ANALYSIS OF SINGLE SERVER QUEUING SYSTEM WITH BULK ARRIVAL AND REST PERIOD

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ABSTRACT

This paper deals with a queuing system where the server taken re-orientation time after each service. A single server serves customer one at one time basically first- come, first – servediscipline.A single- server queuing system is basic queuing system where customers willwait in a line to be served by a single server .Server serves one customer one at a time on a first –come, first-serve basis. When the customer gets service from the server they leave the queue and the number of customers in the system is reduced by one. A fast food shop queue is also an example of single server queue, which works on the first come-first serveprinciple, where the second customer is served after the first one is served and during the intervening period between two services there is rest period during which server re-orient itself after each service. Customers line up in this configuration, which simulates a simple single-server queue, and are served according to arrival order. Customers leave the queue after receiving service, which lowers the overall consumer base in the system.Customers arrive in batches of valuable size and served under general bulk service rule and re-orientation time in arbitrary with distribution function D(x). In this paper we have discussed about the steady state probability distribution, the mean queue length, average queue length and other operational characteristics.

KEYWORDS; Queuing System, Single Server, Bulk Arrival, FCFS, Bulk Service
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INTRODUCTION:

Most of the queuing system studied in the past, in which the server serves the customers within any rest.Ayyappam Govindam (2020) and et al have been studied the repairable single server bulk queue with standby server, two phase heterogeneous service, starting failure and multiple vacation. A bulk service queueing model featuring a primary server, backup server, two service phases, and starting failure under multiple vacations is analyzed in this paper. In order to support seamless production, the standby server runs while the primary server is being repaired for a starting failure. In the event that the queue hits 'a', the server may take a vacation when services are finished. A method using additional variables is suggested to calculate the distribution of queue length. Performance indicators, steady-state probability, specific instances, and stability requirements are defined. The analytical results are validated using numerical experiments.

Nirjana S P (2018) and et al has been studied the A single-server bulk arrival queue system with working vacation and batch size-dependent service is examined in this study. If queue ≥ 'a', the server moves to single service mode; if queue ≥ 'k', where k > a, the server switches to fixed batch service mode. After service, the server takes a working vacation and serves consumers at a reduced rate if the queue falls below 'a'. Using the supplemental variable technique, the queue length probability generating function of the model is obtained. Numerical examples are provided with performance measurements.

The queueing system may be described as follows

i) Arrival of customers follow Poisson process. In section 1 the arrivals occur batches of variable size with rate a is the probability that an arriving batch consists of j units. In section 2, the arrivals occur singly with rate a.

ii) The queue displine is (FCFS)

iii) Service facility consists of single server. In section 2, the service occurs under general bulk service rule, i.e., at each service initiation, the server can take no fewer than 'a' and not more than 'b' customers at a time. In section 2, the units are served one by one. In both the sections, the service time distribution is exponential with parameter ß (irrespective of batch size).

iv) The server takes rest after each service. The rest period is a random variable. In section 2, it is considered to be exponential with parameter Q, whereas in section 3, it is arbitrary with distribution function D(x). It satisfies the following relation.

D(x)=n(x) exp (- $/\int_0^x n$(x)dx)

where n(x)A is the first order probability that the rest period will terminate in time (x, x + A) if it has not been terminated up to time x. It is also assumed in section 2, that the server cannot goin the duties, when there is less than 'a' customer, whereas in section 3, it can join his duties even if there is no customer in the queue. It will wait for anarrival.

2. $M^{(x)}/M^{ab}/I$ queue with exponential rest period

P_n (t) = Probability that at time t, there are n units in the queue and the server is operative (n 0, 1, 2, 3,...)

Q_n (t)= Probability that at time t, there are n units in the queue and the server is idle (n 0, 1, 2,3,...)

Elementary probability reasoning leads to the following set of difference-differential equations.

$Q'_0(t) –αQ_0 (t) + ßP_0 (t)$ (1)

$Q'_n(t)=-aQ_n (t) + ßP_n (t) + a\Sigma^n_{j=1}C_jQ_n\text{-}j^{(t)}n < a$ (2)

$Q'_n(t)= -(a+ß)Q_n (t)+ßP_n (t)+ α \Sigma^n_{j=1}C_j{}^Q{}_n\text{-}j (t)$ $n < a$ (3)

$P'_0(t)=-(a+ ß) P_0(t)+ Ø Σ^b_{j=a}{}^Qj (t)$

(4)

$P_n(t)= -(a+b) P_n (t) + α Σ^b_{j=a}C_jP_n\text{-}j (t) + Ø Q_{n+b}$ $n \geq 1$ (5)

Let the initial condition be

$Q_0 (0) = 1, Q_n (0) = 0 ,P_n(0) = 0$ to reach n

(6)

Define the probability generating functions:

$P(zt)=\sum_{n=0}^{\propto} P_n (t) z''$

$Q(zt)=\sum_{n=0}^{\infty} Ω_n (t) z''$

$C(z)=\sum_{j=0}^{\infty} C_j z^j$

(7)

Using equation (1) to (6), we get

$[s+a+Ø - c(z)] \bar{Q}(zs) ß\bar{P} (zs) = 1=Ø \sum_{n=0}^{a-1} z^nQ_n (s)$

(8)

$[(s+a+ß) z^b - az^b c(z)]$ $_{P^-}$(zs) - $Ø_{Q^-}$(zs) = $Ø \sum_{j=a}^{b-1} z^bQ_j (s)$ –

$\emptyset \sum_{j=0}^{b-1} z^j Q_j(s)$

(9)

where $\bar{Q}_n(s)$ is the Laplace Transform of $Q_n(t)$.
Solving (8) and (.9)

$\emptyset(1=\emptyset \sum_{n=0}^{a-1} z^n Q_n(s)) + (s+a+\emptyset-ac(z))(\emptyset \sum_{j=a}^{b-1} (z^b-z^j) Q_j(s) - \emptyset \sum_{j=0}^{a-1} z^j Q_n(s))$
$\bar{P}$(zs)=--
$(s+a+\beta) z^b - az^b c(z))\quad (s+a+\emptyset-ac(z)) - \beta\emptyset$
(10)
$\beta\emptyset (\sum_{j=a}^{b-1} (z^b-z^j) Q_j(s) - \sum_{j=0}^{a-1} z^j Q_j(s)) + (1+\emptyset \sum_{j=0}^{a-1} z^n Q_n(s))(s+a+\beta) z^b - az^b c(z)$
$\bar{Q}$ (zs) = ---
$((s+a+\beta) z^b c(z)(s+a+\emptyset - ac(z)) - \beta\emptyset$

(11)

$\bar{R}(zs) = \bar{P}(zs) + \bar{Q}(zs)$

(12)

Apply Rouche's Theorem to the denominator in (11). It can be shown that it has 'b' zeros inside and on the unit circle. Since P (z,s) is con- vergent in z≤ 1, the numerator in (2.11) must vanish for these 'b' zeros. Thus we get 'b' equations from which the values of 'b' unknown can be determined.

Steady State Solution:
Using the well known property of Laplace Transform
$\lim_{t\to\infty} f(t) = \lim_{s\to 0} f(s)$ (13)
(10) and (11) reduces to
$\emptyset^2 \sum_{n=0}^{a-1} z^n Q_n + (a+\emptyset-ac(z))(\emptyset \sum_{j=a}^{b-1} (z^b-z^j) Q_j - \emptyset \sum_{j=0}^{a-1} z^j Q_j)$
P (z) = ---
$((a+\beta) z^b - az^b c(z))(a+\emptyset - ac(z)) - \beta\emptyset$

(14)

$\beta(\emptyset \sum_{j=a}^{b-1} (z^b-z^j) Q_j - \emptyset \sum_{j=0}^{a-1} z^j Q_j) + (\emptyset \sum_{n=0}^{a-1} z^n Q_n)((a+\beta) z^b - az^b c(z))$
Q(z) = ---
$((a+\beta) z^b - az^b x(z))(a+\emptyset - ac(z)) - \beta\emptyset$

(15)

Operational Characteristics
i**)** Proportion of Time the System Remains Idle State and Operative State Let E_1, E_2 be the proportion of the time the server remains idle and operative respectively;
E1 = Q(1)

$\beta\emptyset \sum_{j=a}^{b-1} (b-j) Q_j + \emptyset(\beta b - ac'(1))(\sum_{n=0}^{a-1} Q_n)$
=---
$\emptyset(\beta b - ac'(1)) - \alpha\beta$

(16)

E_2 = P (1)
$\emptyset^2 \sum_{j=a}^{b-1} (b-j) Q_j + \alpha\emptyset c'(1) \sum_{j=0}^{a-1} Q_j$
=--

$\emptyset(\beta b - ac'(1)) - \alpha\beta$

(17)

(ii) Mean Queue Length

Let L_q and $L_q$be the mean queue lengths in the idle and operative state respectively

$$l_{q1} = Q'(1) = \frac{D'(1)\, N''_1 - N''_1(1)\, D''(1)}{2[D'(1)]^2} \qquad (18)$$

$$l_{q2} = P'(1) = \frac{D'(1)\, N''_2 - N''_2(1)\, D''(1)}{2[D'(1)]^2}$$

(19)

Where

$N'_1(1) = \beta\emptyset \sum_{j=a}^{b-1} (b-j)\, Q_j + \emptyset(\beta b - ac'(1)) \left(\sum_{n=0}^{a-1} Q_n\right)$

$N''_1(1) = \beta\emptyset \sum_{j=a}^{b-1} (b(b-1) - j(j-1))\, Q_j + \left(\emptyset \sum_{n=0}^{a-1} Q_n\right)(b(\beta(b-1) + 2ac'(1)) - ac''(1))$

$+ 2\left(\emptyset \sum_{n=0}^{a-1} n\, Q_n\right)(\beta b - ac'(1))$

$N'_2(1) = \emptyset^2 \sum_{j=a}^{b-1} (b-j)\, Q_j + a\emptyset c'(1) \sum_{j=0}^{a-1} Q_j$

$N''_2(1) = a\emptyset c''(1) \sum_{j=0}^{a-1} Q_j - (2a\emptyset c'(1)\left(\sum_{j=a}^{b-1} (b-j)\, Q_j - \sum_{j=0}^{a-1} jQ_j\right)$

$D'(1) = \emptyset(\beta b - ac'(1)) - a\beta$

$D''(1) = \emptyset\beta b(b-1) - \emptyset c''(1)(a+\beta) - 2a\, c'(1)(b(\beta+\emptyset) - ac'(1))$

(20)

iii) Average Number to Rest :

Let n be the average number of the rest server takes per unit time

$n = \emptyset(Q(1) - Q_0)$

(21)

Particular Case

When units arrive and serve singly i.e. a = b = 1 Putting a = b = 1 in (14) to (20)

$$P(z) = \frac{a\emptyset\, Q_0}{a^2 z^2 - az(a+\beta+\emptyset) + \beta\emptyset}$$

$$Q(z) = \frac{\emptyset(az - \beta)\, Q_0}{a^2 z^2 - az(a+\beta+\emptyset) + \beta\emptyset}$$

$Q_0 = 1 – p – ψ$

$ψ (1 +p^2 – p^2$
L_{q1} = ---
1 –p ψ

p^2 + ψp (1-p)
L_{q2} =--
1 –p ψ

Φ Q_0 (1 – p)
E_1 =-------------------------------------
Φ (1 – p) – a

Φ Q_0 a
E_2 =--------------------------------------
Φ (1 – p) – a

a Φ Q_0

n = --------------------------------------
Φ (1 – p) – a

where ψ =^{a}-$_{Φ}$ and p = $\frac{a}{\beta}$
Further if the server do not take rest i.e. $ results as that of M/M/I without rest.
Conclusion;

r	M	λ	λ'$_0$	μ	E	$P_o(0)$	$P_n(1)$	$P_n(0)$
9	12	8	2	11	0.5	0.29	0.992	0.008
10	12	8	2	11	0.5	0.29	0.993	0.007
11	12	8	2	11	0.5	0.289	0.994	0.006
9	15	8	2	12	0.5	0.349	0.999	0.001
9	18	8	2	12	0.5	0.348	0.97	0.03
9	12	9	4	11	0.5	0.205	0.934	0.066
9	12	10	4	11	0.5	0.133	0.987	0.018
9	12	10	4	11	0.5	0.133	0.982	0.018

9	12	8	6	11	0.5	0.276	0.977	0.023
9	12	9	2	11	0.5	0.206	0.989	0.011
9	12	9	2	12	0.5	0.267	0.995	0.005
9	12	9	2	13	0.5	0.323	0.995	0.013
9	12	9	4	12	0.5	0.267	0.987	0.011
9	12	9	4	12	1	0.278	0.989	0.09

The value of $P_o(0)$, $P_n(1)$, $P_n(0)$ for different values of r, M , λ, λ'_0, E

$\lambda = 8$, $\mu = 11$

Table -1

R	M	λ	λ'_0	μ	e	L_s
10	12	8	2	11	0.5	4.992
9	12	8	2	11	0.6	3.594
9	12	8	2	11	0.6	3.165
10	14	8	2	11	0.6	2.842
10	11	8	2	11	0.6	2.456
9	13	8	5	12 1	0.6	2.241
9	11	8	2	12	0.6	2.145
9	11	8	2	11	0.6	1.768
9	13	8	2	13	0.6	1.456
9	14	8	2	11	0.6	1.123
9	14	8	2	11	0.6	0.987
9	14	8	2	11	0.6	0.756

10	12	8	2	11	0.6	0.423

e = 0.6, r = 10, λ = 9

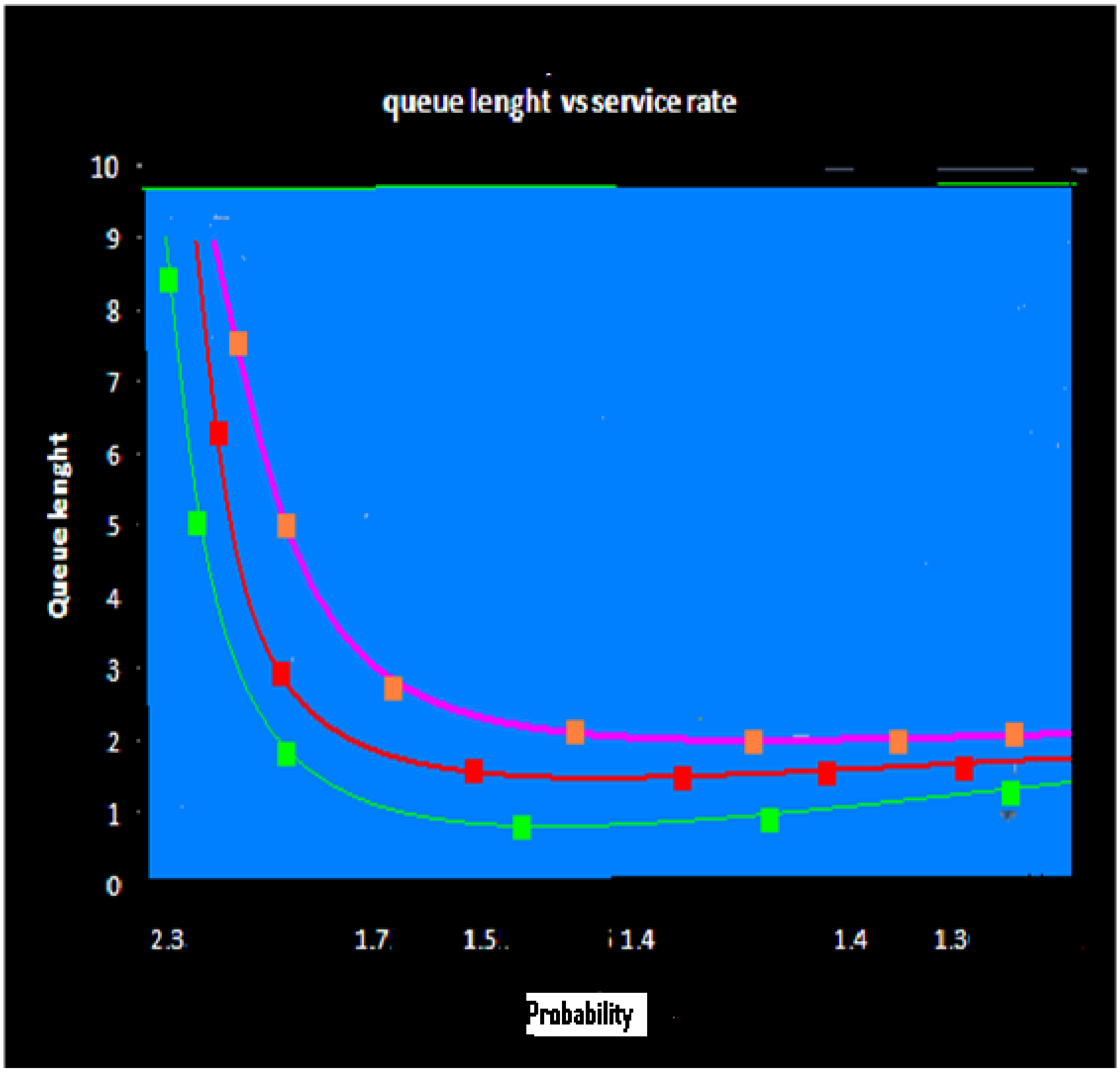
The value of average number of customers in the system is ginen by above table.

Table -2

R	M	λ	λ'_0	μ	E	Ws
10	13	9	3	12	0.6	0.292
10	13	9	3	12	0.6	0.294
10	16	9	3	13	0.6	0.227
10	19	9	3	13	0.6	0.233
10	13	9	4	12	0.6	0.376
10	13	9	5	12	0.6	0.473
10	13	9	5	12	0.6	0.309
10	13	9	7	13	0.6	0.283
10	13	9	3	13	0.6	0.225
10	13	9	3	14	0.6	0.286
10	13	10	3	14	0.6	0.258
10	14	10	5	15	0.6	0.258
10	15	10	5	15	0.6	0.258

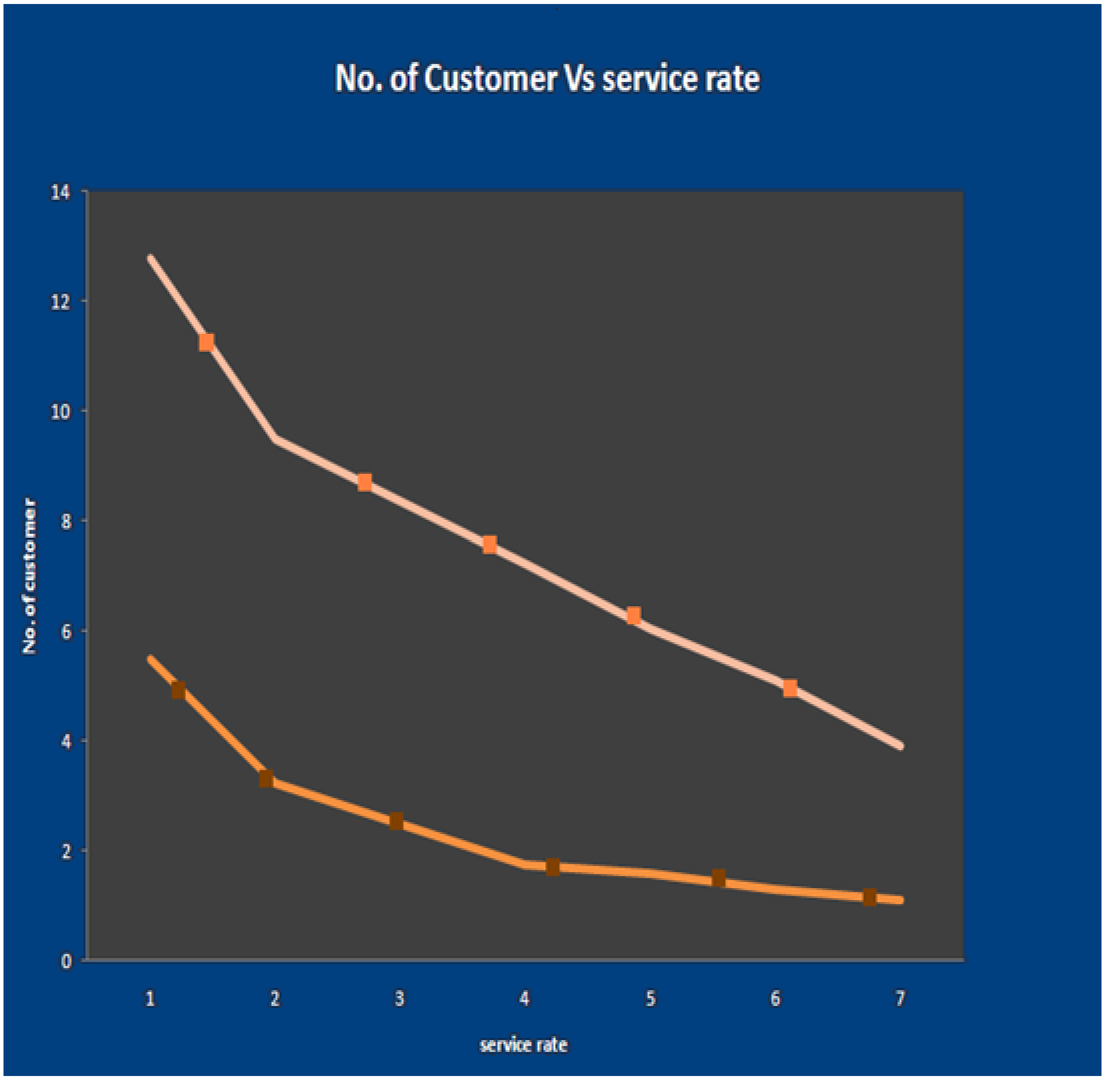
λ = 9, M = 13, e = 0.6, r = 10
The value of average waiting timein the system is given by above table.

Table-3



$P_0(0) = 0.99$ $P_n(1) = 0.29$ $P_0(0) = 0.08$

This graph shows that $P_0(o)$, $P_n(1)$,$P_0(0)$is an decreasing function of e, M,**λ.**



$\lambda'_0 = 9 \lambda = 6$

This graph shows that number of customer decreases when service rate increases.

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