Heat and mass transfer on Unsteady MHD flow through porous medium over an infinite vertical plate

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Abstract:
In this paper, we have considered the unsteady MHD flow through porous medium over an infinite vertical plate under the influence of uniform transverse magnetic field of strength H_0. The flow is induced by a general time dependent movement of vertical plate. The exact solutions for the velocity, temperature and concentration are obtained making use of Laplace transform technique. The influence of various governing flow parameters on the velocity, temperature and concentration is analysed graphically, and numerical solutions for the skin friction, Nusselt number and Sherwood number are also obtained in the tabular forms.

Keywords: Heat and mass transfer, infinite vertical flat plate, Laplace transform technique, MHD flows, porous medium.
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1. Introduction

The unsteady MHD free convection flow with heat and mass transfer past a vertical porous plate is receiving considerable attention of many researchers because of its a range of applications. Permeable porous plates are worned in the filtration processes and also for a heated body to keep its temperature constant and to make the heat insulation of the surface more effective. Sometimes along with the free convection currents caused by difference in temperature the flow is also affected by the differences in concentration or material constitution. The influence of magnetic field has attracted the investigators in view of its various applications in MHD generators, plasma studies, nuclear reactors, geothermal energy extractions and boundary layer control in the field of aerodynamics. Moreover, considerable interest has been shown in radiation interaction with convection for heat and mass transfer in fluids. This is appropriate to the significant role of thermal radiation in the surface heat transfer when convection heat transfer is small, particularly in free convection problems involving absorbing-emitting fluids. Several workers have studied the problem of free convection flow with mass transfer. Singh et al. [1] have studied MHD free convective flow past an accelerated vertical porous plate by finite difference method. Free convection and mass transfer flow through porous medium bounded by an infinite vertical limiting surface with constant suction have been analyzed by Raptis et al [2]. Unsteady free convection interaction with thermal radiation in a boundary layer flow past a vertical porous plate has been discussed by Sattar et al [3]. Das et al [4] have studied numerical solution of mass transfer effects on unsteady flow past an accelerated vertical porous plate with suction. Das et al [5] have studied Mass transfer effects on MHD flow and heat transfer past a vertical porous plate through porous medium under oscillatory suction and heat source. Applied magnetic field on transient convective flow in a vertical channel has been discussed by Jah [6].Kim [7] has investigated the problem of unsteady MHD convective heat transfer past a semi-infinite vertical porous moving plate with variable suction. Soundalgekar et al. [8] have analyzed the transient free convection flow of a viscous dissipative fluid past a semi infinite vertical plate. Mohameda et al [9] have analyzed finite element analysis of hydromagnetic flow and heat transfer of a heat generation fluid over a surface embedded in a non-darcian porous medium in the presence of chemical reaction. The Soret effect on free convective unsteady MHD flow over a vertical plate with heat source has been analyzed by Bhavana et al. [10]. Abd EL–Naby et al [11] employed implicit finite finite-difference methods to study the effect of radiation on MHD unsteady free convection flow past a semi-infinite vertical porous plate but did not take into account the viscous dissipation. Recently, Alam and Rahman [12] have examined Dufour and Soret effects on mixed convection flow past a vertical porous flat plate with variable suction embed medium for a hydrogen-air mixture as the nonchemical reacting fluid pair. Anwa. et al. [13] examined the combined effects of Soret and Dufour diffusion and porous on laminar magneto-hydrodynamic mixed convection heat and mass transfer of an electrically-conducting, Newtonian, Boussinesq fluid from a vertical stretching surface in a Darcian porous medium under uniform transverse magnetic field. Hady et al. [14] studied the problem of free convection flow along a vertical wavy surface embedded in electrically conducting fluid saturated porous media in the presence of internal heat generation or absorption effect. Makinde [15] have discussed Free-convection flow with thermal radiation and mass transfer past a moving vertical porous plate.

The effect of radiation on MHD flows and heat transfer must be considered when high temperatures are reached. The study of heat generation or absorption effects in moving fluids is important in view of several physical problems, such as fluid undergoing exothermic or endothermic chemical reactions. When the mass flux contains a term that depends on the temperature gradient then the soret effect arises focus of our study is the effect on free convection of the addition of a second fluid. Convection in binary fluids is considerably more complicated than that in pure fluids. Both temperature and concentration gradients contribute to the initiation of convection and each may be stabilizing or destabilizing. Even when a concentration gradient is not externally imposed (the thermosolutal problem) it can be created by the applied thermal gradient via the soret effect. Mbeledogu et al. [16] have discussed the unsteady MHD free convection flow of a compressible fluid past a moving vertical plate in the presence of radioactive heat transfer Ahmmed et al. [17] have discussed Numerical Study on MHD free convection and mass transfer flow flat plate. In view of the above investigations, in this paper, we have considered the unsteady MHD flow through porous medium over an infinite vertical plate under the influence of uniform transverse magnetic field.

2. Mathematical Formulation and Solution of the Problem:

We consider the unsteady MHD flow of an electrically conducting viscous incompressible fluid through porous medium over an infinite vertical plate subjected to a uniform transverse magnetic field of strength B_0 normal to plate. The physical configuration of the problem is presented in the Fig. 1.

We choose a Cartesian co-ordinate system $O(x, y, z)$ such that *x*, *y* axes respectively are in the vertical upward and perpendicular directions on the plane. i.e., the *x*-axis is along the plate in the upward direction, *y*-axis normal to it and *z*-axis normal to the *xy*-plane. The fluid is permeated by a uniform transverse magnetic field of strength B_0 applied to along the *y*-axis. For time $t < 0$, the stationary plate and the fluid are at the constant temperature T_∞ and species concentration C_∞.

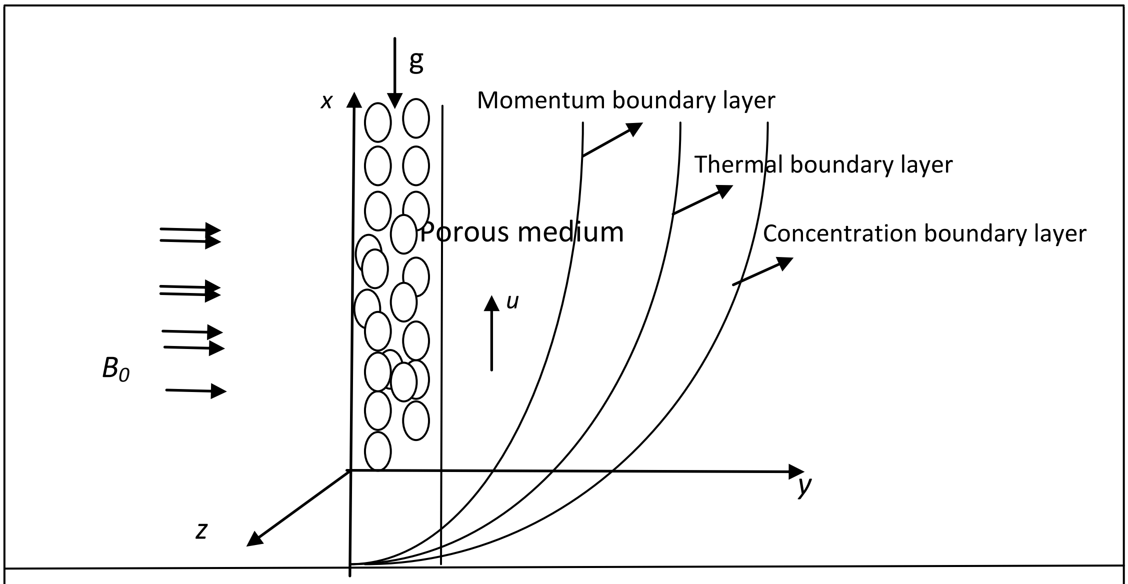

Figure 1: Physical configuration of the problem

At time $t = 0$, the plate begins to move with a time dependent velocity $u_0 \, f(t)$ in its own plane along the *x*-axis and the temperature of the plate is raised or lowered to $T_\infty + (T_w - T_\infty)\frac{t}{t_0}$, when $t < t_0$ and thereafter for $t > t_0$ it is maintained at a uniform temperature T_w. Also for time $t > 0$ species concentration is raised to C_∞. Since the plate is of infinite extent in x and z direction, and is electrically non-conducting, all the physical quantities except pressure, are functions of *y* and *t* only.

Magnetic Reynolds number is small, and hence the induced magnetic field produced by the fluid motion is negligible in comparison to the applied magnetic field $B=(0,B_0,0)$, the governing equations for the unsteady hydro magnetic flow of an electrically conducting viscous incompressible fluid through porous medium over an infinite plate are given by,

$$\frac{\partial u}{\partial t}=\nu\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_0}{\rho}u-\frac{\nu}{K_1}u+g\beta(T-T_\infty)+g\beta^*(C-C_\infty) \quad (1)$$

$$\frac{\partial T}{\partial t}=\frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y^2}-\frac{Q_0}{\rho C_p}(T-T_\infty) \quad (2)$$

$$\frac{\partial C}{\partial t}=D\frac{\partial^2 C}{\partial z^2} \quad (3)$$

Assuming no-slip between the plate and fluid, the initial and boundary conditions are,

$$u=0,\ T=T_\infty,\ C=C_\infty,\ \text{for } t\le 0 \text{ and } y\ge 0 \quad (4)$$

$$u=u_0\, f(t),\ C=C_w,\ \text{at } t>0 \text{ and } y=0 \quad (5)$$

$$T=T_\infty+(T_w-T_\infty)\frac{t}{t_0} \quad \text{at } y=0 \quad \text{for } 0<t\le t_0 \quad (6)$$

$$T=T_\infty \quad \text{at } y=0 \quad \text{for } t>t_0 \quad (7)$$

$$u=0,\ T\to T_\infty,\ C\to C_\infty,\ \text{as } y\to\infty \text{ for } t>0 \quad (8)$$

Introducing the following non-dimensional quantities:

$$y^*=\frac{y}{u_0 t_0},\ u^*=\frac{u}{t_0}, t^*=\frac{t}{t_0},\ T^*=\frac{T-T_\infty}{T_w-T_\infty},\ C^*=\frac{C-C_\infty}{C_w-C_\infty}$$

Making use of non-dimensional quantities (dropping asterisks), the governing equations (1), (2) and (3) can be written as

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial y^2}-\left(M^2+\frac{1}{K}\right)u+G_r T+G_m C \quad (9)$$

$$\frac{\partial T}{\partial t}=\frac{1}{\Pr}\frac{\partial^2 T}{\partial y^2}-\phi T \quad (10)$$

$$\frac{\partial C}{\partial t}=\frac{1}{Sc}\frac{\partial^2 C}{\partial y^2} \quad (11)$$

Where,

$M^2 = \frac{\sigma B_0^2 \nu}{\rho u_0^2}$ is the Hartmann number (Magnetic field parameter), $K = \frac{K_1 u_0^2}{\nu^2}$ is the Porosity parameter, $G_r = \frac{g\beta\nu(T_w - T_\infty)}{u_0^3}$ is the thermal Grashof number, $G_m = \frac{g\beta^*\nu(C_w - C_\infty)}{u_0^3}$ is the mass Grashof number, $\Pr = \frac{\rho\nu C_p}{k}$ is Prandtl parameter, $\phi = \frac{Q_0\nu}{\rho C_p u_0^2}$ is the Heat absorption parameter and $Sc = \frac{\nu}{D}$ is the Schmidt number.

The corresponding non-dimensional boundary conditions

$u = 0, T = 0, C = 0,$ for $t \le 0$ and $y \ge 0$ (12)

$u = f(t), \; C = 1,$ for $t > 0$ and $y = 0$ (13)

$T = t$ for $0 < t \le 1$ at $y = 0$ (14)

$T = 1$ for $t > 1$ at $y = 0$ (15)

$u = 0, T = 0, C = 0,$ for $t > 0$ as $y \to \infty$ (16)

The system of differential equations (9) to (10) together with the initial and boundary conditions (12) to (16) describes our mathematical model for the unsteady MHD free convection heat and mass transfer flow of an electrically conducting viscous incompressible and heat absorbing fluid past a vertical flat plate with ramped wall temperature. The differential equations (9) to (10) subjected to the initial and boundary conditions (12) to (16) are solved analytically making use of Laplace Transform technique.

Transforming equation (11) we get,

$$s\bar{C}(y,s) - C(y,0) = \frac{1}{Sc}\frac{d^2\bar{C}}{dy^2} \quad (17)$$

Using the initial condition (12), we have,

$$\frac{d^2\bar{C}}{dy^2} - s\,Sc\,\bar{C}(y,s) = 0 \quad (18)$$

The solution of the equation (18) is

$$\bar{C}(y,s) = Ae^{\sqrt{sSc}\,y} + Be^{-\sqrt{sSc}\,y} \quad (19)$$

Where *A* and *B* are arbitrary constants.

Again using above boundary conditions (13) and (16), we get,

$$\bar{C}(y,s) = \frac{1}{s}e^{-\sqrt{sSc}\,y} \quad (20)$$

Taking inverse Laplace transforms for the equation (20), we obtain the exact solution for the Species concentration is given by

$$C(y,t) = erfc\left(\frac{y}{2}\sqrt{\frac{Sc}{t}}\right) \tag{21}$$

Also transforming equation (10);

$$s\overline{T}(y,s) - T(y,0) = \frac{1}{\Pr}\frac{d^2\overline{T}}{dy^2} - \phi\overline{\theta}(y,s) \tag{22}$$

Using initial condition (12), it reduces to:

$$\frac{d^2\overline{T}}{dy^2} - (s+\phi)\Pr\,\overline{T}(y,s) = 0 \tag{23}$$

The solution of this equation (23)

$$\overline{T}(y,s) = A_1\, e^{-y\sqrt{(s+\phi)\Pr}} + A_2\, e^{y\sqrt{(s+\phi)\Pr}} \tag{24}$$

Where A_1 and A_2 are arbitrary constants.

Values of A_1 and A_2 can be computed using (14), (15) and (15);

$$\overline{T}(y,s) = \frac{1}{s^2}(1-e^{-s})e^{-y\sqrt{(s+\phi)\Pr}} \tag{25}$$

Taking inverse Laplace transform for the equation (25), we obtain the exact solution for the fluid temperature is given by,

$$T(y,t) = P_1(y,t) - H(t-1)\,P_1(y,t-1) \tag{26}$$

$H(t-1)$ is the Heavisides unit step function.

$$P_1(y,t) = \left(\frac{t}{2} + \frac{\Pr y}{4\sqrt{\Pr\phi}}\right)e^{\sqrt{\Pr\phi}y}erfc\,(t_1) + \left(\frac{t}{2} - \frac{\Pr y}{4\sqrt{\Pr\phi}}\right)e^{-\sqrt{\Pr\phi}y}erfc\,(t_2)$$

Where, $t_1 = \sqrt{\phi t} + \sqrt{\frac{\Pr}{t}}\frac{y}{2}$, $t_2 = -\sqrt{\phi t} + \sqrt{\frac{\Pr}{t}}\frac{y}{2}$

Similarly, taking the Laplace transform to the equation (9) and making use of the initial and boundary conditions (13) to (15), it reduces to

$$\frac{d^2\overline{u}}{dy^2} - \left[s + \left(M^2 + \frac{1}{K}\right)\right]\overline{u}(y,s) = -G_r\, L\{T(y,t)\} - G_m\, L\{C(y,t)\} \tag{27}$$

The solution of the equation (27) is

$$\overline{u}(y,s) = A_3\, e^{-y\sqrt{s+\left(M^2+\frac{1}{K}\right)}} + A_4\, e^{y\sqrt{s+\left(M^2+\frac{1}{K}\right)}} - \frac{Gr}{\Pr-1}\frac{(1-e^{-s})\, e^{-y\sqrt{(s+\phi)\Pr}}}{s^2\left(s+\dfrac{\phi\Pr-(M^2+(1/K))}{\Pr-1}\right)}$$

$$-\frac{G_m}{Sc-1}\frac{e^{-y\sqrt{sSc}}}{s\left(s-\dfrac{M^2+(1/K)}{Sc-1}\right)} \tag{28}$$

Applying the boundary conditions (13) and (16) for (28), i.e.,

The boundary conditions on *u* are

$$u(y,t)=\begin{cases} f(t) & \text{at } \ y=0 \\ 0 & \text{at } \ y\to\infty \end{cases} \tag{29}$$

Then $\bar{u}(y,s)=L\{f(t)\}=\bar{f}(s)$

Since *u* is finite, then $\bar{u}$ is finite. So that $A_4=0$ then equation (28) becomes,

$$\bar{u}(y,s)=A_3\, e^{-y\sqrt{s+\left(M^2+\frac{1}{K}\right)}}-\frac{G_r}{\Pr-1}\frac{(1-e^{-s})\, e^{-y\sqrt{(s+\phi)\Pr}}}{s^2\left(s+\dfrac{\phi\Pr-(M^2+(1/K))}{\Pr-1}\right)}$$

$$-\frac{G_m}{Sc-1}\frac{e^{-y\sqrt{sSc}}}{s\left(s-\dfrac{M^2+(1/K)}{Sc-1}\right)} \tag{30}$$

Making use of the boundary condition (29), to determine A_3 and then the equation (30) reduces to

$$\bar{u}(y,s)=\left\{\ \bar{f}(s)+\frac{G_r\,(1-e^{-s})}{\Pr-1}\frac{1}{s^2\left(s+\dfrac{\phi\Pr-(M^2+(1/K))}{\Pr-1}\right)}+-\frac{G_m}{1-Sc}\frac{1}{s\left(s-\dfrac{M^2+(1/K)}{Sc-1}\right)}\ \right\}$$

$$e^{-y\sqrt{s+\left(M^2+\frac{1}{K}\right)}}-\frac{G_r(1-e^{-s})}{\Pr-1}\frac{1}{s^2\left(s+\dfrac{\phi\Pr-(M^2+(1/K))}{\Pr-1}\right)}e^{-y\sqrt{(s+\phi)\Pr}}$$

$$-\frac{G_m}{Sc-1}\frac{1}{s\left(s-\dfrac{M^2+(1/K)}{Sc-1}\right)}e^{-y\sqrt{sSc}} \tag{30}$$

Taking the inverse Laplace transform to the equation (30), we obtain the exact solution for the velocity of the fluid are given by

$$u(y,t)=P(y,t)+\frac{G_r}{1-\Pr}\left[P_2(y,t)-H(t-1)\,P_2(y,t-1)\right]+\frac{G_m}{1-Sc}P_3(y,t) \tag{31}$$

$$P(y,t)=L^{-1}\left\{\ \bar{f}(s)\, e^{-y\sqrt{s+\left(M^2+\frac{1}{K}\right)}}\ \right\}$$

$$P_2(y,t)=\left(\frac{at-1}{2a^2}+\frac{\Pr y}{4a\sqrt{\Pr\phi}}\right)e^{\sqrt{\Pr\phi}y}\ erfc(t_1)+\left(\frac{at-1}{2a^2}-\frac{\Pr y}{4a\sqrt{\Pr\phi}}\right)e^{-\sqrt{\Pr\phi}y}\ erfc(t_2)+$$

$$+\frac{e^{-at}}{2a^2}\left(e^{\sqrt{d-a\Pr}y}erfc(t_3)-e^{-\sqrt{d-a\Pr}y}erfc(t_4)\right)-$$

$$-\left(\frac{at-1}{2a^2}+\frac{y}{4a\sqrt{M^2+(1/K)}}\right)e^{\sqrt{(M^2+(1/K))}y}\ erfc(t_5)-$$

$$-\left(\frac{at-1}{2a^2}-\frac{y}{4a\sqrt{M^2+(1/K)}}\right)e^{-\sqrt{(M^2+(1/K))}y}\ erfc(t_6)-$$

$$-\frac{e^{-at}}{2a^2}\left(e^{\sqrt{(M^2+(1/K))-ay}}erfc(t_7)-e^{-\sqrt{(M^2+(1/K))-ay}}erfc(t_8)\right)-$$

$$P_3(y,t)=\frac{1-Sc}{M^2+(1/K)}erfc(t_9)-\frac{e^{-bt}}{2b}\left[e^{\sqrt{-aby}}\ erfc(t_{10})+e^{-\sqrt{-aby}}\ erfc(t_{11})\right]-$$

$$-\frac{1}{2b}\left[e^{\sqrt{M^2+(1/K)}y}\ erfc(t_5)+e^{-\sqrt{M^2+(1/K)}y}\ erfc(t_6)\right]$$

$$+\frac{e^{-bt}}{2b}\left[e^{\sqrt{M^2+(1/K)-by}}\ erfc(t_{12})+e^{-\sqrt{M^2+(1/K)-by}}\ erfc(t_{13})\right]$$

The equations (21), (26) and (31) represent the analytical solutions for the unsteady MHD free convection heat and mass transfer flow of an electrically conducting viscous incompressible and heat absorbing fluid past a vertical flat plate with ramped temperature in the presence of uniform transverse magnetic field. In order to highlight the effects of the ramped wall temperature of the field, It is worthwhile to compare such a flow with the flow near moving plate with constant temperature. The solution for species concentration is given by the equation (21). However the fluid temperature and velocity for free convection near an isothermal plate has the following form τ

$$T(y,t)=\frac{1}{2}\left[e^{\sqrt{\Pr\phi}y}erfc(t_1)+e^{-\sqrt{\Pr\phi}y}erfc(t_2)\right] \tag{32}$$

$$u(y,t)=P(y,t)+\frac{G_m}{1-Sc}\ P_3(y,t)+\frac{G_r}{1-\Pr}P_4(y,t) \tag{33}$$

Where,

$$P_4(y,t)=\frac{1}{2a}\ \left[e^{\sqrt{\Pr\phi}y}\ erfc(t_1)+e^{-\sqrt{\Pr\phi}y}\ erfc(t_2)\right]-$$

$$-\frac{e^{-at}}{2a}\left(e^{\sqrt{\Pr\phi-a\Pr}y}erfc(t_3)+e^{-\sqrt{\Pr\phi-a\Pr}y}erfc(t_4)\right)-$$

$$-\frac{1}{2a}\left(e^{\sqrt{M^2+(1/K)}y}erfc(t_5)+e^{-\sqrt{M^2+(1/K)}y}erfc(t_6)\right)+$$

$$+\frac{e^{-at}}{2a}\left(e^{\sqrt{M^2+(1/K)-ay}}erfc(t_7)+e^{-\sqrt{M^2+(1/K)-ay}}erfc(t_8)\right)$$

The physical quantities of engineering interest are the Skin friction (τ), the Nusselt number *Nu* and Sherwood number *Sh*. The Nusselt number measures the rate of heat transfer at the plate and for a ramped temperature is,

$$Nu=-\left(\frac{\partial T}{\partial y}\right)_{y=0}=-\left[F_1(t)-H(t-1)\ F_1(t-1)\right] \tag{34}$$

Where,

$$F_1(t)=-\left[\sqrt{\Pr\phi}+\frac{\Pr}{2\sqrt{\Pr\phi}}\right]erf\ (\sqrt{\phi t})-\sqrt{\frac{\Pr t}{\pi}}\ e^{-\phi t}$$

In case of an isothermal plate, The Nusselt number is

$$Nu=-\ (\sqrt{\Pr\phi})\ erf\ (\sqrt{\phi t})-\sqrt{\frac{\Pr t}{\pi}}\ e^{-\phi t} \tag{35}$$

The Sherwood number measures the rate of mass transfer at the plate and for a ramped temperature is,

$$Sh=-\left(\frac{\partial C}{\partial y}\right)_{y=0}=-\sqrt{\frac{Sc}{\pi t}} \tag{36}$$

The equations (21), (26), (31), (32) and (33) represent the analytical solutions for the flow induced by a general time dependent movement of the vertical flat plate. To gain some practical understanding of the flow dynamics, some particular or special cases of time dependent movements of the plate are discussed below.

Assuming that the plate moves with uniform velocity $f(t)=H(t)$, the fluid velocity for ramped temperature plate is obtained as

$$u(y,t)=P_5(y,t)+\frac{G_r}{1-\Pr}\left[P_2(y,t)-H(t-1)\,P_2(y,t-1)\right]+\frac{G_m}{1-Sc}P_3(y,t) \tag{37}$$

while the isothermal plate has the velocity,

$$u(y,t)=P_5(y,t)+\frac{G_r}{1-\Pr}\,P_4(y,t)+\frac{G_m}{1-Sc}P_3(y,t) \tag{38}$$

where, $P_5(y,t)=\frac{1}{2}\left(e^{\sqrt{M^2+(1/K)}y}erfc(t_5)+e^{-\sqrt{M^2+(1/K)}y}erfc(t_6)\right)$

The Skin friction for the ramped temperature plate is

$$\tau_{11}=\left(\frac{\partial u}{\partial y}\right)_{y=0}=Q_1(t)+\frac{G_r}{1-\Pr}\left[F_2(t)-H(t-1)\;F_2(t-1)\right]+\frac{G_m}{1-Sc}F_3(t) \tag{39}$$

and for the isothermal plate

$$\tau_{12}=Q_1(t)+\frac{G_r}{1-Pr}\,F_4(t)+\frac{G_m}{1-Sc}F_3(t) \tag{40}$$

Where,

$$Q_1(t)=-\sqrt{M^2+\frac{1}{K}}\;erf\left(\sqrt{\left(M^2+\frac{1}{K}\right)t}\right)-\frac{e^{-\left(M^2+\frac{1}{K}\right)t}}{\sqrt{\pi t}}$$

$$F_2(t)=-\left(\left(\frac{at-1}{a^2}\right)\sqrt{\Pr\phi}+\frac{\Pr}{2a\sqrt{\Pr\phi}}\right)\;erf\,(\sqrt{\phi t})-\left(\frac{\sqrt{(\phi-a)\Pr}}{a^2}\right)erf\left(\sqrt{(\phi-a)t}\right)e^{-at}+$$
$$+\left(\left(\frac{at-1}{a^2}\right)\sqrt{M^2+\frac{1}{K}}+\frac{1}{2a\sqrt{M^2+(1/K)}}\right)\;erf\left(\sqrt{\left(M^2+\frac{1}{K}\right)t}\right)+$$
$$+\left(\frac{\sqrt{M^2+(1/K)-a}}{a^2}\right)\;erf\left(\sqrt{\left(M^2+\frac{1}{K}-a\right)t}\right)e^{-at}-\frac{e^{-\phi t}}{a}\sqrt{\frac{\Pr t}{\pi}}+\frac{e^{-(M^2+(1/K))t}}{a}\sqrt{\frac{t}{\pi}}$$

$$F_3(t)=\frac{1}{b}\left\{\sqrt{-Scb}\;\;erf\,(\sqrt{-bt})e^{-bt}+\sqrt{M^2+\frac{1}{K}}\;erf\left(\sqrt{\left(M^2+\frac{1}{K}\right)t}\right)-\right.$$

$$-\sqrt{M^2+\frac{1}{K}-b}\ erf\left(\sqrt{\left(M^2+\frac{1}{K}-b\right)t}\right)e^{-bt}\ \}$$

$$F_4(t)=-\frac{\sqrt{\Pr\phi}}{a}\ erf\,(\sqrt{\phi t})+\frac{\sqrt{(\phi-a)\Pr}}{a}\ erf\left(\sqrt{(\phi-a)t}\right)e^{-at}+$$

$$+\frac{\sqrt{M^2+(1/K)}}{a}\ erf\left(\sqrt{\left(M^2+\frac{1}{K}\right)t}\right)-$$

$$-\frac{\sqrt{M^2+(1/K)-a}}{a}\ erf\left(\sqrt{\left(M^2+\frac{1}{K}-a\right)t}\right)\ e^{-at}$$

3. Results and Discussion:

The results of various flow parameters on the MHD heat and mass transfer in the fluid flow past an infinite vertical plate, the velocity, temperature and species concentration profiles are given in Figures (17), (18-23) and (24-25) respectively when $\omega = \pi/2$ for both ramped temperature and isothermal plates. The numerical values of the skin friction, Nusselt number and Sherwood number are presented in Tables (1-3) again for both ramped temperature and isothermal plates. The computational results carried out using Mathematica Software.

Figures (2-3 &10-15) shown that for both ramped temperature and isothermal plates, the fluid velocity decreases with an increase in M, φ, Sc or Pr, whereas it increases with an increase in K, Gr or Gm (Figs 4-9). The application of a transverse magnetic field on an electrically conducting fluid gives rise to a resistive-type of force called the Lorentz force. This force has the propensity to reduce speed for the motion of the fluid in the boundary layer. Thus it follows that heat absorption and the magnetic field tend to retard the fluid flow, whereas thermal diffusion, mass diffusion, thermal buoyancy force and mass buoyancy force have the opposite effect. Figures (16-17) shows that for both ramped temperature and isothermal plates, the fluid velocity increases with an increase in *t* throughout the fluid region. The fluid velocity in the case of a flow past an isothermal plate is higher than that of a flow past a ramped temperature plate.

The computational results for the temperature profiles for variation in t, ϕ and Pr are shown in Fig. (18-23) while fixing the other parameters. We noticed that for both ramped temperature and isothermal plates, the fluid temperature increases with t and decreases with an increase in either φ or Pr. Since the Prandtl number is the ratio of viscosity to the thermal diffusivity, an increase in Pr implies a decrease in thermal diffusivity. This implies that heat absorption tends to reduce the fluid temperature, whereas thermal diffusion and time have the contrary effect.

The computational results for the Concentration profiles for variation in t, and Sc are shown in Fig. (24-25). We noticed that, for increasing Pr. In the presence of porous matrix the concentration boundary layer became thinner and thinner as Schmidt number increases. It is observed that the concentration profile is asymptotic in nature. Concentration increases with time whereas it reduces with Schmidt number increases throughout the fluid region.

The numerical computation of skin friction coefficient, Nusselt number and Sherwood number is obtained and presented in Table (1-3). The skin friction coefficient for both ramped temperature and isothermal plates increases with an increase in time. It is evident that the skin friction, for both ramped temperature and isothermal plates, increases with φ, Pr, or M, while it decreases with an increase in K, Sc, Gr or Gm, which implies that heat absorption and the magnetic field have a propensity to increase the shear stress at the plate, while the thermal diffusion, thermal buoyancy force and mass buoyancy force have the reverse effect on it (Table 1).

The values of the Nusselt number are shown in Table 2. The Nusselt number increases with increasing for the all parameters. The similar behaviour is observed at the isothermal plate. This is a significant increase is remarked in case of Sherwood number when there is an increase in the value of the parameters, but the reverse trend is well marked from the table (3). The Sherwood number increase with increasing Sc but it reduces with time.

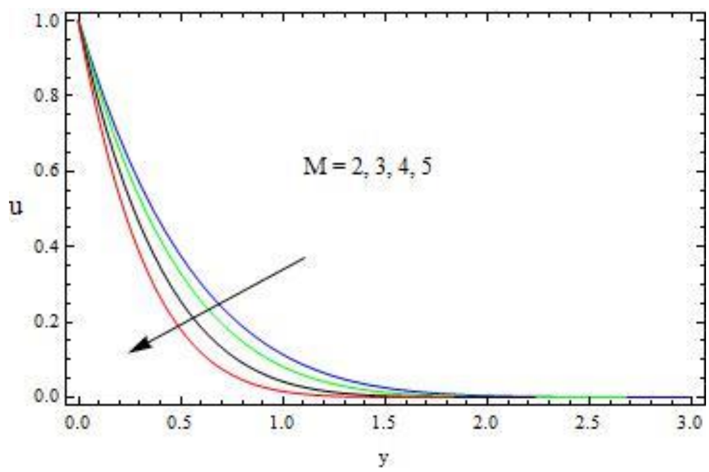

Fig. 2 The velocity profile against M for ramped temperature with

K=1, Pr=0.71, Gr=2, Gm=2, $\phi=1$, Sc=0.22,t=0.2

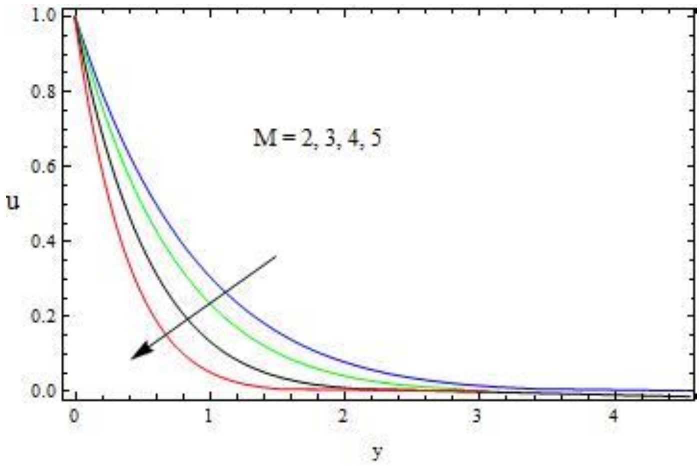

Fig. 3 The velocity profile against M for isothermal plate with
K=1, Pr=0.71, Gr=2, Gm=2, $\phi=1$, Sc=0.22,t=0.2

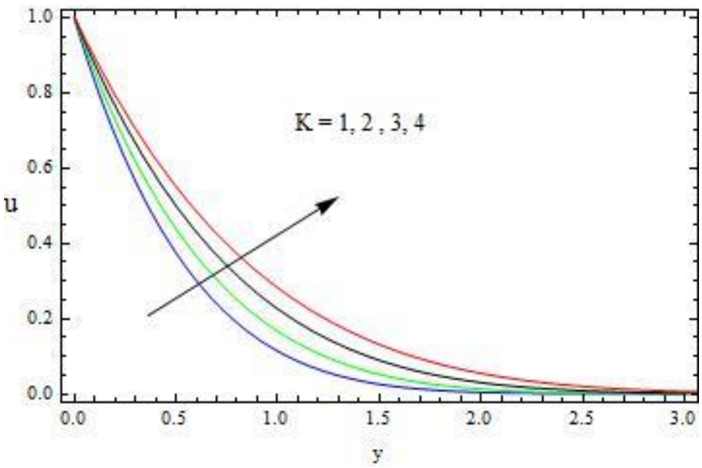

Fig. 4 The velocity profile against K for ramped temperature with
M=2, Pr=0.71, Gr=2, Gm=2, $\phi=1$, Sc=0.22,t=0.2

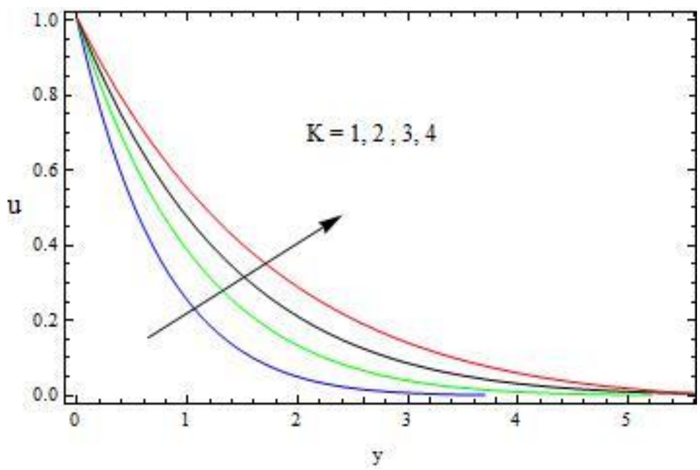

Fig. 5 The velocity profile against K for isothermal plate with

M=2, Pr=0.71, Gr=2, Gm=2, $\phi=1$, Sc=0.22,t=0.2

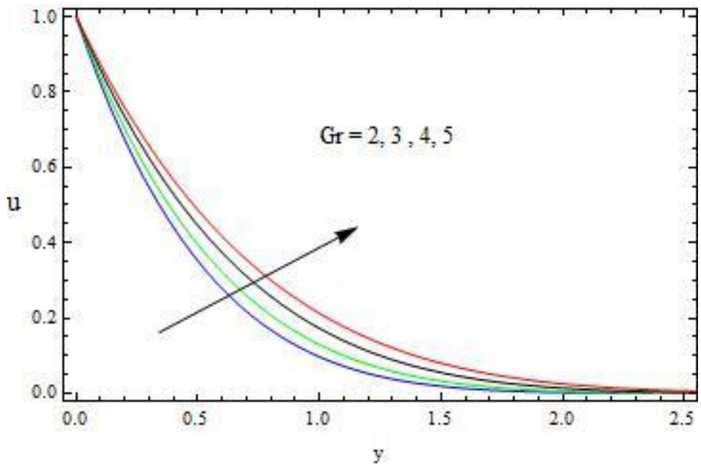

Fig. 6 The velocity profile against Gr for ramped temperature with
M=2, Pr=0.71, K=1, Gm=2, $\phi=1$, Sc=0.22,t=0.2

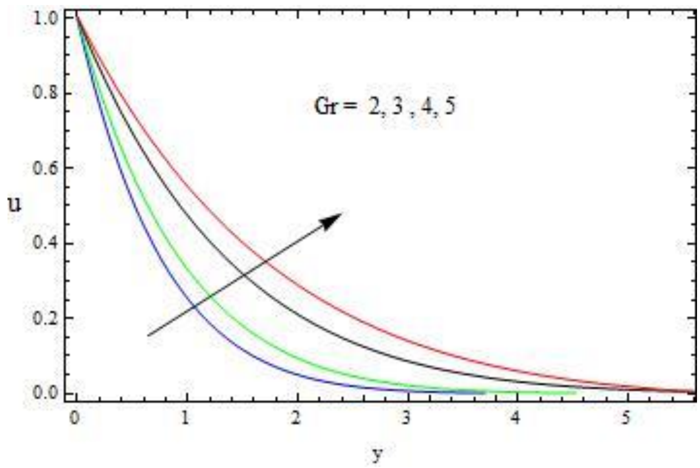

Fig. 7 The velocity profile against Gr for isothermal plate with
M=2, Pr=0.71, K=1, Gm=2, $\phi=1$, Sc=0.22,t=0.2

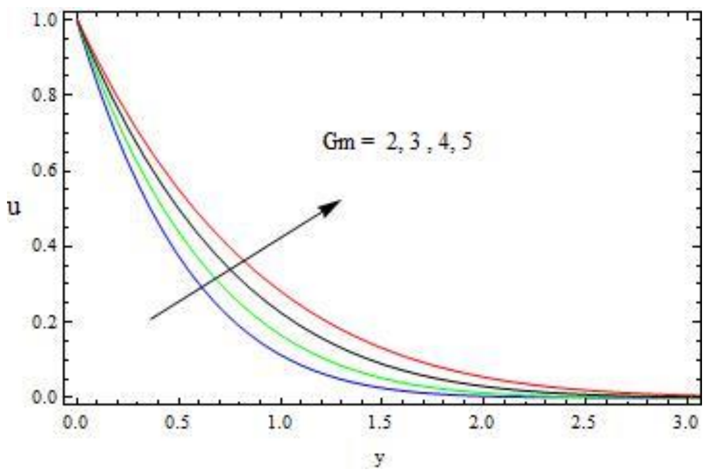

Fig. 8 The velocity profile against Gm for ramped temperature with

M=2, Pr=0.71, K=1, Gr=2, $\phi=1$, Sc=0.22,t=0.2

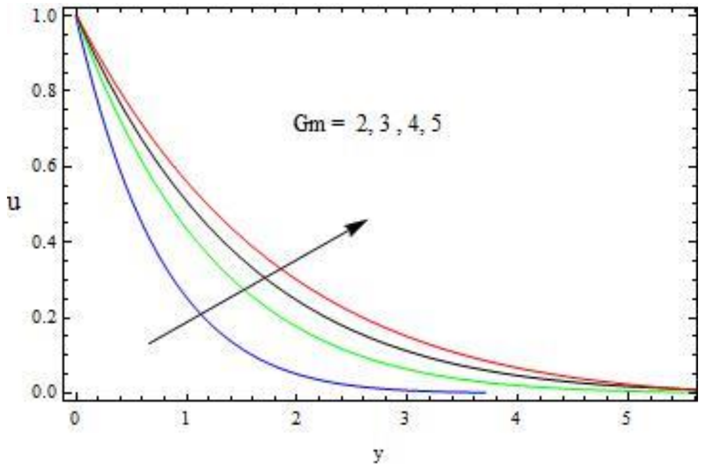

Fig. 9 The velocity profile against Gm for isothermal plate with
M=2, Pr=0.71, K=1, Gr=2, $\phi=1$, Sc=0.22,t=0.2

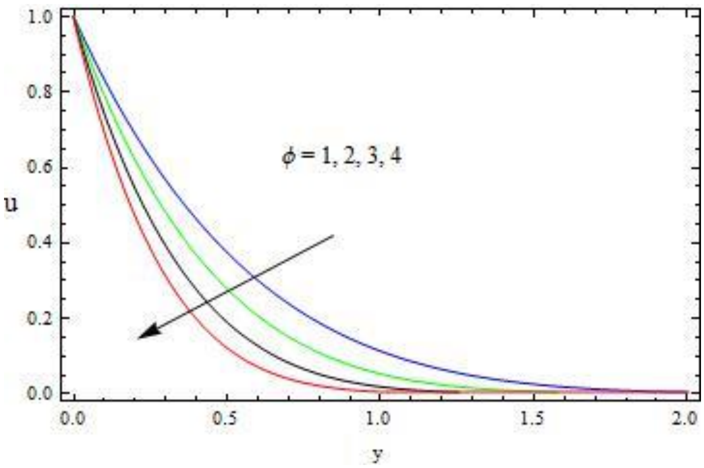

Fig. 10 The velocity profile against ϕ for ramped temperature with
M=2, Pr=0.71, K=1, Gr=2, Gm=2, Sc=0.22,t=0.2

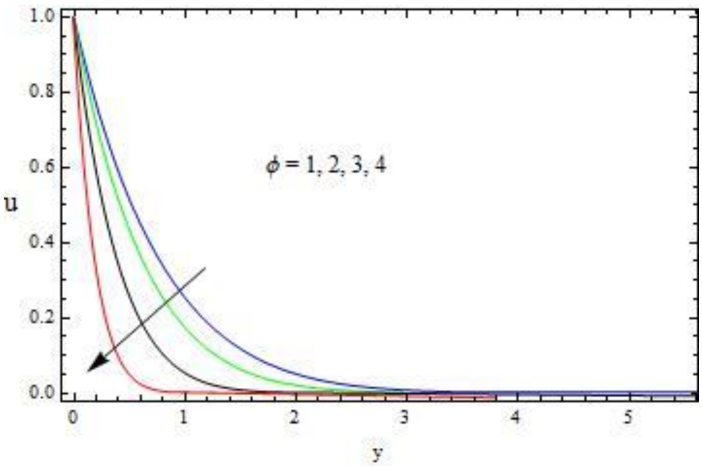

Fig. 11 The velocity profile against ϕ for isothermal plate with
M=2, Pr=0.71, K=1, Gr=2, Gm=2, Sc=0.22,t=0.2

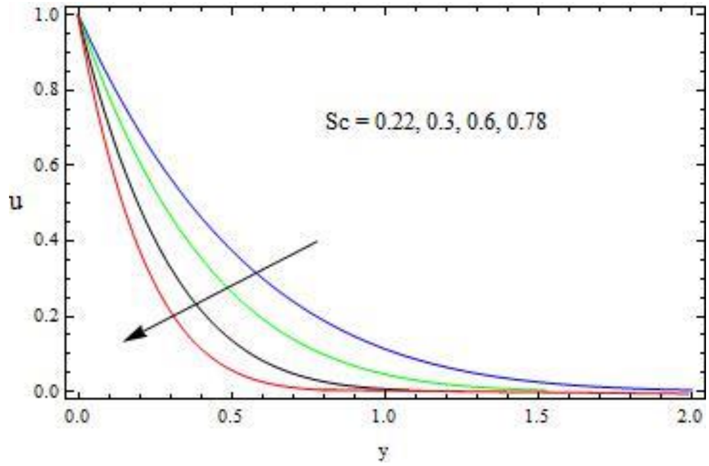

Fig. 12 The velocity profile against Sc for ramped temperature with M=2, Pr=0.71, K=1, Gr=2, Gm=2, $\phi = 1$,t=0.2

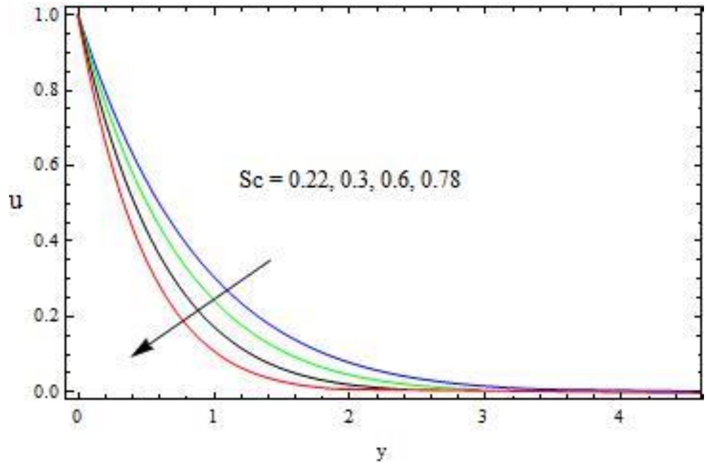

Fig. 13 The velocity profile against Sc for isothermal plate with M=2, Pr=0.71, K=1, Gr=2, Gm=2, $\phi = 1$,t=0.2

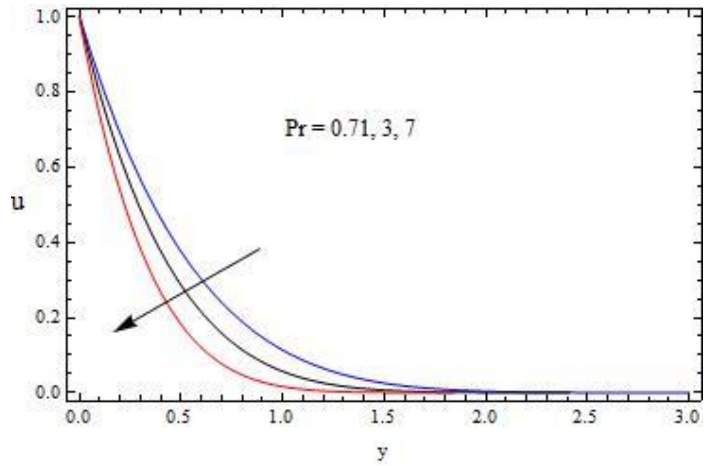

Fig. 14 The velocity profile against Pr for ramped temperature with M=2, Sc=0.22, K=1, Gr=2, Gm=2, $\phi = 1$,t=0.2

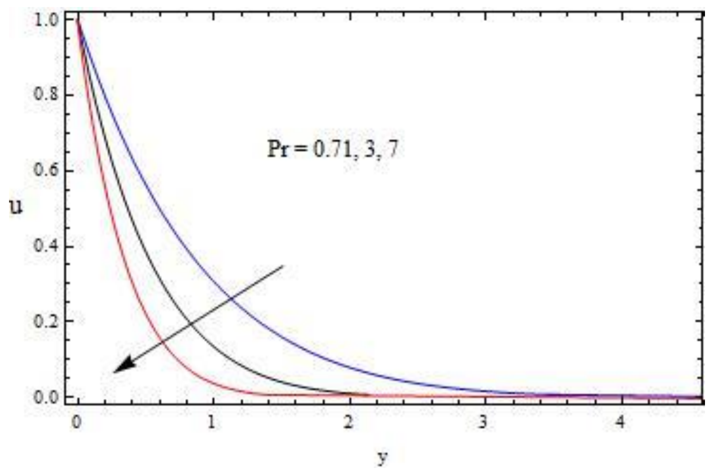

Fig. 15 The velocity profile against Pr for isothermal plate with M=2, Sc=0.22, K=1, Gr=2, Gm=2, $\phi=1$,t=0.2

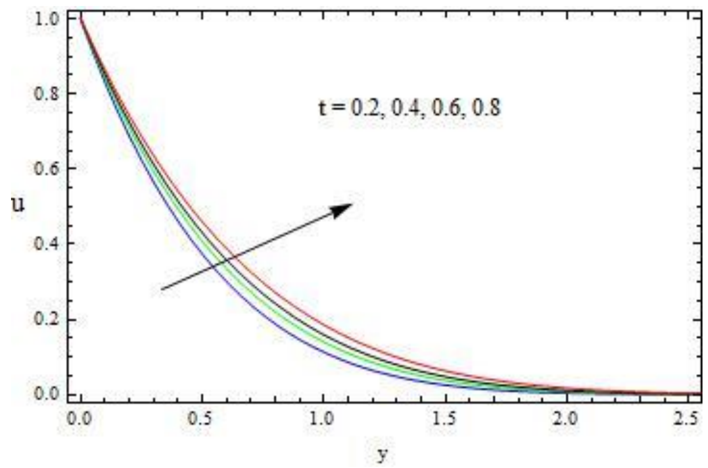

Fig. 16 The velocity profile against t for ramped temperature with M=2, Sc=0.22, K=1, Gr=2, Gm=2, $\phi=1$, Pr=0.71

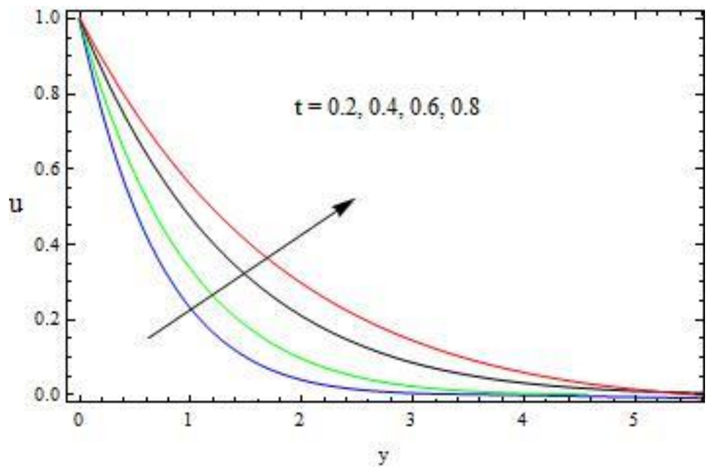

Fig. 17 The velocity profile against t for isothermal plate with M=2, Sc=0.22, K=1, Gr=2, Gm=2, $\phi=1$, Pr=0.71

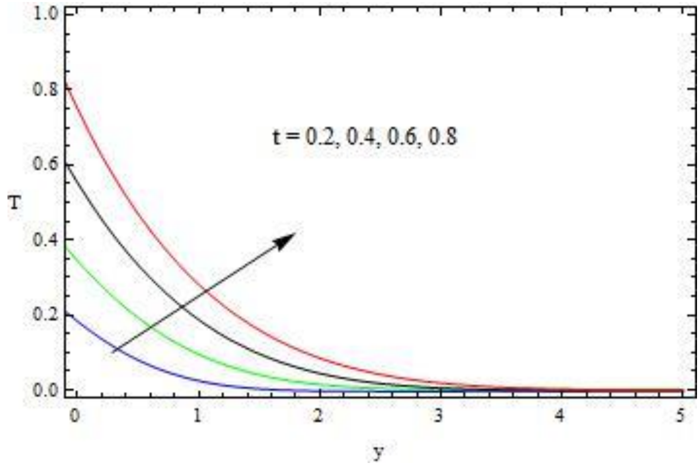

Fig. 18 The temperature profile against t for ramped temperature with $\phi = 1$, Pr=0.71

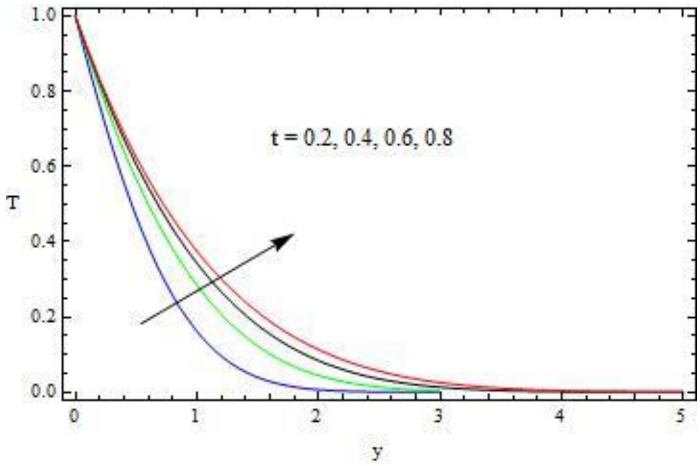

Fig. 19 The temperature profile against t for isothermal plate with $\phi = 1$, Pr=0.71

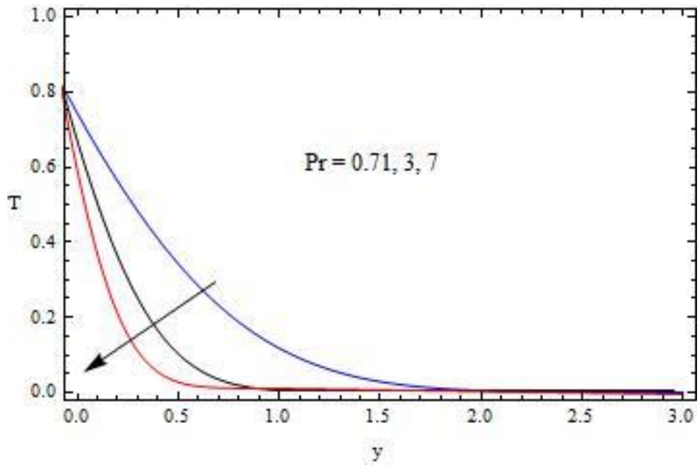

Fig. 20 The temperature profile against Pr for ramped temperature with $\phi = 1$, t=0.8

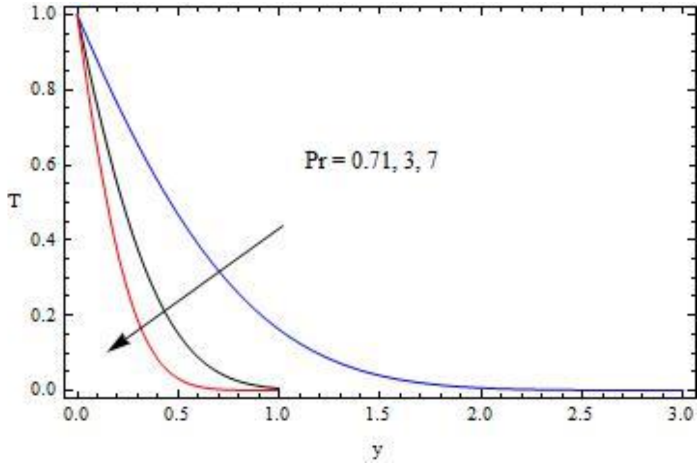

Fig. 21 The temperature profile against Pr for isothermal plate with $t = 0.2$, t=0.2

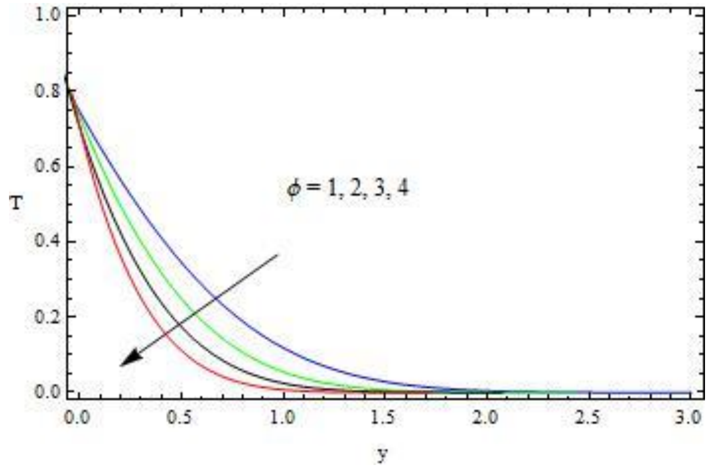

Fig. 22 The temperature profile against ϕ for ramped temperature with t=0.8, Pr=0.71

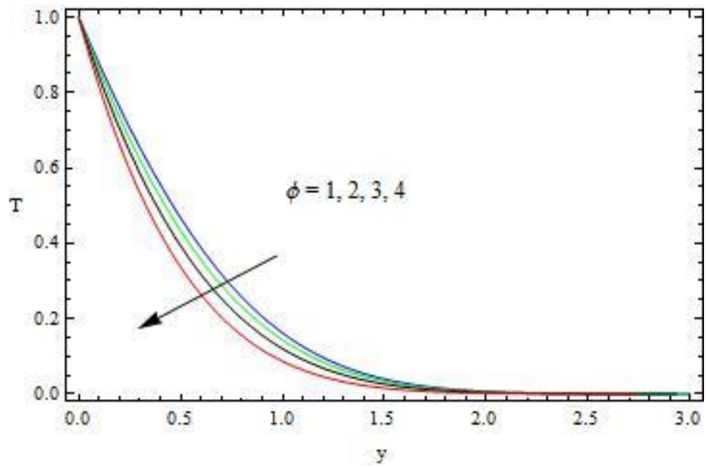

Fig. 23 The temperature profile against ϕ for isothermal plate with t=0.2, Pr=0.71

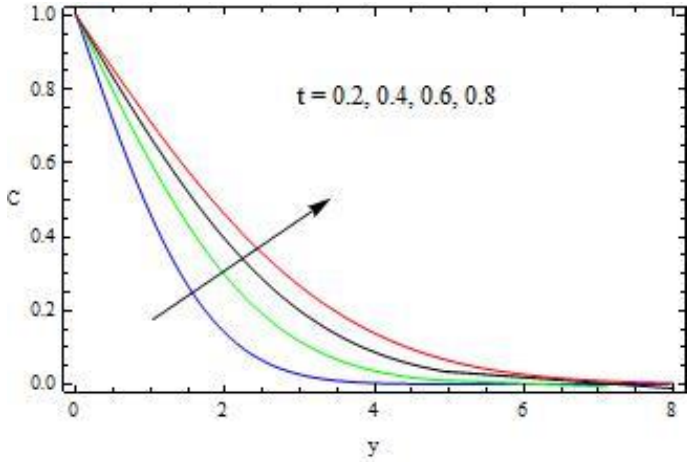

Fig. 24 The Concentration profile against ϕ with Sc=0.22

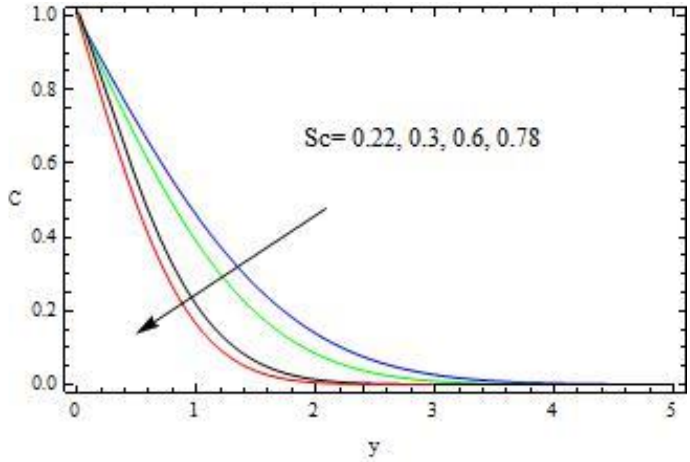

Fig. 25 The Concentration profile against Sc with ϕ

Table. 1 Skin friction

M	K	φ	Pr	Gr	Gm	Sc	t	τ_{11}	τ_{12}
2	1	1	0.71	2	2	0.22	0.2	-0.807404	-0.025145
3	1	1	0.71	2	2	0.22	0.2	-0.725546	-0.014528
4	1	1	0.71	2	2	0.22	0.2	-0.624584	-0.000547
2	2	1	0.71	2	2	0.22	0.2	-0.747512	-0.014256
2	3	1	0.71	2	2	0.22	0.2	-0.602546	-0.009885
2	1	2	0.71	2	2	0.22	0.2	-1.225407	-0.035526
2	1	3	0.71	2	2	0.22	0.2	-2.870058	-0.041785
2	1	1	3	2	2	0.22	0.2	-0.902585	-0.074855
2	1	1	7	2	2	0.22	0.2	-1.255468	-0.144152
2	1	1	0.71	3	2	0.22	0.2	-1.255468	-0.125496
2	1	1	0.71	4	2	0.22	0.2	-1.355895	-0.859552
2	1	1	0.71	2	3	0.22	0.2	-2.854795	-0.998546
2	1	1	0.71	2	4	0.22	0.2	-3.558044	-1.845522

2	1	1	0.71	2	2	0.6	0.2	-0.690478	-0.009452
2	1	1	0.71	2	2	0.78	0.2	-0.360252	-0.004985
2	1	1	0.71	2	2	0.22	0.4	-1.589045	-0.085546
2	1	1	0.71	2	2	0.22	0.6	-2.401045	-0.102563

Table 2. Nusselt number

t	ϕ	Pr	At Ramped temperature (Nu)	At isothermal plate (Nu)
0.2	1	0.71	0.857396	−0.658155
0.4	1	0.71	1.243430	−0.978467
0.6	1	0.71	1.589440	−1.283285
0.2	2	0.71	1.253952	−1.066595
0.2	3	0.71	1.624698	−1.447939
0.2	1	3	1.762434	−1.352882
0.2	1	7	2.692163	−2.066561

Table 3. Sherwood number

t	Sc	Sh
0.2	0.22	1.597310
0.4	0.22	1.445966
0.6	0.22	1.371005
0.8	0.22	1.324355
0.2	0.3	1.671532
0.2	0.6	1.833021
0.2	0.78	1.884904

4. Conclusions

We have examined the unsteady MHD free convection heat and mass transfer flow of a viscous, incompressible, electrically conducting flow past an infinite vertical plate. The flow was induced by a time-dependent movement of the plate. We consider the movement of the plate with uniform velocity; exact solutions of the governing equations found making use of Laplace transform technique. The conclusions are made as follows:

1. Heat absorption reduces the fluid temperature, whereas thermal diffusion and time have the opposite effect,

2. Heat absorption and the magnetic field tend to retard the fluid flow, whereas thermal diffusion, mass diffusion, thermal buoyancy force and mass buoyancy force and porosity have the opposite effect.
3. The mass diffusion rate and time tend to increase species concentration,
4. Heat absorption and the magnetic field tend to increase the shear stress at the plate, whereas thermal diffusion, thermal buoyancy force and mass buoyancy force have the reverse effect,
5. Heat absorption and time tend to increase the rate of heat transfer at the plate, whereas thermal diffusion has the reverse effect, and mass diffusivity and time tend to reduce the rate of mass transfer at the plate.

Nomenclatures:

u the velocity component along x direction,

T the temperature of the fluid,

C the species concentration,

ρ the density of the fluid,

σ the electrical conductivity of the fluid,

K_1 the permeability of the porous medium,

B_0 the uniform magnetic field of strength,

ν the coefficient of kinematic viscosity,

k the thermal conductivity of the fluid,

C_p the specific heat of the fluid at constant pressure,

β the volumetric coefficient of the thermal expansion,

β^* the volumetric coefficient of the thermal expansion with concentration,

g the acceleration due to gravity,

D the chemical modular diffusivity of the fluid and

Q_0 the heat absorption coefficient.

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