IMPACT OF AWARENESS AND MOSQUITO CONTROL EFFORTS/EFFECTS/STRATEGIES ON THE SPREAD OF DENGUE INFECTION IN THE HUMAN POPULATION

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ABSTRACT
In this paper, a compartmental model is proposed to study the impact of awareness on the dynamics of dengue with the role of mosquito control strategies. The awareness is the responsiveness against the disease while control strategies are the effort due to which the mosquitoes are killed at constant rate. It is assumed that due to awareness of the disease some susceptible and all recovered individuals are entering into the aware class by following necessary precautionary measures to protect themselves from mosquito bites. It is clearly seen that the increase in the awareness rate decreases the densities of infectious populations of human as well as mosquitoes. But the elimination of the disease depends only on the killing rate of mosquitoes due the mosquito control efforts.

KEYWORDS: Awareness, controlling strategy; awareness; stability.
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1.Introduction.

Preventing or reducing dengue virus transmission depends entirely on control of the mosquito vectors orinterruption of human–vector contact. Today, therefore, the main aim of most dengue resistance programs is to reduce the densities of vector populations as much as possible and to maintain them at low levels. Activities to control transmission should target *Ae. Aegypti* (the main vector) in the habitats of its immature and adult stages in the household and immediate vicinity, as well as othersettings where human–vector contact occurs (e.g. schools, hospitals and workplaces). The mosquitoesproliferates in many purposely-filled household containers such as those used for domestic water storage and for decorative plants, as well as in a multiplicity of rain-filled habitats – including used tires, discarded foodand beverage containers, blocked gutters and buildings under construction. Typically, these mosquitoes do not fly far, the majority remaining within 100 meters of where they emerged. They feed almost entirely on humans, mainly during daylight hours, and both indoors and outdoors.Integrated vector management (IVM) is the strategic approach to vector controlpromoted by WHO and includes control of the vectors of dengue. The control of *Ae.Aegypti*is mainly achieved by eliminating container habitats which permit the development of the mosquitoes and their stages.

The habitats are eliminated by preventing access by mosquitoes to these containersor by frequently emptying and cleaning them, by removing the developing stagesusing insecticides or biological control agents, by killing the adult mosquitoes usinginsecticides, or by combinations of these methods.Where feasible,efforts may also be made to reduce the longevity of adult female mosquitoes by the useof insecticidal methods in order to lessen the risk of virus transmission.In selecting the most appropriate vector control method, or combination of methods,consideration should be given to the local ecology and behavior of the target species, theresources available for implementation, the cultural context in which control interventionsare carried out, the feasibility of applying them in a timely manner, and the adequacyof coverage.

Some man-made container habitats produce largenumbers of adult mosquitoes, whereas others are less productive. Consequently, controlefforts should target the habitats that are most productive and hence epidemiologicallymore important rather than all types of containers. Such targeted strategies require a thorough understanding of the local vectorecology and the attitudes and habits of residents pertaining to the containers.

The role of these controlling strategies/efforts can be studied by using mathematical models. They can provide a useful tool to better understand the dynamics of dengue epidemic. Different mathematical models have been proposed to study the spread of dengue. In general, the compartmental structure in which the susceptible, exposed, infectious and removed compartments for human and susceptible and infective compartments for mosquito are considered. But the spread of the disease in the population make the people to change their behavior to protect themselves from the disease [4]. Such change of behavior and attitude can be considered as awareness towards the disease and it makes vital change in the dimension of the population due to their preventive attempts. So it is necessary consider the effects such change of behavior to study and control infectious disease. When the dengue infected person gets recovered from the disease he or she have experienced the bitterness of the disease and keen not to have previous experience again. So takes all the necessary precautionary measures like to use mosquito nets, repellents etc. and eliminate the possibility of mosquito bite. To incorporate this idea an assumption of aware class is taken into account. Transfer in the aware class is not from only infected class but also from susceptible class due to the reason that some people learn to protect from the experience of the other and follow the precautionary measures.In other words, infection of one member makes the whole family to take necessary steps to be protected. Feng and Vealsco proposed and model to study the population dynamics of vector transmitted disease with two pathogen strains[5]. A model for the transmission of dengue fever in a constant human population and variable vector population is discussed by Estava and Vargas [6]. The same authors proposed another model [7] considering vertical and mechanical transmission in the vector population, to study the effects on the dynamics of the disease. In 2003 an SIR model is proposed by L. Allen et al. to study the influence of age structure in the human population for the transmission of Dengue hemorrhagic fever

(DHF) [8]. Some more mathematical models have been developed in the literature to gain insights into the transmission dynamics of dengue in a community (see, for instance, [9-18]).

2. Mathematical Model

The total human population of interest is divided into three mutually disjoint classes. The members of susceptible class (C_1) are at the risk of contracting DF.

Each member of aware class (C_2) is aware of the risk of DF and very much cautious to protect themselves from mosquito bite. The members of Dengue infected class (C_3) are getting infection due to infected mosquito bite.

Let $S(t)$, $S_h(t)$ and $I(t)$ be the number of individuals in three compartments respectively such that the total human population at time t is $N(t) = S(t) + S_h(t) + I(t)$.

Let $M(t)$ be the total population of mosquitoes which is classified into susceptible population $S_1(t)$ and infected population $I_1(t)$ at time t such that $M(t) = S_1(t) + I_1(t)$.

Further, for constructing the model some more assumption are taken as follows:

Let ω be the constant recruitment rates while d be the natural death rates in human population. Som susceptible are transferring the compartment from C_1 to C_2 at the rate m_1. This incorporates the direct transfer from susceptible to aware class. Infected human are treated and recovered at the rate γ. It is assumed that after recovery the infected person becomes aware as he or she understands the risk factors for the disease. The decay in the life expectancy of human population due to dengue virus is ignored.

Let b_1 and d_1 be the birth rate and death rate respectively in the mosquito population. The susceptible mosquitoes become dengue infected following the bite dengue infected human with the transmission coefficient α_2. The infected mosquito transmits the virus to susceptible human when it bites. Also, the vertical transmission of dengue infection is considered in mosquito population and there is no change in the birth rate and death rate of mosquitoes due to dengue infection. The constant d_2 is the induced death rate in the mosquito population due to mosquito control strategies. The model is developed according to the above discussed assumptions.

$$\begin{aligned}
\frac{\mathrm{dS}}{\mathrm{dT}} &= \omega - m_1 S - \alpha_1 S I_1 - dS \\
\frac{\mathrm{d}S_h}{\mathrm{dT}} &= m_1 S + \gamma I - dS_h \\
\frac{\mathrm{d}I}{\mathrm{dT}} &= \alpha_1 S I_1 - (\gamma + d) I \\
\frac{\mathrm{dS_1}}{\mathrm{dT}} &= (b_1 - d_1)\left(1 - \frac{S_1 + I_1}{K}\right) S_1 - \alpha_2 S_1 I - d_2 S_1 \\
\frac{\mathrm{dI_1}}{\mathrm{dT}} &= (b_1 - d_1)\left(1 - \frac{S_1 + I_1}{K}\right) I_1 + \alpha_2 S_1 I - d_2 I_1
\end{aligned} \tag{2.1}$$

All parameters are assumed to be nonnegative. The initial conditions associated with the system (2.1) are:

$$\begin{aligned}
&S(0) = S_0 \geq 0,\ S_h(0) = S_{h0} \geq 0,\ I(0) = I_0 \geq 0, \\
&S_1(0) = S_{10} \geq 0, I_1(0) = I_{10} \geq 0
\end{aligned} \tag{2.2}$$

The feasible region for (2.1) is the positive octant of R^5. The model (2.1) is obviously well-posed. Further, by adding the first three equations in (2.1), we have

$$\frac{\mathrm{dN}}{\mathrm{dT}} = \omega - DN$$

Also, it is clear from the last two equation of the system (2.1)

$$\frac{\mathrm{dM}}{\mathrm{dT}} = (b_1 - d_1)\left(1 - \frac{M}{K}\right) M - d_2 M$$

Therefore, the following set Ω is positive invariant.

$$\Omega = \left\{ \left(S,\ S_h,\ I,\ S_1,\ I_1\right) \in R^5 : 0 \leq S + S_h + I \leq \frac{\omega}{d},\ 0 \leq I_1 \leq \frac{(b_1 - d_1 - d_2)K}{b_1 - d_1} \right\}$$

From the equation (2.2), it is clear that

for $t \to \infty,\ S_1 + I_1 \to \dfrac{(b_1 - d_1 - d_2)K}{b_1 - d_1}$

For the limiting population, if the following substitution is made in the fifth equation of the system (2.1) then there is no need to consider the fourth equation of the system explicitly.

$$S_1 = \frac{(b_1 - d_1 - d_2)K}{b_1 - d_1} - I_1$$

Also, S_h does not appear in the remaining other equations except second equation it is sufficient to consider only first third and fifth equations to determine the dynamics of the system (2.1). So in the rest of this paper, the following three-dimensional nonlinear system (2.3) will be studied and analyzed. Taking $g_1 = b_1 - d_1$.

$$\begin{aligned}\frac{dS}{dT} &= \omega - m_1 S - \alpha_1 S I_1 - DS \\ \frac{dI}{dT} &= \alpha_1 S I_1 - (\gamma + D) I \\ \frac{dI_1}{dT} &= \alpha_2 \left(\frac{(g_1 - d_2)K}{g_1} - I_1 \right) I\end{aligned} \tag{2.3}$$

The following equilibrium points of the non-linear dynamical system (2.4) are obtained:

The disease free equilibrium point $E_0(S^*, 0, 0)$=$E_0\left(\frac{\omega}{(d+m_1)}, 0, 0\right)$ always exists.

The endemic equilibrium $E_1(\bar{S}, \bar{I}, \bar{I}_1)$ is obtained by solving the three simultaneous equations:

$$\omega - m_1 S - \alpha_1 S I_1 - dS = 0 \tag{2.3a}$$

$$\alpha_1 S I_1 - (\gamma + d) I = 0 \tag{2.3b}$$

$$\alpha_2 \left(\frac{(g_1 - d_2)K}{g_1} - I_1 \right) I = 0 \tag{2.3c}$$

Solving (2.3c) for I_1, gives

$$\bar{I}_1 = \frac{(g_1 - d_2)K}{g_1}. \tag{2.4a}$$

Substituting the value of (2.4a) in (2.3b) and solving it for S gives

$$\bar{S} = \frac{g_1 \omega}{\eta};\ \eta = g_1(A_1 K + d + m_1) - A_1 d_2 K \tag{2.4b}$$

Substitution of (2.4a) and (2.4b) in the equation (2.3a) yields

$$\bar{I} = \frac{A_1 (g_1 - d_2) \omega K}{(d + \gamma)\eta} \tag{2.4c}$$

Therefore, the system (2.3) has a unique endemic equilibrium point $E_1(\bar{S}, \bar{I}, \bar{I}_1)$, if

$$g_1 > d_2, \eta > 0$$

(2.5)

3 Stability Analysis

The local stability of different equilibrium points of (2.3) can be discussed on the basis of stability matrix at $\mathrm{E}(\mathrm{S}, I, I_1)$. It is computed as

$$J = \begin{bmatrix} -(d+m_1+A_1 I_1) & 0 & -A_1 S \\ A_1 I_1 & -(d+\gamma) & A_1 S \\ 0 & A_2\left\{\dfrac{(g_1-d_2)K}{g_1} - I_1\right\} & -A_2 I \end{bmatrix}$$

The stability results are stated below in the form of theorems.

Theorem 1 The disease-free equilibrium $E_0\left(\mathrm{S}^*, 0, 0\right)$ is locally asymptotically stable when

$$g_1 < d_2 \qquad (3.1)$$

Proof: The characteristic equation of stability matrix at E_0 is obtained as

$$(\lambda + d + m_1)(\lambda^2 + a_1\lambda + a_2) = 0;$$
$$a_1 = d + \gamma > 0, a_2 = \frac{A_1 A_2 (d_2 - g_1)\omega K}{(d+m_1)g_1} > 0$$

Clearly by using Routh Hurwitz's criteria it is easy to check that the eigenvalues of the stability matrix J at E_0 have negative real part whenever $g_1 < d_2$ .Hence, the result is proved.

□

Clearly, when $g_1 < d_2$ there will be no disease in the population.

Remark: It may be noted from (2.5) that the local stability of E_0 ensures the non-existence of other equilibrium point E_1. Therefore, when $g_1 < d_2$, one may expect E_0 to be globally stable which is proved by using geometric approach for the global stability in the following theorem using geometric approach for the global stability [19].

Theorem 2 The locally stable disease-free equilibrium, $E_0(\mathrm{S}^*, 0, 0)$ is always globally asymptotically stable if

$$(2d + \gamma + m_1) > \frac{A_1\omega}{d+m_1},\ g_1 < d_2. \qquad (3.2)$$

Proof: The Jacobean matrix of the system (2.3) around a equilibrium point $E_0(\mathrm{S}^*, 0, 0)$ is

$$J(E_0) = \begin{bmatrix} -(d+m_1) & 0 & -\frac{A_1\omega}{(d+m_1)} \\ 0 & -(d+\gamma) & \frac{A_1\omega}{(d+m_1)} \\ 0 & A_2\left\{\frac{(g_1-d_2)K}{g_1}\right\} & 0 \end{bmatrix}$$

The second additive compound matrix is obtained as

$$J^{(2)}(E_0) = \begin{bmatrix} -(2d+m_1+\gamma) & \frac{A_1\omega}{(d+m_1)} & \frac{A_1\omega}{(d+m_1)} \\ A_2\left\{\frac{(g_1-d_2)K}{g_1}\right\} & -(d+m_1) & 0 \\ 0 & 0 & -(d+\gamma) \end{bmatrix}$$

Define a function $P \in C^1$ as

$$P(S,I,I_1) = diag\{S,S,S\}$$

Let f be a vector field of (2.3), it may be noted that

$$P_f P^{-1} = diag\left\{\frac{\dot{S}}{S},\frac{\dot{S}}{S},\frac{\dot{S}}{S}\right\} \text{ and } PJ^{(2)}P^{-1} = J^{(2)}$$

Taking $B = P_f P^{-1} + PJ^{(2)}P^{-1}$

$$B = \begin{bmatrix} \frac{\dot{S}}{S} - (2d+m_1+\gamma) & \frac{A_1\omega}{(d+m_1)} & \frac{A_1\omega}{(d+m_1)} \\ A_2\left\{\frac{(g_1-d_2)K}{g_1}\right\} & \frac{\dot{S}}{S} - (d+m_1) & 0 \\ 0 & 0 & \frac{\dot{S}}{S} - (d+\gamma) \end{bmatrix}$$

Considering the block matrix as $B = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix}$, where

$$B_{11} = \frac{\dot{S}}{S} - (2d + m_1 + \gamma) \ , \qquad B_{12} = \begin{bmatrix} \frac{A_1\omega}{(d+m_1)} & \frac{A_1\omega}{(d+m_1)} \end{bmatrix},$$

$$B_{21} = \begin{bmatrix} A_2 \left\{ \frac{(g_1 - d_2)K}{g_1} \right\} \\ 0 \end{bmatrix}, \qquad B_{22} = \begin{bmatrix} \frac{\dot{S}}{S} - (d+m_1) & 0 \\ 0 & \frac{\dot{S}}{S} - (d+\gamma) \end{bmatrix}$$

The Lozinski norm $\mu(B)$ with respect the norm $|.|$ can be estimated as follows [20]:

$$\mu(B) \le \max\{g_1, \ g_2\} = \max\{\mu(B_{11}) + |B_{12}|, \mu(B_{22}) + |B_{21}|\}$$

$$g_1 = \frac{\dot{S}}{S} - (2d + \gamma + m_1) + \frac{A_1\omega}{d + m_1},$$

$$g_2 = \frac{\dot{S}}{S} - d + \max\{-m_1, -\gamma\} + A_2 \left\{ \frac{(g_1 - d_2)K}{g_1} \right\}$$

If the conditions(3.2)hold then there exist a number $w > 0$ such that

$$\mu(B) \le \frac{\dot{S}}{S} - w$$

Hence, $\frac{1}{t}\int_0^t \mu(B)ds \le \frac{1}{t}\frac{\log(S)}{S_0} - w$.

The Bendixson's criterion given by [21] is thus verified if the conditions given by (3.2) are true and the result is proved.

□

Theorem 3 The equilibriumpoint $E_1(\overline{S}, \overline{I}, \overline{I}_1)$, if it existsis locally asymptotically stable.

Proof: The characteristic equation of stability matrix at E_1 is obtained as

$$(\lambda + d + \gamma)\left[\lambda + \frac{\eta}{g_1}\right]\left[\lambda + \frac{A_1A_2(g_1 - d_2)K\omega}{(d+\gamma)\eta}\right] = 0$$

Clearly, all the roots of the characteristic equation (3.6) are negative if the equilibrium point E_1 exists.

Hence, the result is proved. □

Theorem 4 The endemic equilibrium point $E_1(\overline{S}, \overline{I}, \overline{I}_1)$ is globally asymptotically stable if

$$\begin{aligned}&\frac{\eta}{g_1}+(d+\gamma)>\frac{A_1 g_1\omega}{\eta}\\&\frac{A_1A_2(g_1-d_2)\omega K}{(d_1+\gamma)\eta}>\max\{-\frac{\eta}{g_1}+\frac{A_1(g_1-d_2)K}{g_1},-(d+\gamma)\}\end{aligned}. \tag{3.3}$$

Proof: The Jacobian matrix of the system (2.3) around the equilibrium point $E_1(\bar{S},\bar{I},\bar{I}_1)$ is

$$J(E_1)=\begin{bmatrix}-\frac{\eta}{g_1} & 0 & -\frac{A_1g_1\omega}{\eta}\\ \frac{A_1(g_1-d_2)K}{g_1} & -(d+\gamma) & \frac{A_1g_1\omega}{\eta}\\ 0 & 0 & -\frac{A_1A_2(g_1-d_2)\omega K}{(d_1+\gamma)\eta}\end{bmatrix}$$

The second additive compound matrix is

$$J^{(2)}(E_1)=\begin{bmatrix}-\frac{\eta}{g_1}-(d+\gamma) & \frac{A_1g_1\omega}{\eta} & \frac{A_1g_1\omega}{\eta}\\ 0 & -\frac{\eta}{g_1}-\frac{A_1A_2(g_1-d_2)\omega K}{(d_1+\gamma)\eta} & 0\\ 0 & \frac{A_1(g_1-d_2)K}{g_1} & -(d+\gamma)-\frac{A_1A_2(g_1-d_2)\omega K}{(d_1+\gamma)\eta}\end{bmatrix}$$

Define a function $P\in C^1$ as

$$P(S,I,I_1)=diag\{S,S,S\}$$

Let f be a vector field of (2.3), it may be noted that

$$P_fP^{-1}=diag\left\{\frac{\dot{S}}{S},\frac{\dot{S}}{S},\frac{\dot{S}}{S}\right\} \text{ and } PJ^{(2)}P^{-1}=J^{(2)}$$

Taking

$$B=P_fP^{-1}+PJ^{(2)}P^{-1}$$

$$B = \begin{bmatrix} \frac{\dot{S}}{S} - \frac{\eta}{g_1} - (d+\gamma) & \frac{A_1 g_1 \omega}{\eta} & \frac{A_1 g_1 \omega}{\eta} \\ 0 & \frac{\dot{S}}{S} - \frac{\eta}{g_1} - \frac{A_1 A_2 (g_1 - d_2)\omega K}{(d_1+\gamma)\eta} & 0 \\ 0 & \frac{A_1 (g_1 - d_2) K}{g_1} & \frac{\dot{S}}{S} - (d+\gamma) - \frac{A_1 A_2 (g_1 - d_2)\omega K}{(d_1+\gamma)\eta} \end{bmatrix}$$

Considering the block matrix as $B = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix}$, where

$$B_{11} = \frac{\dot{S}}{S} - \frac{\eta}{g_1} - (d+\gamma), \qquad B_{12} = \begin{bmatrix} \frac{A_1 g_1 \omega}{\eta} & \frac{A_1 g_1 \omega}{\eta} \end{bmatrix},$$

$$B_{21} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \; B_{22} = \begin{bmatrix} \frac{\dot{S}}{S} - \frac{\eta}{g_1} - \frac{A_1 A_2 (g_1 - d_2)\omega K}{(d_1+\gamma)\eta} & 0 \\ + \frac{A_1 (g_1 - d_2) K}{g_1} & \frac{\dot{S}}{S} - (d+\gamma) - \frac{A_1 A_2 (g_1 - d_2)\omega K}{(d_1+\gamma)\eta} \end{bmatrix}$$

The Lozinski norm $\mu(B)$ with respect the norm $|.|$ can be estimated as follows [20]:

$$\mu(B) = \max\{g_1, g_2\} = \max\{\mu(B_{11}) + |B_{12}|, \mu(B_{22}) + |B_{21}|\}$$

$$g_1 = \frac{\dot{S}}{S} - \frac{\eta}{g_1} - (d+\gamma) + \frac{A_1 g_1 \omega}{\eta},$$

$$g_2 = \frac{\dot{S}}{S} - \frac{A_1 A_2 (g_1 - d_2)\omega K}{(d_1+\gamma)\eta} + \max\{-\frac{\eta}{g_1} + \frac{A_1 (g_1 - d_2) K}{g_1}, -(d+\gamma)\}$$

If the conditions(3.3) hold than there exist a number $w > 0$ such that

$$\mu(B) \le \frac{\dot{S}}{S} - w$$

Hence, $\frac{1}{t}\int_0^t \mu(B) ds \le \frac{1}{t} \frac{\log(S)}{S_0} - w$.

The Bendixson's criterion given by [21] is thus verified if the conditions given in (3.3) are true and the result is proved.

□

From the above theorems it can be concluded that for the sufficient awareness the disease free equilibrium points remains globally stable and the disease will disappear from the population.

4. Conclusion

A five dimensional model is proposed and analyzed to get the insights of dengue transmission in the human population. It is assumed that some susceptible and all the recovered individuals are convinced that the dengue infection is very dangerous.So, they join aware compartment and are taking sufficient care to protect themselves from the mosquito bites.The model has two equilibrium points namely the disease-free equilibrium and endemic equilibrium point. The disease-free equilibrium remains stable if the growth (birth minus death) rate of mosquitoes is smaller than the death caused by mosquito controlling strategies. For this case the disease will be eliminated from the human population and there will be no infected mosquitoes in the environment. But if the growth (birth minus death) rate of mosquitoes exceeds than the death caused by mosquito controlling strategies then the equilibrium with the disease comes into existence and remain stable which indicates the disease persists in the population. Increase in the rateawareness decreases density of infected human as well as infected mosquitoes but for the elimination of the disease death rate due to mosquito control strategies is more important.

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