{V}_4, \mathcal{V}_5\}\}$, take $\mathfrak{I} = \{\emptyset, \{\mathcal{V}_2\}\}$ Similarly from Figure 2 $V(\mathbb{H}) = \{U_1, U_2, U_3, U_4, U_5, U_6\}$, $N(U_1) = \{U_1, U_4\}$, $N(U_2) = \{U_2, U_5\}$, $N(U_3) = \{U_3, U_4, U_6\}$, $N(U_4) = \{U_2, U_4\}$, $N(U_5) = \{U_1, U_3, U_5\}$ and $N(U_6) = \{U_1, U_2, U_6\}$. Let $\mathcal{Y} = \{U_1, U_4\}$, then $\mathcal{L}(\mathcal{Y}) = \{U_1\}, \mathcal{U}(\mathcal{Y}) = \{U_1, U_2, U_4\}$ and $\mathcal{B}(\mathcal{Y}) = \{U_2, U_4\}$, which mean that $\mathcal{N}' = \{V(\mathbb{H}), \emptyset, \{U_1\}, \{U_1, U_2, U_4\}\{U_2, U_4\}\}$. Take $= \{\emptyset, \{U_1\}\}$.

Define a function $\varphi: (\mathcal{U}, \mathcal{N}, \mathfrak{I}) \to (\mathcal{V}, \mathcal{N}', \mathfrak{K})$such that $\varphi(\mathcal{V}_1) = U_5, \varphi(\mathcal{V}_2) = U_1, \varphi(\mathcal{V}_3) = U_6, \varphi(\mathcal{V}_4) = U_2, \varphi(\mathcal{V}_5) = U_4 \text{ and } \varphi(\mathcal{V}_6) = U_3$. Therefore, φ is nano continuous,$n\mathfrak{I}$-wg -continuous and $n\mathfrak{I}$-wg- irresolute, and this function is nano open, and $n\mathfrak{I}$-wg -open also this function is one to one and onto, therefore it is nano homeomorphism and $n\mathfrak{I}$-wg -homeomorphism. Therefore, the graph $\mathbb{G}$ and $\mathbb{H}$ are isomorphic in structure.

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