Analysis of Redundant System Subject to Degradation During Repair

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Abstract

In this paper, we have studied a two-unit cold standby redundant system subject to degradation during repair. In case, the repairman is unable to repair the system on his own, help from expert repairman is taken. The expert repairman helps in the repair of the system by providing instructions to the repairman. Using path analysis and regenerative point technique, various efficiency measures like MTSF, Availability analysis, Busy period of the repairman and the expert repairman, expected number of visits by the expert repairman. Graphical analysis has also been done.

Keywords: Failed unit, repaired unit, degradation, instruction, MTSF, Availability.
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1. Introduction

In this paper, we have considered a two-unit redundant system which may degrade further during the repair. By redundant system, we mean the use of more units than required for the satisfactory working of the system so as to enhance the reliability of the system. Here, we have considered a cold standby redundancy i.e. the extra unit won't fail while it is not operating. There is a repair facility available for the repair of the failed unit. In case, the repairman is unable to repair the failed unit, help from the expert is taken. The expert repairman does not repair the system on his own but gives instructions to the repairman to repair the failed unit. There might be stage during the system failure that the system degrades further while the repairman is repairing the failed unit, in that case the expert reanalyzes the situation and gives a new set of instruction to repair the failed unit.

Many researchers have worked on redundant systems. A single-unit model's profit was examined by S. Chander and R.K. Bansal (2005) when repairs were made at various failure scenarios. A reliability model for a two-out-of-three redundant system in which a unit deteriorated after repair was examined by S. Chander & Mukernder Singh (2009). One server handled both the inspection and repair duties. The system was considered to be in an up-state condition if any of the two original or deteriorated units functioned. The server looked at each unit that broke to see whether it could be fixed. A new unit was put in place of the damaged one if it could not be fixed. The original (new) unit superseded the deteriorating unit when it was in operation. A two-unit cold standby system with two repair phases was studied by El-Said & El-Sherbeny (2010). The full repair process for the failed unit was only supposed to be completed in the second phase. The duration that elapsed between two repair procedures is known as the waiting period. They developed a model and claimed that their results extended to the case of generic time distributions in all cases. Nevertheless, it appeared that they were unable to substantiate their claim because their model lacked a non-regenerative stage. In their work, Amit Manocha & Gulshan

Taneja (2015) addressed the aforementioned limitation. They believed that all distributions were general. A stochastic model for a system of producing utensils with preventative maintenance that is impacted by changes in temperature-dependent maintenance was analyzed by M. Gaba, D. Singh, and K. Sachdeva (2023). As a result, they took additional efficiency measures into account in addition to system availability during the summer and winter.

2. Description of model and Assumptions:

i. System is made up of two identical units: operating and cold standby.
ii. In the event that one unit fails, the backup unit will start up immediately while the malfunctioning unit is being repaired.
iii. In case of failure of both the units, the system will be in failed state.
iv. After the repair, the failed unit will behave like new one.
v. The time to failure for each unit is in exponential distribution and the repair time and instruction time are in arbitrary distribution.
vi. All the random variables are mutually independent.

3. Nomenclature

p probability that the repairman repairs system without instructions
q probability that the repairman fails to repair system without instructions
λ constant failure rate of the operative unit
O operative unit
CS cold standby
$g_1(t)$ p.d.f. of time to repair by repairman before instructions are given
$g_2(t)$ p.d.f. of time to repair by repairman after instructions are given
$G_1(t)$ c.d.f. of time to repair by repairman before instructions are given
$G_2(t)$ c.d.f. of time to repair by repairman after instructions are given
$i(t)$ p.d.f. when expert gives instructions to the repairman
$I(t)$ c.d.f. when expert gives instructions to the repairman
$\mathrm{F_{bi}}$ failed unit under the repair of repairman before getting instructions
$\mathrm{F_{ai}}$failed unit under the repair of repairman after getting instructions
$\mathrm{F_{BI}}$ repair by the repairman is continuing from the previous state before getting instructions
$\mathrm{F_{AI}}$ repair by the repairman is continuing from the previous state after getting instructions
$\mathrm{F_{I\alpha}}$ expert is giving instructions to the repairman
$\mathrm{F_{I\beta}}$ instructions by the expert are continuing from the previous state
$\mathrm{F_{wbi}}$ failed unit waiting for repair before getting instructions
$\mathrm{F_{wai}}$ failed unit waiting for repair after getting instructions

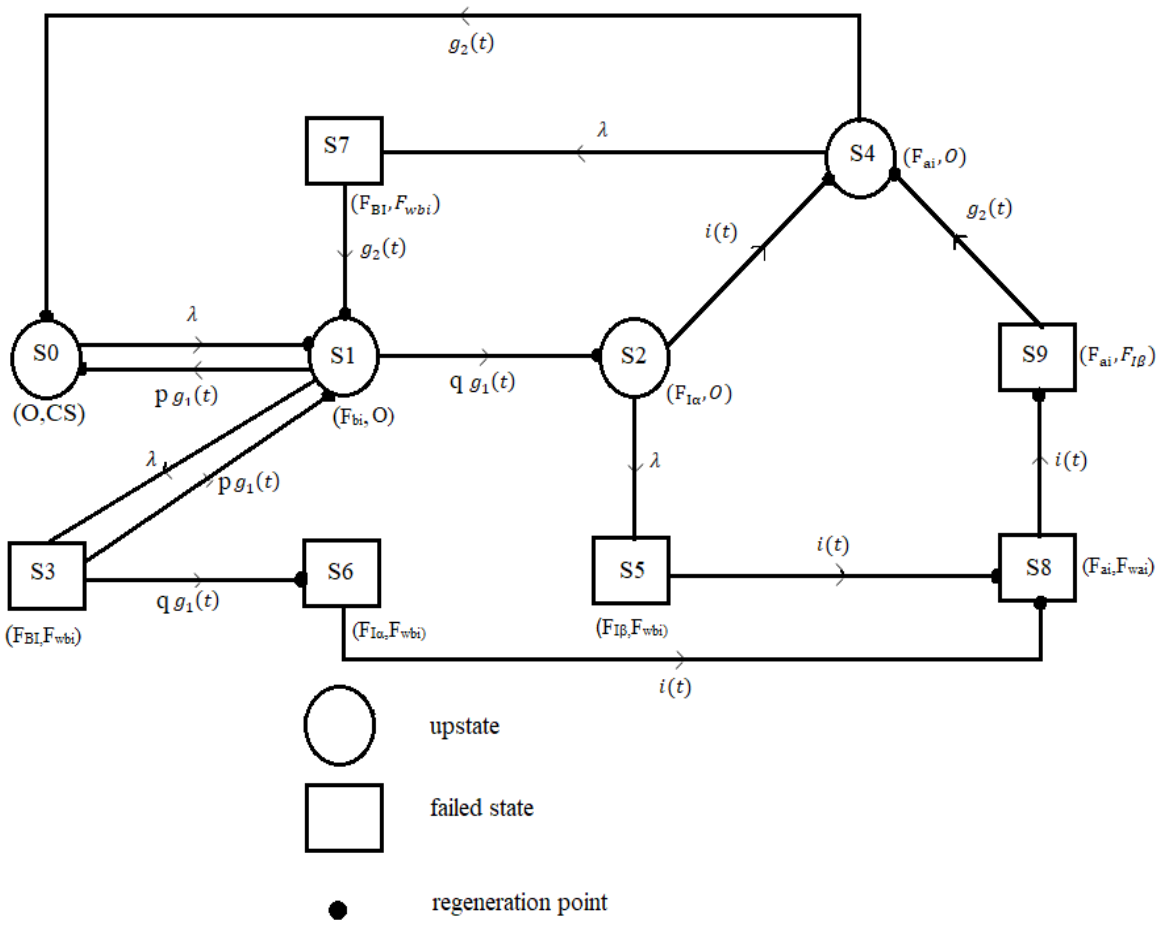

Fig. 1 (Transition Diagram)

4. Transition probabilities

The transition probabilities are:

$dQ_{01}(t) = \lambda e^{-\lambda t} dt$
$dQ_{10}(t) = p g_1(t) e^{-\lambda t} dt$
$dQ_{12}(t) = q g_1(t) e^{-\lambda t} dt$
$dQ_{13}(t) = \lambda e^{-\lambda t} \overline{G_1(t)} dt$
$dQ_{11}^{(3)}(t) = (\lambda e^{-\lambda t} \copyright p) g_1(t) dt = p(1 - e^{-\lambda t}) g_1(t) dt$
$dQ_{16}^{(3)}(t) = (\lambda e^{-\lambda t} \copyright q) g_1(t) dt = q(1 - e^{-\lambda t}) g_1(t) dt$
$dQ_{24}(t) = i(t) e^{-\lambda t} dt$
$dQ_{25}(t) = \lambda e^{-\lambda t} \overline{I(t)} dt$
$dQ_{28}^{(5)}(t) = (\lambda e^{-\lambda t} \copyright 1) i(t) dt = (1 - e^{-\lambda t}) i(t) dt$
$dQ_{40}(t) = g_2(t) e^{-\lambda t} dt$
$dQ_{41}^{(7)}(t) = (\lambda e^{-\lambda t} \copyright 1) g_2(t) dt = (1 - e^{-\lambda t}) g_2(t) dt$
$dQ_{47}(t) = \lambda e^{-\lambda t} \overline{G_2(t)} dt$
$dQ_{68}(t) = i(t) dt$
$\mathrm{d}Q_{89}(\mathrm{t}) = i(t) dt$
$dQ_{94}(t) = g_2(t) dt$

The non-zero elements p_{ij} are as follows:

$p_{01} = 1, \quad p_{10} = p g_1^*(\lambda), \quad p_{12} = q g_1^*(\lambda), \quad p_{13} = 1 - g_1^*(\lambda), \quad p_{11}^{(3)} = p(1 - g_1^*(\lambda)), p_{16}^{(3)} = q(1 - g_1^*(\lambda)), p_{24} = i^*(\lambda), p_{25} = 1 - i^*(\lambda), p_{28}^{(5)} = 1 - i^*(\lambda), p_{40} = g_2^*(\lambda), p_{41}^{(7)} = 1 - g_2^*(\lambda), p_{47} = 1 - g_2^*(\lambda), p_{58} = p_{68} = p_{71} = p_{89} = p_{94} = 1$

From the transition probabilities, it can be verified that

$p_{01} = 1$

$p_{10} + p_{12} + p_{13} = p_{10} + p_{12} + p_{11}^{(3)} + p_{16}^{(3)} = 1$
$p_{24} + p_{25} = p_{24} + p_{28}^{(5)} = 1$
$p_{40} + p_{47} = p_{40} + p_{41}^{(7)} = 1$
$p_{68} = p_{89} = p_{94} = 1$

5. Mean Sojourn Time

If T denotes mean sojourn time in state 0, then

$$\mu_0 = \int P(T > t)dt = \frac{1}{\lambda}$$
$$\mu_1 = \frac{1 - g_1^*(\lambda)}{\lambda}$$
$$\mu_2 = \frac{1 - i^*(\lambda)}{\lambda}$$
$$\mu_4 = \frac{1 - g_2^*(\lambda)}{\lambda}$$
$$\mu_8 = \int_0^\infty \overline{I(t)}dt = \int_0^\infty tdI(t) = \text{Mean instruction time}$$
$$\mu_9 = \int_0^\infty \overline{G_2(t)}dt$$

The unconditional mean time taken by the system to transit to any regenerative state i when it is counted from the epoch of entrance into that state is, mathematically, stated as

$$m_{ij} = \int_0^\infty tdQ_{ij}(t) = -\frac{d}{ds}q_{ij}^*|_{s=0}$$

Thus,

$m_{01} = \mu_0$
$m_{10} + m_{12} + m_{13} = \mu_1$
$m_{24} + m_{25} = m_{24} + m_{28}^{(5)} = \mu_2$
$m_{40} + m_{47} = m_{40} + m_{41}^{(7)} = \mu_4$
$m_{10} + m_{12} + m_{11}^{(3)} + m_{16}^{(3)} = k_1$

6. Mean time to system failure

To determine the MTSF of the system, we regard the failed states of the system as absorbing. By probabilistic arguments, we have

$\phi_0(t) = Q_{01}(t)Ⓢ\phi_1(t)$
$\phi_1(t) = Q_{13}(t) + Q_{12}(t)Ⓢ\phi_2(t) + Q_{10}(t)Ⓢ\phi_0(t)$
$\phi_2(t) = Q_{25}(t) + Q_{24}(t)Ⓢ\phi_4(t)$
$\phi_4(t) = Q_{47}(t) + Q_{40}(t)Ⓢ\phi_0(t)$

Taking the L.S.T. of the above equations and solving them for $\phi_0^{**}(s)$, we have

$$\phi_0^{**}(s) = \frac{Q_{01}^{**}(s)[Q_{13}^{**}(s) + Q_{12}^{**}(s)Q_{25}^{**}(s) + Q_{12}^{**}(s)Q_{24}^{**}(s)Q_{47}^{**}(s)]}{1 - Q_{01}^{**}(s)[Q_{10}^{**}(s) + Q_{12}^{**}(s)Q_{24}^{**}(s)Q_{40}^{**}(s)]}$$

Now the MTSF, given that the system started at the beginning of state 0 is

$$T_0 = \lim_{s\to 0}\frac{1 - \phi_0^{**}(s)}{s} = \frac{\mu_{0+}\mu_1 + P_{12}\mu_2 - P_{12}p_{24} + P_{12}p_{24}\mu_4}{p_{01} - p_{10} - p_{12}p_{24}p_{40}}$$

7. Availability Analysis

$M_i(t)$ denotes the probability that the system starting in up regenerative state is up at time t without passing through any regenerative state. Thus, we have

$M_0(t) = e^{-\lambda t}$
$M_1(t) = e^{-\lambda t}\overline{G_1(t)}$
$M_2(t) = e^{-\lambda t}\overline{I(t)}$
$M_4(t) = e^{-\lambda t}\overline{G_2(t)}$

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Taking Laplace transform of the above equations and solving them for $s \to 0$, we get
$M_0^*(0) = \mu_0, M_1^*(0) = \mu_1, M_2^*(0) = \mu_2, M_4^*(0) = \mu_4$
Using the arguments of the theory of regenerative processes, the availability $A_i(t)$ is seen to satisfy
$A_0(t) = M_0(t) + q_{01}(t) \odot A_1(t)$
$A_1(t) = M_1(t) + q_{10}(t) \odot A_0(t) + q_{11}^{(3)}(t) \odot A_1(t) + q_{12}(t) \odot A_2(t) + q_{16}^{(3)}(t) \odot A_6(t)$
$A_2(t) = M_2(t) + q_{24}(t) \odot A_4(t) + q_{28}^{(5)}(t) \odot A_8(t)$
$A_4(t) = M_4(t) + q_{40}(t) \odot A_0(t) + q_{41}^{(7)}(t) \odot A_1(t)$
$A_6(t) = q_{68}(t) \odot A_8(t)$
$A_8(t) = q_{89}(t) \odot A_9(t)$
$A_9(t) = q_{94}(t) \odot A_4(t)$
The steady state availability of the system is given by

$$A_0 = \lim_{s \to 0} s A_0^*(s) = \frac{N_1}{D_1}$$

Where $N_1 = \mu_0 \left[1 - p_{11}^{(3)} - p_{12}p_{28}^{(5)}p_{41}^{(7)} - p_{12}p_{24}p_{41}^{(7)} - p_{16}^{(3)}p_{41}^{(7)}\right] + \mu_1 + \mu_2 p_{12} + \mu_4 \left[p_{12}p_{24} + p_{12}p_{28}^{(5)} + p_{16}^{(3)}\right]$
and $D_1 = k_1 + \mu_0[p_{10} + p_{40}] + \mu_2 p_{12} + \mu_4 \left[p_{12} + p_{16}^{(3)}\right] + \mu_8 \left[p_{12}p_{28}^{(5)} + 2p_{16}^{(3)}\right] + \mu_9 \left[p_{12}p_{28}^{(5)} + p_{16}^{(3)}\right]$

8. Busy period analysis

Busy period of the repairman:

$B_0^r(t) = q_{01}(t) \odot B_1^r(t)$
$B_1^r(t) = W_1(t) + q_{10}(t) \odot B_0^r(t) + q_{11}^{(3)}(t) \odot B_1^r(t) + q_{12}(t) \odot B_2^r(t) + q_{16}^{(3)}(t) \odot B_6^r(t)$
$B_2^r(t) = q_{24}(t) \odot B_4^r(t) + q_{28}^{(5)}(t) \odot B_8^r(t)$
$B_4^r(t) = W_4(t) + q_{40}(t) \odot B_0^r(t) + q_{41}^{(7)}(t) \odot B_1^r(t)$
$B_6^r(t) = q_{68}(t) \odot B_8^r(t)$
$B_8^r(t) = q_{89}(t) \odot B_9^r(t)$
$B_9^r(t) = W_9(t) + q_{94}(t) \odot B_4^r(t)$
Where $W_1(t) = e^{-\lambda t}\overline{G_1(t)} + [\lambda e^{-\lambda t} \odot 1]\,\overline{G_1(t)}$, $W_4(t) = e^{-\lambda t}\overline{G_2(t)} + [\lambda e^{-\lambda t} \odot 1]\,\overline{G_2(t)}$,
$W_9(t) = \overline{G_2(t)}$
Taking Laplace transform of the above equations and solving them for $s \to 0$, we get
$W_1^*(0) = k_1, W_4^*(0) = \mu_4 = W_9^*(0)$
In the steady-state, the total fraction of time for which repairman is busy is

$$B_0^r = \lim_{s \to 0} s\, B_0^{r^*} = \frac{N_2}{D_1}$$

Where $N_2 = k_1 + \mu_4 \left[p_{12}p_{24} + 2p_{12}p_{28}^{(5)} + 2p_{16}^{(3)}\right]$ and D_1 is already mentioned above.

Busy period analysis for expert repairman

$B_0^i(t) = q_{01}(t) \odot B_1^i(t)$
$B_1^i(t) = q_{10}(t) \odot B_0^i(t) + q_{11}^{(3)}(t) \odot B_1^i(t) + q_{12}(t) \odot B_2^i(t) + q_{16}^{(3)}(t) \odot B_6^i(t)$
$B_2^i(t) = W_2(t) + q_{24}(t) \odot B_4^i(t) + q_{28}^{(5)}(t) \odot B_8^i(t)$
$B_4^i(t) = q_{40}(t) \odot B_0^i(t) + q_{41}^{(7)}(t) \odot B_1^i(t)$
$B_6^i(t) = W_6(t) + q_{68}(t) \odot B_8^i(t)$
$B_8^i(t) = W_8(t) + q_{89}(t) \odot B_9^i(t)$
$B_9^i(t) = q_{94}(t) \odot B_4^i(t)$
Where $W_2(t) = e^{-\lambda t}\overline{I(t)} + [\lambda e^{-\lambda t} \odot 1]\,\overline{I(t)} = \overline{I(t)}, W_6(t) = \overline{I(t)} = W_8(t)$
Taking Laplace transform of the above equations and solving them for $s \to 0$, we get

$$W_2^*(0) = W_6^*(0) = W_8^*(0) = \mu_2$$

In steady state, the total fraction of time for which the expert repairman is busy in giving instructions is

$$B_0^i = \lim_{s\to 0} s\, B_0^{i^*} = \frac{N_3}{D_1}$$

Where$N_3 = \mu_2\left[p_{12} + 2p_{16}^{(3)} + p_{12}p_{28}^{(5)}\right]$ and D_1 is same as above.

9. Expected number of visits by expert repairman

Using probabilistic arguments, we have the following recursive relations:

$V_0(t) = Q_{01}(t)\textcircled{S}[1 + V_1(t)]$

$V_1(t) = Q_{10}(t)\textcircled{S}V_0(t) + Q_{12}(t)\textcircled{S}V_2(t) + Q_{16}^{(3)}(t)\textcircled{S}V_6(t) + Q_{11}^{(3)}(t)\textcircled{S}V_1(t)$

$V_2(t) = Q_{21}(t)\textcircled{S}V_1(t) + Q_{24}(t)\textcircled{S}[1 + V_4(t)] + Q_{28}^{(5)}(t)\textcircled{S}V_8(t) + Q_{29}^{(5)}(t)\textcircled{S}V_9(t)$

$V_4(t) = Q_{40}(t)\textcircled{S}V_0(t) + Q_{41}^{(7)}(t)\textcircled{S}V_1(t)$

$V_6(t) = Q_{68}(t)\textcircled{S}V_8(t) + Q_{69}(t)\textcircled{S}[1 + V_9(t)]$

$V_8(t) = Q_{81}(t)\textcircled{S}V_1(t) + Q_{86}(t)\textcircled{S}V_6(t)$

$V_9(t) = Q_{91}(t)\textcircled{S}V_1(t)$

In steady-state, the number of visits per unit time is given by taking $s \to 0$ and $t \to \infty$

$$V_0 = \lim_{t\to\infty}\frac{V_0(t)}{t} = \lim_{s\to\infty}[sV_0^{**}(s)] = \frac{N_4}{D_1}$$

Where $N_4 = [1 - p_{68}p_{86}]\left[1 - p_{11}^{(3)} - p_{12}p_{21} + p_{12}p_{24}p_{40}\right] - p_{12}\left[p_{29}^{(5)} + p_{28}^{(5)}p_{81}\right] - p_{28}^{(5)}p_{69}p_{86}[1 - p_{12}] + p_{68}\left[p_{29}^{(5)}p_{86} - p_{16}^{(3)}p_{81}\right]$

And D_1 is same as above.

10. Cost-Benefit analysis

The expected cost-profit of the system in steady state is given by

$$P = C_0A_0 - C_1B_0^r - C_2B_0^i - C_3V_0$$

Where C_0 is revenue per unit up-time of the system.

C_1 is cost per unit time for which the assistant repairman is busy.

C_2 is cost per unit for which expert repairman is busy.

C_3 is cost per visit of expert repairman.

11. Graphical Analysis

AVAILABILITY VS. FAILURE RATE

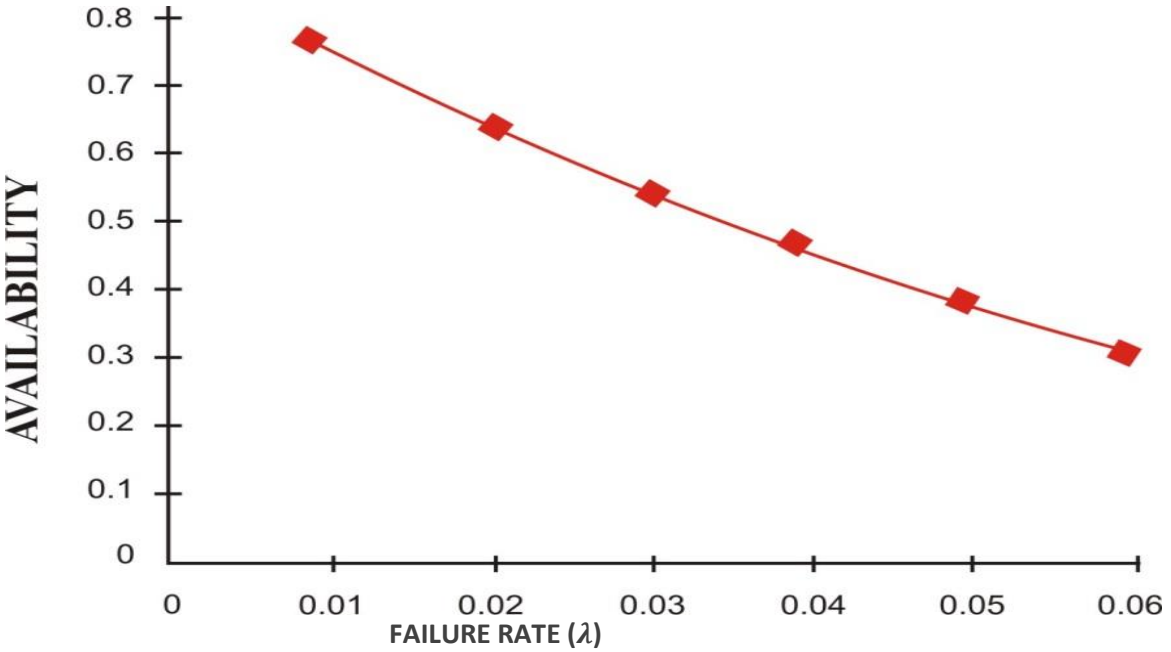

Fig. 2

AVAILABILITY VS. REPAIR RATE (α_1)

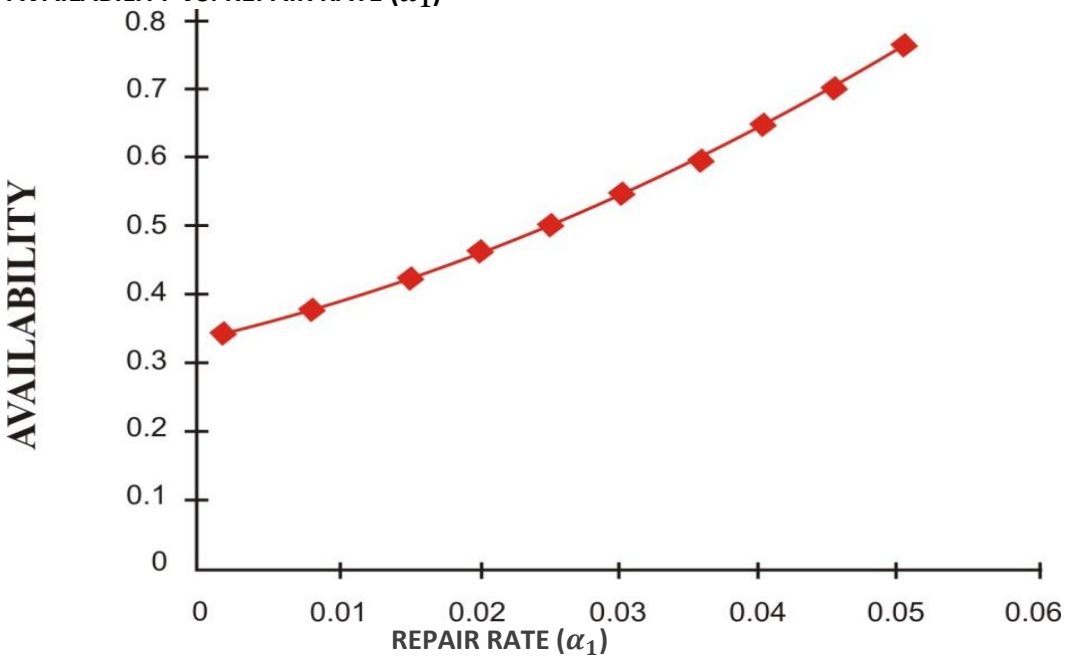

Fig.3

PROFIT Vs. REPAIR RATE (α_1)

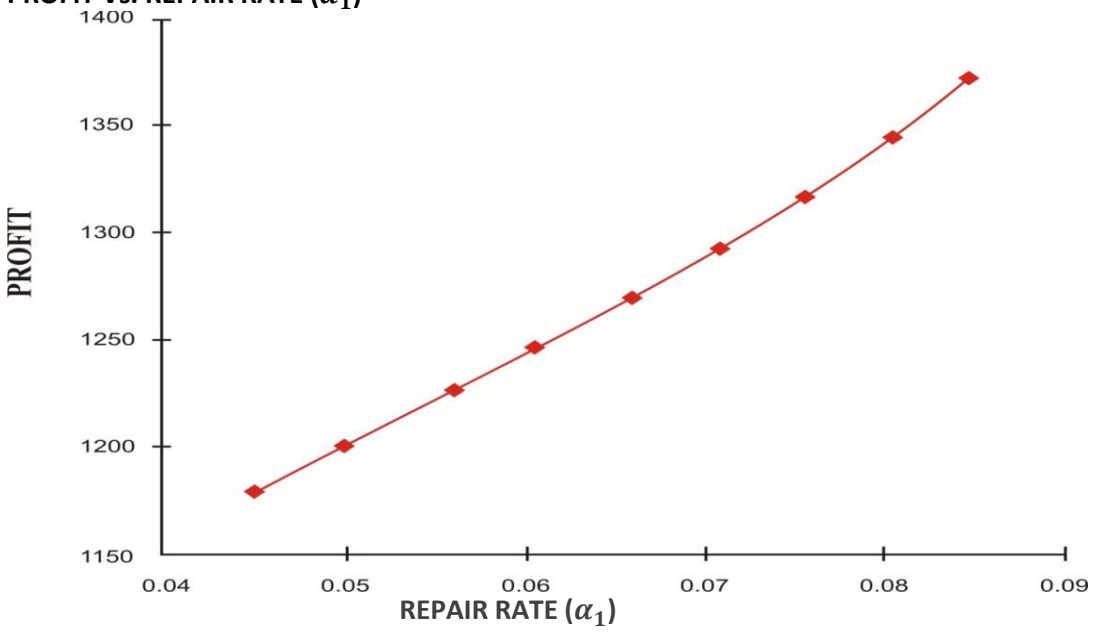

Fig.4

MTSF Vs. FAILURE RATE (λ) FOR DIFFERENT VALUES OF REPAIR RATE (α_1)

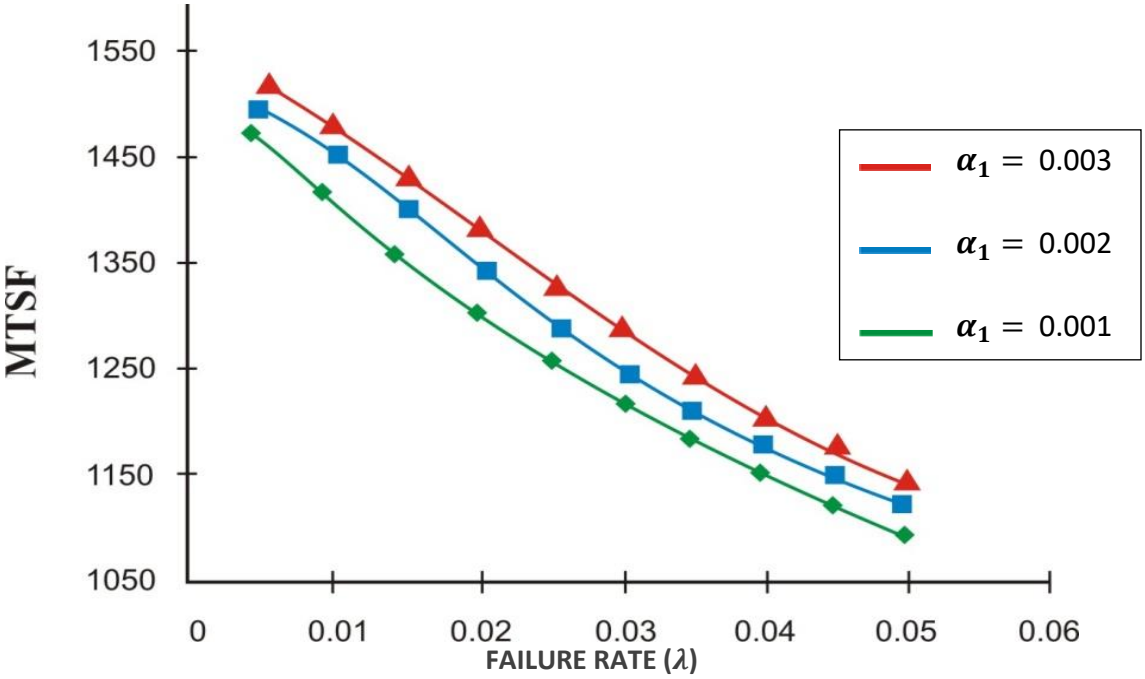

Fig.5

MTSF Vs REPAIR RATE (α_1) FOR DIFFERENT VALUES OF FAILURE RATE(λ)

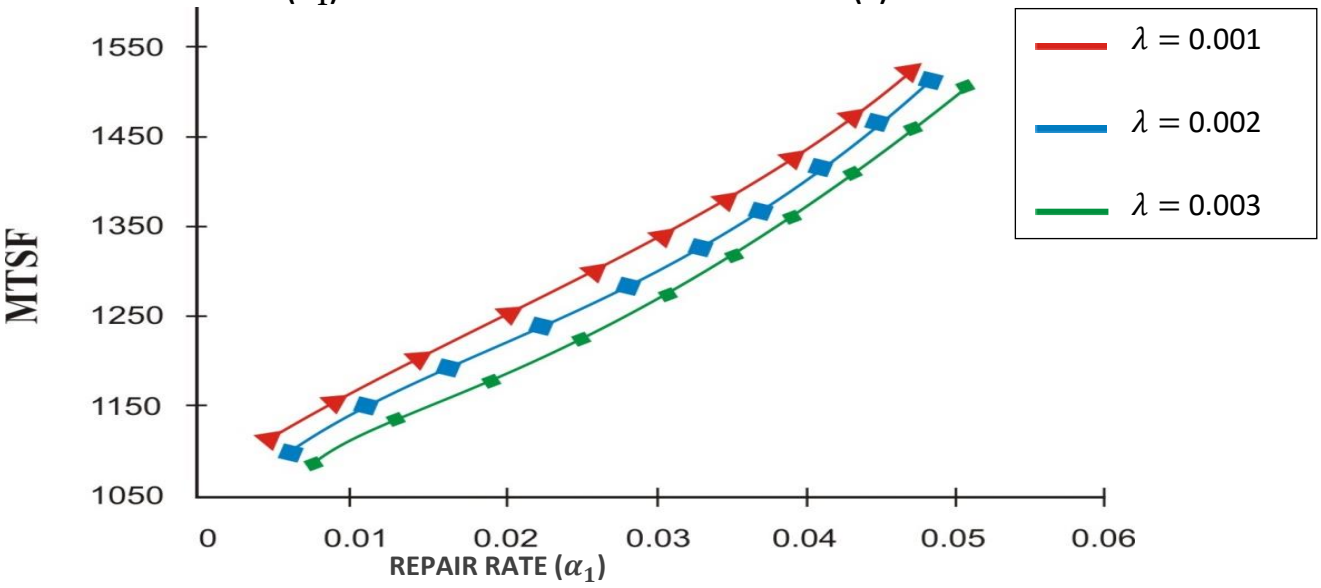

Fig.6

12. Conclusion

For a clearer view of the system characteristics w.r.t. various parameters involved, we plot various curves.

Fig. 2 and 3 shows the behavior of Availability with respect to the failure rate (λ) and repair rate (α_1). It is observed that Availability decrease with increase of failure rate and increases with increase of repair rate.

Fig. 4 reveals the pattern of Profit with respect to the repair rate (α_1). It is clear from the graph that Profit increase with increase of repair rate.

Fig. 5 shows the behavior of MTSF with respect to failure rate (λ) for different values of repair rate (α_1). It is observed that MTSF decrease with increase of failure rate and higher for higher values of repair rate.

Fig. 6 shows that MTSF increase with increase of repair rate rate (α_1)and higher for lower values of failure rate (λ).

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