



A DESCRIPTIVE SYNTHESIS OF MATHEMATICS APPLIED TO ECONOMICS AND JURIS DOCTOR.

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Abstract

Change and diversity in the initial training of students entering university lead to a series of basic questions: What criteria should be used to decide on the most appropriate syllabus and teaching methodology in a situation of change? What are the peculiarities of teaching mathematics in economics? This last question introduces us to the issue we are analyzing here: to what extent should economics be present in mathematics classes? Is it necessary for students to have a solid mathematical background before studying economics? For some teachers, economic problems are simply an appendix to the practical part of the subject, for others economic models form the core of the subject. It is clear that one or the other approach has a decisive impact on our syllabus and teaching methodology. In this paper, we have analyzed the mathematics-economics relationship by reflecting on its historical development and on the experience, we have developed over several years in the subject of mathematics and economics. for several years in the subject of Business and Financial Mathematics in the first year of the course of the Diploma in Business Studies.

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1. Introduction

A large number of publications deal with the role of mathematics in economics. This subject, which is not without controversy, not only has an impact on the development of economics as a social science but also has a decisive influence on the teaching of mathematics for economics. Already the educators of the 18th and 19th centuries thought that the training of the economist was forged in the basic subjects (Microeconomics, Macroeconomics, and Statistics), while the specialized subjects played the dual role of complementing this training and providing professional knowledge. In accordance with this approach, the learning and teaching of Mathematics should be conceived essentially as a support for the analysis of basic knowledge, with hardly any initial reference to possible specializations. [Santos 1997].

In this article, we intend to put ourselves in the point of view of the mathematician (or mathematics teacher) and reflect on the role that economics should play in mathematics classes. Is a solid mathematical background necessary before studying the basic subjects of economics? Or should we justify each mathematical concept with its corresponding economic application? Or is a joint study of both disciplines more interesting for the student? It is clear that these and other reflections have an important impact on the programming of our subjects and their methodology.

In section 1 of this paper, we analyze the students with whom we are going to work, trying to understand what the transition from secondary school to university entails. In section 2, we highlight some characteristics of the mathematics/economics relationship with a view to the framework we are interested in analyzing. In section 3, we show the results of the specific analysis we have carried out comparing a series of problems and questions with economic content, asked in mid-term or final exams in the last three academic years, with problems and questions with

purely mathematical content. Finally, we draw some initial conclusions.

2. The transition from high school to university

Schematically, the most notable difference between secondary and university mathematics organizations is the shift from demonstrative mathematics to demonstrative mathematics. In secondary school, the demonstration is practically absent and the justifications that may arise in the classes serve only as ornamentation for the teacher's discourse (in the recommendations for university entrance exams, teachers are told that it is sufficient to cite the theorems and justify them graphically). The student can ignore them without any problem. Accordingly, precise definitions are not necessary either. An approximation to the concept would suffice: "Definitions serve more to describe previously known objects than to logically construct new objects". [Gascón 1997].

In university education, on the contrary, proof becomes the main mathematical activity. It is implicitly established that at university all statements must be justifiable by the student. He has to verify the hypotheses of a theorem in order to justify its applicability. He must be able to check whether or not an object satisfies a certain definition. He must accept that the property shown by the graphs has no demonstrative value... It is intended that the student feels the need for demonstration and consequently the importance of specifying the definitions.

With regard to problem-solving activities, and in coherence with this change from demonstrative mathematics to demonstrative mathematics, there is a shift from a strong preponderance of problems to be solved in secondary school to a significant presence of problems to be demonstrated at university. In coherence with the above, the passage from one mathematical organization to the other obliges the student to change from being a student with little autonomy and minimal mathematical responsibility to being co-responsible for his or her training process. [Gascón et al. 2004]. The majority of students in

Economics and Business degrees move from a mathematical organization based mainly on the subject Mathematics Applied to Social Sciences to one based on Linear Algebra and Calculus in one and several variables.

The organization of algebra at the secondary level is centered on solving systems of linear equations and simple linear programming problems. Since the study of determinants has been dispensed with, after modeling statement problems, reduction methods or the so-called Gaussian method are used to solve them. Paradoxically, the algebraic instrument is used in university teaching as if it had been known for a lifetime. Suddenly, logical symbols are needed, axioms are introduced, and theorems are justified and proved based on the aforementioned axioms and new definitions. In short, the study of algebra in university mathematics is so complete and natural from the outset that students must quickly overcome all the outstanding stages of their cognitive development.

On the other hand, the organization of calculus at the secondary level is based on the study of concrete functions, whereas at the university level, it is based on the study of classes of functions. While geometric intuition was sufficient in secondary school, one of the main aims of the study of calculus at the university level is precise to show that this intuition is not only insufficient but also misleading. Although concrete problems of minimizing costs and maximizing profits are solved at the secondary level, integrals are not studied and, therefore, there is no global framework to provide an integrated view of all elementary economic functions.

This background invites us to reflect on the criteria that should be used to decide the contents and methodology of the subject of Business and Financial Mathematics that we teach in the first year of the Diploma in Business Studies, at the same time as we question whether it is only the teachers of the subject who are responsible for carrying out the changes that we see as necessary.

In any case, what are their limits? As the actual changes that are ultimately experienced are small, we have limited ourselves to studying the relationship between economics and mathematics in the teaching of our subject. We are invited by the emergence of new approaches and texts in Mathematics for Economics and Business where economic topics are developed in more and more depth. [Simon & Blume 1994] [Renshaw 2005] [Werner & Sotskov 2006].

3. Economymathematic relationships

From a teaching point of view, and with different nuances, we think that there are two essential ways of approaching mathematical-economics relations: 1) deepening the study of mathematics in order to consolidate a basis for studying economic models; 2) justifying each advance in mathematics with its application to a related topic in economics. One or the other way of thinking implies different ways of approaching the subject. Economics/mathematics relationships over time can be a framework for reflection on the appropriate teaching approach. For this purpose, we select from among many observable facts a small sample that will be the "relevant facts" in our study.

First, an analysis of the use of mathematical methods and procedures in economics. This relationship can be divided into three periods: the marginalist period, from 1838 to 1947 (publication of P. Samuelson's Fundamentals of Economic Analysis), the conjunct period, 1948-1959 (publication of G. Debreu's Theory of Value), and the integration period, from 1960 to the present day (Arrow & Intriligator 1981).

Marginalist, based on the logic of infinitesimal calculus, develops, despite their limitations, the mathematical foundations of the theories of production, consumption, and general equilibrium. Its reference is physics, trying to transfer the concepts and formulations of theoretical mechanics to the language of economics (Jevons, Walras, Edgeworth, Fisher, Pareto, etc...). Coinciding with the crisis of



Newtonian physics and the appearance of the theory of relativity, a new generation of economists appeared who used topology (especially centered around Brouwer's fixed point theorem) to give a new formulation to economic theory. The topological-conjunctural approach was used to establish the existence of equilibrium, while the approach based on infinitesimal calculus was used to study the stability of equilibrium. This was the era of Leontief's linear models. From the 1960s to the present day, the use of mathematical methods and tools in economics has become widespread so that practically all branches of mathematics have penetrated some field of economics, which could justify the name of this period. Topology, geometry, classical and functional analysis, and dynamic equations (differential, in differences, in partial derivatives, etc.), are involved in a variety of economic reasoning. Likewise, incursions have been made in economics using the most recent mathematical developments such as Catastrophe Theory, Fuzzy Sets, and fuzzy logic, ... [Barragán 2002] [Barragán 2002].

Secondly, more recent research in economics. A comparison of today's academic journals with those of, say, fifty years ago reveals a dramatic increase in a mathematical expression. The same can also be said of economics textbooks at all levels. However, at the same time as the recognition of the importance of mathematics is gaining ground in the world of economics professionals, and the teaching of mathematics subjects in economics and business is questioned by some sectors linked to the academic world. [Ovejero 2005].

Thirdly, the scientific relations between mathematics and economics. The first economists who introduced calculus in the mathematization of economic theory took the relationship between physics and mathematics as a common thread. It was claimed that mathematics in general, and differential calculus in particular, would contribute to the legitimization of neoclassical economic theory as a supposed social physics. However, the

relationship between mathematics/economics is of a very different nature than that between mathematics/physics. [Barragán 2002].

An example is the subject of the derivative. When, in physics, we calculate the instantaneous velocity as the limit of the average velocity, we do the same as when, in mathematics, we calculate the slope of the tangent as the limit of the slope of the secant. This is not the case in economics, where the concept of marginality entails a leap from the discrete to the continuous that is justifiable only to use the full power of infinitesimal calculus. Economists, such as Samuelson, criticize the lack of realism of the mathematical assumptions used in neoclassical economic theory. They criticize the fact that in the process of elaborating some models, the reality of economic processes is often forced to adapt or adapt to the formal requirements of mathematical analysis. Even the very definition of some mathematical functions and theorems can be instrumentalized in favor of the demonstration of certain economic results.

4. Test

With the premises indicated in the two previous sections, we have adapted our teaching program and methodology in recent academic years along the lines of studying the economic applications of the mathematical concepts developed. To evaluate the results, we have analyzed a series of economic problems and questions asked in mid-term or final exams in the last three academic years. We aim to analyze the student's response to the more traditional mathematical problem, where the use of economic concepts is not required.

The student, now, not only has to know the mathematical method for solving the exercise presented in the exam, but it is also necessary to carry out certain mathematical modeling that allows him/her to transfer the economic statement to a known mathematical model susceptible to applying resolution techniques analyzed in class.

For the purposes of our study, we differentiated between the economic content of algebra questions and that of calculus questions within the economic topics covered:

- 1) Economic contents of linear algebra:
 - a. Systems of Linear Equations: Production Problems, IS- L M Model.
 - b. Leontieff Linear Models. Input-Output matrices.
 - c. Markov processes.
 - d. Models of finite difference equations (recursive models).

2) Economic contents of the differential and integral calculus:

- a. Marginalism.
- b. Elasticity.
- c. Consumer surplus and producer surplus.

The following table shows the percentage of questions graded with a pass mark out of the total number of questions asked in the different exams, differentiating between those with mathematical content and those with economic content. We also differentiate between Linear Algebra and Calculus (see Table 1):

Table 1. Differentiate between Linear Algebra and Calculus

Item	Algebra	Calculation
Economy	30,38%	49,05%
Mathematics	45,56%	54,64%

The statistics show that while 45.56% of students pass the mathematical content questions in linear algebra, this only happens in 30.38% of the cases of economic content questions, a difference of more than 15 percentage points. In calculus, on the other hand, we see that the percentages are much closer, although the percentage for mathematical content questions remains higher.

The data show that, despite the fact that the aim is to approximate the case of business students with the content of their degree, the results obtained in the objective tests are not as positive as would be expected. The following table shows how many students have left the questions blank or have been marked with zero (see Table 2):

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Table 2. Statistics of answers to the questions evaluated.

Item	Algebra	Calculation
Economy	30,38%	49,05%
Mathematics	45,56%	54,64%

We observe that the trend has reversed, with mathematical questions now more often receiving a mark of zero (or no answer) than economic questions. Questions with economic content are initially more attractive to students, even if they are much more difficult for them to solve well.

to the Calculus part. We will now focus on a separate analysis of the results obtained in the two types of exams. First, we considered a collection of questions asked in the Linear Algebra exams and analyzed the average number of students who obtained different numerical marks (between 0 and 1.5 points per question). The results were as follows (see Table 3):

In general, students are more reluctant or show a greater aversion, to the Linear Algebra part than



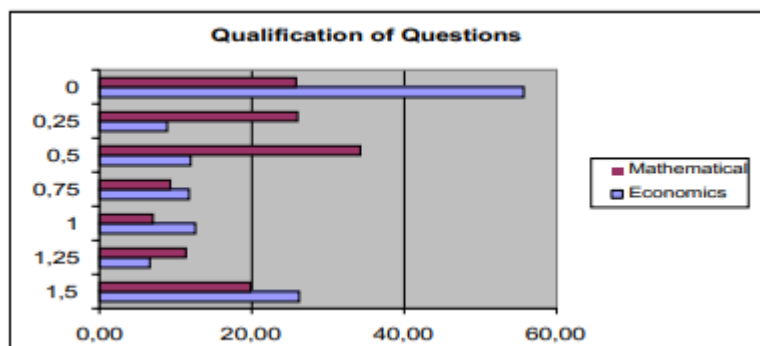


Figure 1. Qualification statistics of the questions analyzed.

Table 3. The average number of students achieving the qualification

Linear Algebra Qualification	The average number of students achieving the qualification	
	Mathematics	Economy
1,5	26,17	19,75
1,25	6,67	11,25
1	12,5	7,0
0,75	11,67	9,25
0,5	12,0	34,25
0,25	8,83	26,0
0	55,67	25,75

Although, in principle, questions with mathematical content obtain a greater number of zeros, it is also true that questions on economics concentrate their marks in low values, while it is more common for passes to be obtained in questions with mathematical content (see Figure 1). We may think that students are more likely to answer a question with economic content, although their answers are not usually correct, while questions with mathematical content discourage those students who do not understand them from the outset.

Secondly, we performed the same analysis for the questions in the calculus exams. Now we also considered a set of different questions that were asked in the calculus exams and whose numerical grades ranged from 0 to 1.75 points. We counted the average number of times the corresponding grade was repeated and the results are shown in the following table which counts the average number of times we gave the students the corresponding grade (see Table 4):

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We see that the results are now reversed and that students obtain a higher number of positive scores on the economic questions (see Figure 2).

Table 4. The average number of students achieving the qualification

Linear Algebra Qualification	The average number of students achieving the qualification	
	Mathematics	Economy
1,75	21,67	26,00
1,5	32,75	18,0
1,25	31,4	40,0
1,0	14,80	12,0
0,75	11,0	25,33
0,5	24,00	9,67
0,25	9,0	2,0

0,0	43,60	56,67
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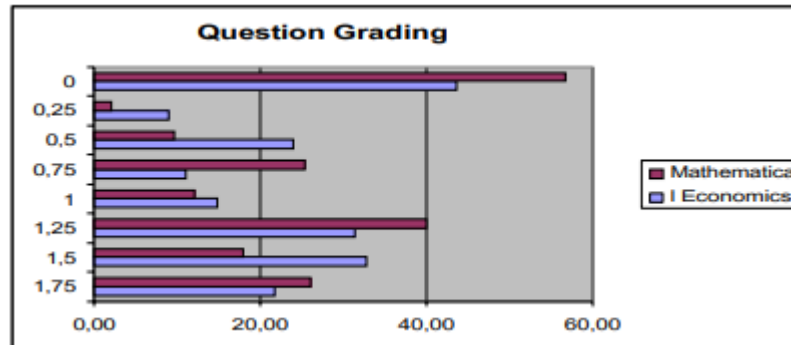


Figure 2. Qualification corresponds to the questions with the highest scores.

There seem to be significant differences between the reaction to economic questions in linear algebra and in calculus. In linear algebra, the student is perhaps surprised, although he does not reject them, but obtains better results in questions with economic content. This tendency is reversed in differential and integral calculus. Explanatory factors include the following:

- 1) In linear algebra, the effort of modeling the exercise is much greater than in calculus questions. The techniques used in calculus are much simpler and do not require a strong effort to interpret the statement, unlike in linear algebra, where the interpretation of the statements has a very important influence on the final results.
- 2) Linear algebra questions with a mathematical content are more common since they have been repeated systematically over the years,

and the degree of variation in the answers is also much lower than in calculus questions where, although the student knows the type of question, there is a greater degree of variation in the specific resolution of the question.

- 3) The student sees much more of the mathematics-economics relationship in the part of the calculus, which is closer to economic theory. On the other hand, the examples of linear algebra are new and are not usually found in other subjects in the business studies degree.

The following table shows the evolution of the number of passes depending on whether or not questions with economic content predominated in the exam as opposed to questions with mathematical content (see Table 5):

Table 5. Percentage Approved

Item	Algebra	Calculation
Economy	20,10%	45,69%
Mathematics	32,58%	54,00%

We can see that the success figures (number of passes by number of candidates) show that those exams where mathematical questions are in the majority (they account for more than 50% of the overall grade) have better percentages than those exams where the economic content is higher.

To reiterate what has been said so far, we note that the difference is also greater in the case of the algebra exams (12 points below), compared to

the calculus exams, where the difference is less than 9 percentage points.

The overall difference in success between the algebra and calculus exams is explained by the fact that the algebra exam is the first one taken in this subject (in December) while the calculus exam is always taken later (in April). The evolution of the student body shows that the number of



students taking the algebra exam is higher than the number taking the calculus exam.

5. Conclusions.

The above study seems to indicate that the introduction of economics in the mathematics syllabus is, in principle, a break from the traditional scheme expected by students and clearly affects the results they obtain in the examinations. However, there is no rejection of this type of format on the part of the student.

On the other hand, their negative impact seems to be much more important in the linear algebra midterm than in the differential and integral calculus midterm. This could be explained by the fact that algebra makes a greater effort of abstraction than differential and integral calculus in the study of mathematics for business.

The contents of differential and integral calculus are best included within economics, given the basic underpinning character they have for the economic theory itself. Basically, students learn to use the concepts of elasticity, surplus, or marginalism in a very natural way, similar to the BARRAGAN M . (2002). "Economics and Mathematics. Productivity, Labour and Income Distribution. A Critical Study". Ph. D. Thesis. Complutense University. Madrid.
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way they do for the other usual concepts of calculus. This fact explains why in differential and integral calculus the results are similar in mathematical and economic questions.

The economic content of linear algebra is based on the application, especially of systems of linear relations, to economics, which implies the need to learn to carry out the processes of abstraction and modeling, a fact for which our students may not yet be prepared and which requires a greater effort. This is perhaps the main reason for the results obtained in the economic questions. Even so, we believe that the procedure to follow involves trying to increase students' ability to carry out these processes of abstraction and modeling, since, although we could do without them in linear algebra, they will be absolutely necessary for the last part of the subject, financial mathematics, where any explanation that does not involve increasing their ability to interpret the problem statements is doomed to failure and means perpetuating the teaching of recipes as opposed to the teaching of methods.

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