Solving Triangular Fuzzy Sequencing Problem with three Different Rankings

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Abstract:
In this paper job sequencing problem is taken in which processing times are taken as symmetric triangular fuzzy numbers. The main aim of sequencing problem is to find the optimal sequence of the jobs on given machines that helps to minimize the total completion time of all the jobs. To achieve this aim in better way sequencing problem is solved with three different rankings: Yager's ranking, Uthra's ranking and Proposed ranking.
Key Words: Triangular Fuzzy Numbers, Yager's Ranking, Uthra's Ranking and Sequencing Problem.
DOI NUMBER: 10.48047/NQ.2022.20.1.NQ22434 NEUROQUANTOLOGY 2022; 20(1):1423-1429

Introduction

A fuzzy set is defined as the function whose domain is a set X and range is closed unit interval. Zadeh [14,15] gave the concept of fuzzy set in 1965. He introduced this concept by taking into consideration the imprecisely defined classes, for example “the class of beautiful woman” or “the class of tall men”. It can be used in a wide range of domains where information is in complete and imprecise. Bellman and Zadeh [1] used fuzzy theory in decision making in which goals and constraints were fuzzy in nature. Jain [5] was first who proposed ranking of fuzzy numbers and used in fuzzy decision making where vague quantities are represented as fuzzy sets. Jain [6] proposed method to solve qualitative data quantitatively with the help of fuzzy sets. In the method presented by Jain final rating of each alternative is calculated. Arithmetic operations on fuzzy quantities were given and the method used was explained with the help of numerical example. Yager [11] in 1978 illustrated the ordering of fuzzy numbers. For this procedure was given in which regular fuzzy numbers are mapped into real numbers came out with meaningful ordering of fuzzy numbers. Dubois and Prade [3,4] had defined fuzzy numbers as a fuzzy subset of the real line. It is a quantity which is imprecise rather than exact and is a subset of the real line whose membership vales cluster around the value called mean value. Dubois and Prade in this paper extended usual operations of real numbers to fuzzy numbers with the help of fuzzification principle. Yager [12,13] in 1980 introduced a class of connectives, intersection and union for fuzzy sets. The properties of this class are also studied by comparing with union and intersection of ordinary sets. He also defined a function which helped in the ordering of fuzzy subsets of the unit interval and its properties. Chen et al. [2] used fuzzy approach in multiobjective scheduling for finding optimal processing sequence.After introduction of fuzzy theory, it was assumed that processing times would be more accurately taken in the form of intervals in which approximate completion time would lie near the middle of the interval. This theory was best used in fuzzy numbers. Keeping this in view McCahon and Lee [8] modified algorithms to solve job sequencing problem. Kripa and Govindarajan [7] also worked with triangular fuzzy numbers and by using defined rankings found optimal sequence of jobs. Sowmiya and Selvakumari[10] proposed method to find shortest route of network problems with the help of Johnson's algorithm. Ponnalagu and Mounika [9] did comparative study between fuzzy sequencing problem and crisp problem. There are many more researchers who worked in the field of triangular fuzzy numbers.

Preliminaries

In this section some basic definitions related to triangular fuzzy numbers that are needed to solve sequencing problem are presented.

Definition 1 (Crisp Set)

A crisp set or a classical set A is defined as a collection of distinct and distinguishable objects. The objects are called elements of A .

Definition 2 (Characteristic Function)

A crisp set A, defined on the universal set *X*, can also be represented by $A=\{(x,\mu_A(x));x\in X\}$ where $\mu_A : X \to \{0,1\}$ is called characteristic function defined by

$$\mu_A(x) = \begin{cases} 1 & if x \in A \\ 0 & if x \notin A \end{cases}$$

Definition 3 (Membership Function)

The characteristic function μ_A of a crisp set $A \subseteq X$ assigns a value either 0 or 1 to each member in X . This function can be generalized to a function $\mu_{\tilde{A}}$ such that the value assigned to the element of the universal set X fall within a specified range $[0,1]$ i.e.$\mu_{\tilde{A}}: X \to [0,1]$. The assigned values indicate the membership grade of the element in the set A . The function $\mu_{\tilde{A}}$ is called the membership function and the set $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)): x \in X\}$defined by $\mu_{\tilde{A}}$ for each $x \in X$ is called a fuzzy set.

Definition 4 (Fuzzy Number)

A fuzzy set $\tilde{A}$,defined on the universal set of real numbers R is said to be a fuzzy number if its membership function has the following characteristics:

1.$\mu_{\tilde{A}}: R \to [0,1]$ is continuous.
2. $\mu_{\tilde{A}}(x) = 0$ for all $x \in (-\infty, c] \cup [d, \infty)$.
3. Is strictly increasing on $[c, a]$ and strictly decreasing on$[b, d]$.
4. $\mu_{\tilde{A}}(x) = 1$ for all $x \in [a, b]$.

Definition 5 (Triangular Fuzzy Number)

A fuzzy number $\tilde{A} = (a, b, c)$, shown in Fig. 3.1, is said to be a **triangular fuzzy number** if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x-a}{b-a}, & a \le x \le b \\ \dfrac{x-c}{b-c}, & b \le x \le c \\ 0, & otherwise \end{cases}$$

Where $a, b and c \in R$.

Definition 6 (Symmetric Triangular Fuzzy Number)

If $b = c$, then the triangular fuzzy number $A = (a, b, c)$ is called symmetric triangular fuzzy number.

Proposed Ranking

If $A = (a, b, c)$ is a triangular fuzzy number then its proposed ranking function is defined as

$$R(A) = \frac{3a + b + c}{5}$$

Method to solve Sequencing Problem

In this section the algorithm used to find optimal sequence of jobs in job sequencing problem is presented in brief.

Johnson's algorithm is used to solve the job sequencing problem. Steps are as follows:

- Write processing times of all the jobs.
- Select job with minimum time. If that time is for first machine, then place that job first. If that time is for second machine then place that job in the last.
- Eliminate scheduled job from further procedure.
- Repeat the above two steps until all jobs have been placed.

Numerical Examples

In this section to show the shortcomings of existing rankings and effectiveness of proposed ranking numerical example is solved as follows:

Example 3.5.1 *There are 5 jobs each of which must go through the two machines M_1 and M_2 in the order M_1M_2 with following processing times. Find optimal sequence of jobs that minimizes the total elapsed time required to complete all jobs.*

Jobs→ Machines↓	J_1	J_2	J_3	J_4	J_5
M_1	(2,3,3)	(2,7,7)	(2,5,5)	(3,6,6)	(0,3,3)
M_2	(4,9,9)	(4,8,8)	(0,5,5)	(1,4,4)	(2,13,13)

Solution:

With Yager's Ranking

Step 1: Find ranking function of all processing times.

Jobs→ Machines↓	J_1	J_2	J_3	J_4	J_5
M_1	2.75	5.75	4.25	5.25	2.25
M_2	7.75	7	3.75	3.25	10.25

As minimum processing time corresponding to above table is 2.25 so job sequence of this table is

J_5	-	-	-	

Step 2: Reduced table is

Jobs→ Machines↓	J_1	J_2	J_3	J_4
M_1	2.75	5.75	4.25	5.25
M_2	7.75	7	3.75	3.25

As minimum processing time corresponding to above table is 2.75 so job sequence of this table is

J_5	J_1	-	-	

Step 3: Reduced table is

Jobs→ Machines↓	J_2	J_3	J_4
M_1	5.75	4.25	5.25
M_2	7	3.75	3.25

As minimum processing time corresponding to above table is 3.25 so job sequence of this table is

J_5	J_1	-	-	J_4

Step 4: Reduced table is

Jobs→ Machines↓	J_2	J_3
M_1	5.75	4.25
M_2	7	3.75

As minimum processing time corresponding to above table is 3.75 so job sequence of this table is

J_5	J_1	-	J_3	J_4

Final Sequence is

J_5	J_1	J_2	J_3	J_4

Step 5: Find total elapsed time

Job Sequence	M_1		M_2	
	Time in	Time out	Time in	Time out
J_5	0	2.25	2.25	12.5
J_1	2.25	5.0	12.5	20.25
J_2	5.0	10.75	20.25	27.25
J_3	10.75	15.0	27.25	31.0
J_4	15.0	20.25	31.0	34.25

Thus the minimum total elapsed time is 34.25 hours.

With Uthra's Ranking

Step 1: Find ranking function of all processing times.

Jobs→ Machines↓	J_1	J_2	J_3	J_4	J_5
M_1	2.67	5.33	4.0	5.0	2.0
M_2	7.33	6.67	3.33	3.0	9.33

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As minimum processing time corresponding to above table is 2.0 so job sequence of this table is

J_5	-	-	-	

Step 2: Reduced table is

Jobs→ Machines↓	J_1	J_2	J_3	J_4
M_1	2.67	5.33	4.0	5.0
M_2	7.33	6.67	3.33	3.0

As minimum processing time corresponding to above table is 2.67 so job sequence of this table is

J_5	J_1	-	-	

Step 3: Reduced table is

Jobs→ Machines↓	J_2	J_3	J_4
J_1	5.33	4.0	5.0
J_2	6.67	3.33	3.0

As minimum processing time corresponding to above table is 3.0 so job sequence of this table is

J_5	J_1	-	-	J_4

Step 4: Reduced table is

Jobs→	J_2	J_3

Machines↓		
J_1	5.33	4.0
J_2	6.67	3.33

As minimum processing time corresponding to above table is 3.33 so job sequence of this table is

J_5	J_1	-	J_3	J_4

Final Sequence is

C	J_1	J_2	J_3	J_4

Step 5: Find total elapsed time

Job Sequence	M_1		M_2	
	Time in	Time out	Time in	Time out
J_5	0	2.0	2.0	11.33
J_1	2.0	4.67	11.33	18.66
J_2	4.67	10.0	18.66	25.33
J_3	10.0	14.0	25.33	28.66
J_4	14.0	19.0	28.66	31.66

Thus the minimum total elapsed time is 31.66 hours.

With Proposed Ranking

Step 1: Find ranking function of all processing times.

Jobs→ Machines↓	J_1	J_2	J_3	J_4	J_5
M_1	2.4	4.0	3.2	4.2	1.2
M_2	6.0	5.6	2.0	2.2	6.4

As minimum processing time corresponding to above table is 1.2 so job sequence of this table is

J_5	-	-	-	

Step 2: Reduced table is

Jobs→ Machines↓	J_1	J_2	J_3	J_4
M_1	2.4	4.0	3.2	4.2
M_2	6.0	5.6	2.0	2.2

As minimum processing time corresponding to above table is 2.0 so job sequence of this table is

J_5	–	-	-	J_3

Step 3: Reduced table is

Jobs→ Machines↓	J_1	J_2	J_4
M_1	2.4	4.0	4.2
M_2	6.0	5.6	2.2

As minimum processing time corresponding to above table is 2.2 so job sequence of this table is

J_5	-	-	J_4	J_3

Step 4: Reduced table is

Jobs→ Machines↓	M_1	M_2
M_1	2.4	4.0
M_2	6.0	5.6

As minimum processing time corresponding to above table is 2.4 so job sequence of this table is

J_5	J_1	-	J_4	J_3

Final Sequence is

J_5	J_1	J_2	J_4	J_3

Step 5: Find total elapsed time

Job Sequence	M_1		M_2	
	Time in	Time out	Time in	Time out
J_5	0	1.2	1.2	7.6
J_1	1.2	3.6	7.6	13.6
J_2	3.6	7.6	13.6	19.2
J_4	7.6	11.8	1.9.2	21.4
J_3	11.8	15.0	21.4	23.4

Thus, the minimum total elapsed time is 23.4 hours.

Comparison

The following table shows the result obtained by using three rankings namely: Yager's ranking, Uthra's ranking and Proposed ranking. It shows that proposed ranking is better to use to find total elapsed time in job sequencing problem.

S. No.	Rankings	Total Elapsed Time
1.	Yager's Ranking	34.25 hours
2.	Uthra's Ranking	31.66 hours
3.	Proposed Ranking	23.4 hours

Conclusion

On the basis of presented study, it can be concluded that proposed ranking is better and provided the least total elapsed time. In this chapter job sequencing problem is solved in which processing times are taken as symmetric triangular fuzzy numbers. Total elapsed time is calculated by changing processing times into crisp numbers with the help of three different rankings and proposed ranking provided better result.

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