# Heat and Mass Transport Characteristics of MHD Nano fluid Flow over Nonlinear Stretching Surface

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**Abstract**

This study investigates the heat generation as well as the rate of heat and mass transfer in nano -fluid flow in an exponentially stretching surface with mass suction. In the present study two different situations are analyzed in exponential order. It can be easily seen that the fundamental laws of motion and heat transfer. The transformation is used to transform the proposed equation into non-linear ordinary differential equations. Furthermore, numerical analysis is performed to validate the analysis. Graphically, all analytical results are valid. This shows that the rate of heat transfer decreases with an increase in Brownian motion.

**Keywords:** Heat transfer; mass transport; MHD nanofluid; nonlinear surface; nonlinear differential equations

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## 1. Introduction

The real world situation suggests that the surface should be linear so that it can be easily stretched in plastic sheet. In previous decades significant interest is shown in the area of scientific research of industrial processing glass fiber, metallic wires, paper production, etc. Sakiadis has first studied boundary layer in stretched surface including two dimensions [1]. In addition many researchers have done tremendous work in the boundary layer on a linear stretch surface [2–5]. Caesar d. et al. Convection studied mass transfer and Newtonian fluid flow of Walters-B nanofluid [6]. In addition, the existence of mass and heat transfer in nanofluid flow of three-dimensional coordination Haque et al., [7]. In addition, thermal boundary layer flow was studied along with the effects of viscous dissipation and ohmic heating on a hot plate, internal heat generation/absorption, diffusion and viscous diffusion. Afify (2004) investigated MHD free convective heat and fluid flow passing over a surface stretched with chemical reaction [8]. Naramgari and Sulochana (2016) outlined the mass and heat transfer of thermophoretic fluid flow from an exponentially expanding surface inserted in porous media in the presence of internal heat generation/absorption, condensation and viscous diffusion [9–11]. A numerical analysis of the unsupervised MHD boundary layer flow of a nanofluid past a diffuse surface in porous media was performed by Baig et al. [12, 13].

Mathematically, the current flow problem is configured according to the fundamental laws of motion and heat transfer. Similarity transformations have been used to convert the governing equations into nonlinear ODEs. The numerical solutions for the resulting non-linear ODEs have been found with the use of connected boundary conditions. The behavior of the resulting equations of the problem under the influence of various flow parameters is checked graphically. These graphical results are obtained for various physical flow parameters in terms of velocity, temperature, skin friction coefficient, local Nusselt number and Sherwood number until the required accuracy level is achieved. It is anticipated that the results of the current work will prove beneficial in future research to advance developments in scientific and engineering fields.

**Basic Differential Equations:**

Consider steady two-dimensional Williamson nanofluid flow over a stretched porous exponential surface. It is assumed that the sheet expands exponentially with different velocity Uw in the x-direction as well as the fluid that occupies the y-direction is governed by the velocity Uw. Furthermore, an external aligned magnetic field of intensity B0 is applied along the stretched surface to the suction vw with an angle β of incidence and the existence of heat generation/absorption is considered. With these preconditions, the main boundary layer MHD equations for the continuum, momentum, energy, as well as concentration are defined as [12,13], respectively.

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}= 0 \quad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=v\frac{\partial^2 u}{\partial y^2}+\sqrt{2}v\Gamma\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2}-v\frac{u}{k_1}-\frac{\sigma B^2}{\rho}sin^2 \quad (2)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{Q_0}{(\rho C_p)_f}(T-T_\infty)+\frac{(\rho C_p)_p}{(\rho C_p)_f}\left[D_B\frac{\partial T}{\partial y}\frac{\partial C}{\partial y}+\frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right], \quad (3)$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_B\left(\frac{\partial^2 C}{\partial y^2}\right)+\frac{D_T}{T_\infty}\left(\frac{\partial^2 u}{\partial y^2}\right) \quad (4)$$



The boundary conditions connected to (1–4) are

$u = u_w = U_0 e^{(x/t)}$, $v_w = -y(x)$, $T = T_w$, $C = C_w$ at $y = 0$

$u = u_e \to 0$, $T\to T_\infty$ , $C\to C_\infty$ , as $y \to \infty$

Here u and vw denote the two parts of the velocity in the x and y paths individually. In addition to k1, ν, ρ, σ, T, T∞, C, C∞, Q0 separately the permeability of the porous medium, kinematic viscosity, density, electrical conductivity, temperature, ambient fluid temperature, concentration, ambient nanoparticle volume fraction and provides. The initial value of the heat output coefficient (heat source). Similarly, Γ,α, (ρCp)p, (ρCp)f , DBand DT deliver the shear stress, thermal diffusivity, heat capacity, fluid heat capacity, coefficient of Brownian and thermophoresis diffusion, respectively.

2.1 Equivalence solution of the governing equations

The governing equations (1)-(4) are non-linear PDEs. We use the similarity transformation below to convert a non-linear PDE to a non-linear ODE

$$u = U_0 e^{\frac{x}{l}} f'(\eta), \eta = \sqrt{\frac{U_0}{2\nu l}}\, y e^{\frac{x}{2l}}$$

$$v_w = -\sqrt{\frac{\nu U_0}{2l}}\, e^{\frac{x}{2l}}\left[f(\eta)+\eta f'(\eta)\right]$$

The boundary conditions for the two cases of PEHF and PEST associated with the above equations (1)-(4) are given as follows.

2.2. PEST case

$$T = T_\infty + (T_w - T_\infty)e^{\frac{x}{2l}}\theta(\eta), h = \frac{C-C_\theta}{C_w-C_\theta}. \quad (7)$$

2.3. PEHF case

$$T = T_\infty + \frac{(T_w-T_\infty)}{K} e^{\frac{x}{2l}} \sqrt{\frac{U_0}{2\nu l}}\,\theta(\eta), h = \frac{C-C_\theta}{C_w-C_\theta}. \quad (8)$$

Considering the similarity transformation defined above, equation (1) satisfies in a similar way as well as equations (2)-(4) reduce to a subsequent set of non-linear ODEs

2.4. PEST case

$$\theta'' + Pr\left(f\theta' - f'\theta + N_b g'\theta' + N_t\theta'^2 + Q\theta\right) = 0, \quad (9)$$

$h'' + LePr(fh') + \frac{N_t}{N_b}\theta'' = 0.$ (10)

2.5. PEHF case

$\varphi'' + Pr(f\varphi' - f'\varphi + N_b g'\varphi' + N_t\varphi'^2 + Q\varphi)$ (11)

$h'' + LePr(fh') + \frac{N_t}{N_b}\varphi'' = 0.$ (12)

Using the equality transformation in the boundary conditions (5), we get

f(0) = $v_w$ , f′ (0) = 1 (13)

f′ $(\eta) \rightarrow 0 \; as \; \eta \rightarrow \infty.$

Boundary conditions for PEST case

$\theta(0) = -1, h(0) = 1,$ (14)

$\theta(\eta) \rightarrow 0, h(\eta) = 0 \; as \; \eta \rightarrow \infty.$

Boundary conditions for PEHF

$\varphi'(0) = -1, h(0) = 1$ (15)

$\varphi(\eta) \rightarrow 0, h(\eta) = 0 \; as \; \eta \rightarrow \infty.$

The similarity parameters appeared in the above Eq. 9–16 are Nt, Nb, Le, Pr, M, λ, K, Q and Re representing thermophoresis and Brownian motion parameters, Lewis number, Prandtl number, Hartmann number, Williamson parameter, porosity parameter, heat source/sink, respectively We do. parameter and Reynolds number. These parameters are defined

$N_t = D_B \frac{(\rho c)_p}{(\rho c)_f}(C_w - C_\infty), N_b = \frac{D_T}{T_\infty}\frac{(\rho c)_p}{(\rho c)_f}\frac{(T_w - T_\infty)}{v}, L_e = \frac{a}{D_B}, P_r = \frac{v}{a},$

M = $\frac{\sigma B_0}{\rho U_0 e^{\frac{x}{l}}}, \lambda = \left(\frac{U_0}{2}\right)^{\frac{2}{3}} \frac{\Gamma}{\sqrt{l}} e^{\frac{x}{2l}},$

**Numerical Analysis**

Comparison of results with previous studied

| $N_t$ | $N_b$ | Khan and Pop (2011) | Beg et. al (2014) | Present Result |
|---|---|---|---|---|
| **0.1** | **0.1** | **0.9524** | **0.9523768** | **0.952376800** |
| | **0.2** | **0.6932** | **0.6931743** | **0.695177510** |
| | **0.3** | **0.5201** | **0.5200790** | **0.595198510** |
| | **0.4** | **0.4026** | **0.4025808** | **0.485174510** |
| | **0.5** | **0.3211** | **0.3210543** | **0.785186510** |
| **0.2** | | **0.5056** | **0.5055814** | **0.505581300** |
| **0.3** | | **0.2522** | **0.2521560** | **0.252155500** |
| **0.4** | | **0.1194** | **0.1194059** | **0.119405400** |
| **0.5** | | **0.0543** | **0.0542534** | **0.054252970** |



Figures 1, 2, 3 and 4 show the effect of the magnetic parameter on the temperature distribution and the concentration distribution, respectively. It can be seen from the figures that the thickness of the boundary layer increases with the increase of the magnetic parameter. This is due to the Lorentz force that is created by applying a magnetic field to a conductive fluid. The Lorentz force has a tendency to reduce the flow speed which supports our results. By applying a magnetic field to a fluid, the resistance of the fluid particles increases, resulting in an increase in temperature.

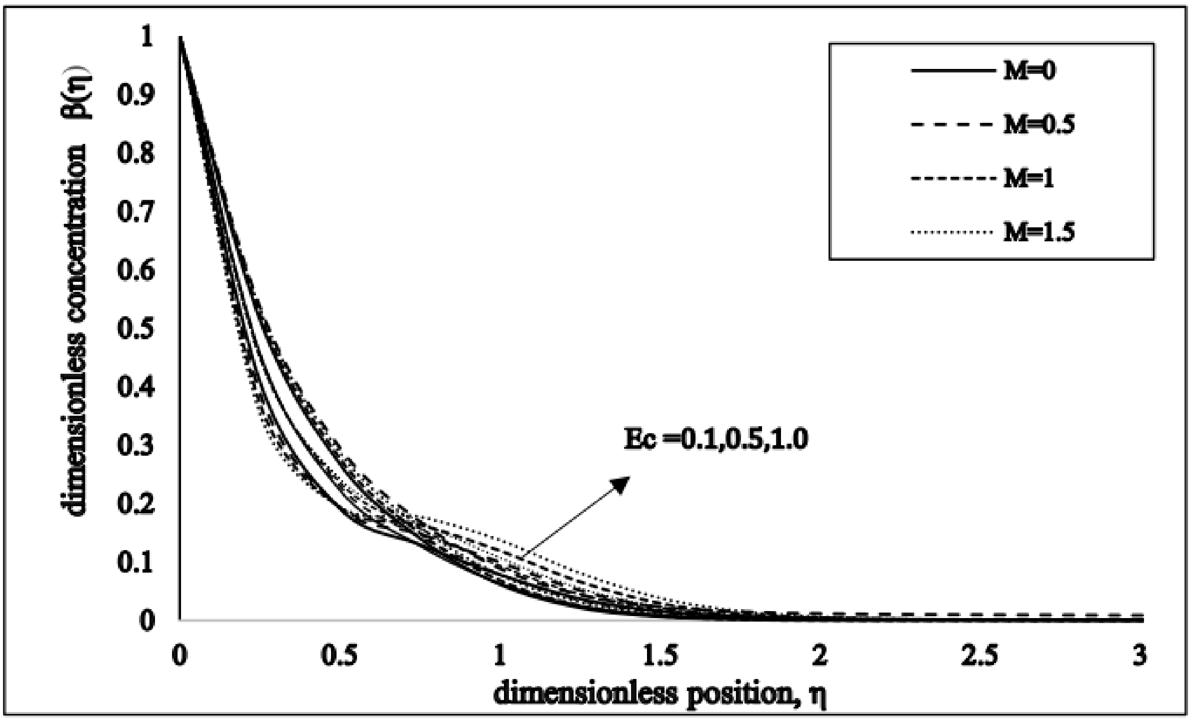


**Fig 1**: Effect of dimensionless concentration $\beta(\eta)$ with respect to $\eta$.



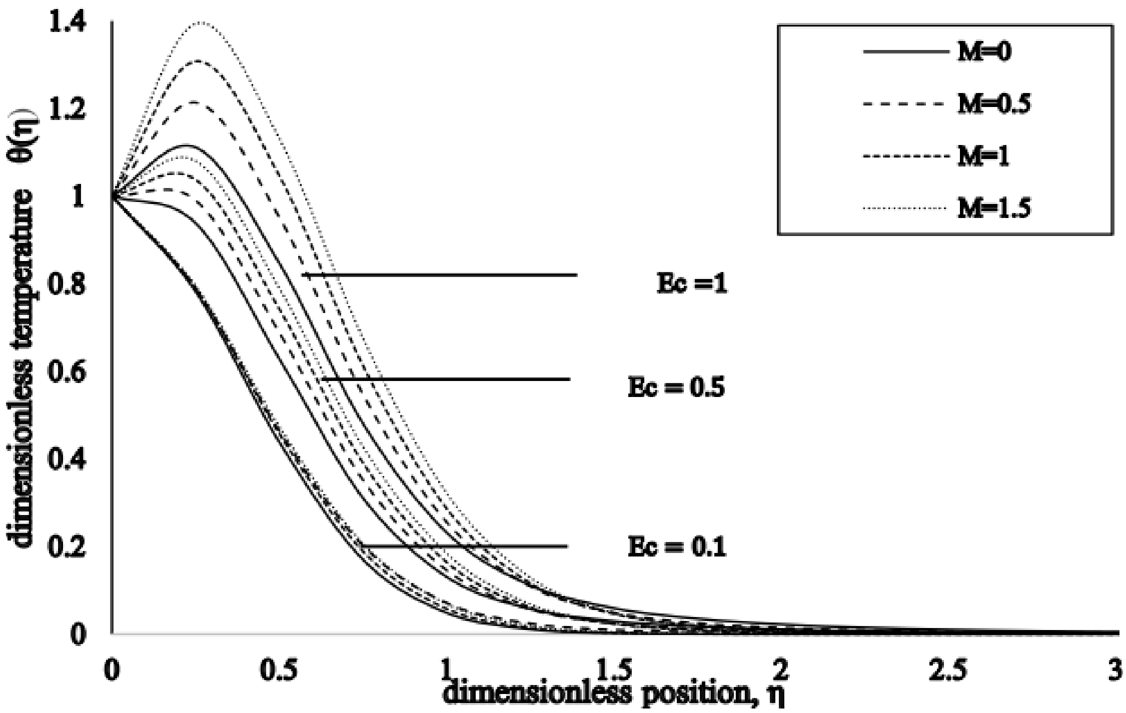


**Fig 2**: Effect of dimensionless temperature $\theta(\eta)$ with respect to $\eta$.

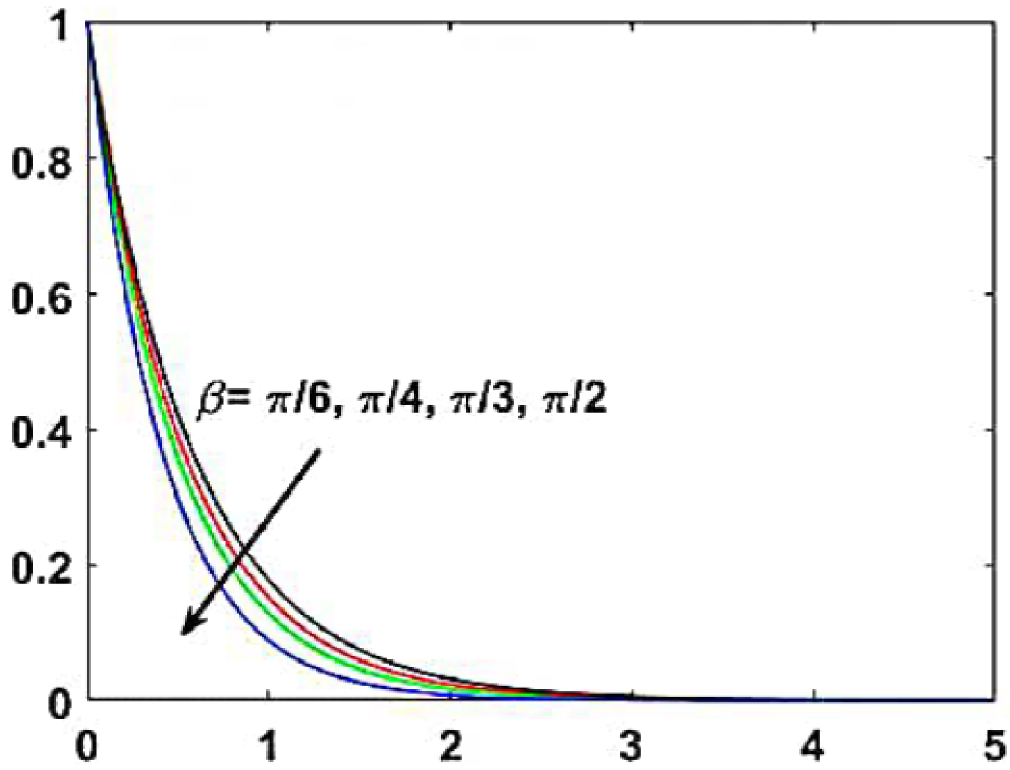




**Fig 3**: Effect of different values of $\beta$on temperature distribution.

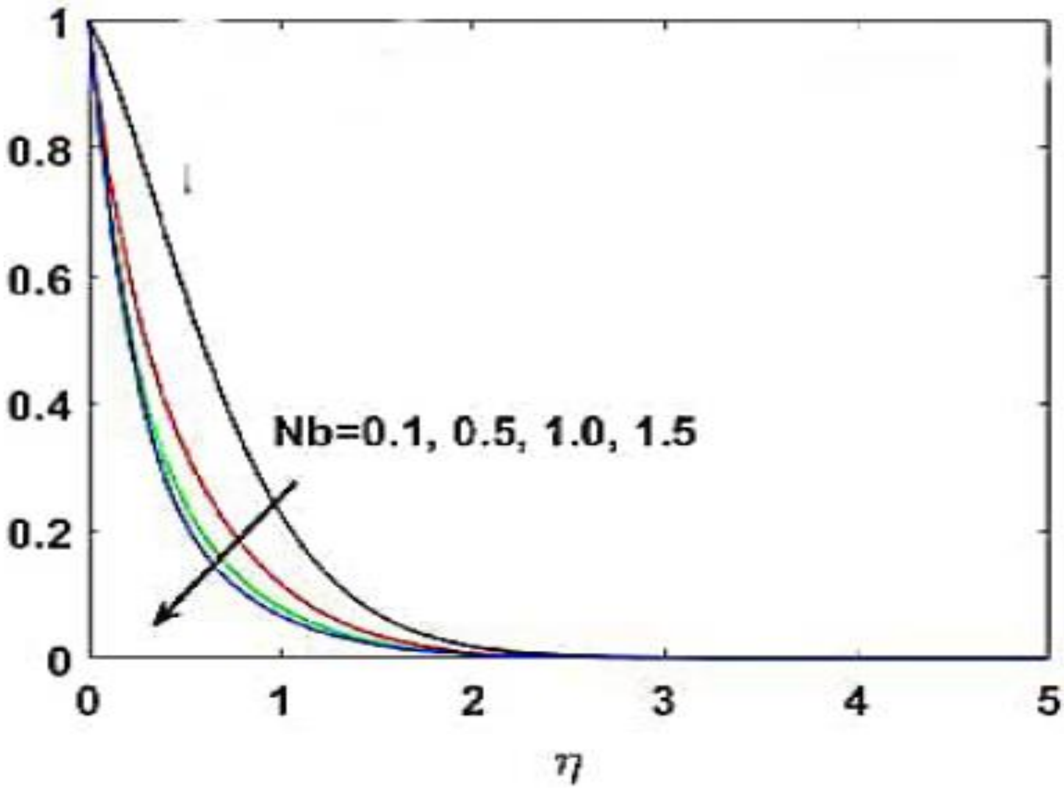


**Fig 4**: Variation of temperature distribution with respect to $\eta$.

## Conclusion

We have proposed heat and mass transfer in nanofluid flow in an exponential stretching surface with heat and mass suction. This method is easy to exploit in real world problems of heat and mass transfer. In this study, we have presented and analyzed the different conditions of exponential order. Fundamental laws of motion and heat transfer are validated successfully. Numerical methods along with figure to see the effect of different parameters are presented. Our main findings are that the heat transfer rate decreases as the Brownian motion rises.

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