Using Green's Function, Maxwell's and Schrodinger beside Plank Hypotheses for Photon Mass and Propagation as a Travelling Wave Obeying the Potential Expression

Najwa Idris A. Ahmed[a], Sonia Soltani[b] ,Mona A. Alamri[c], Mubarak Dirar Abd-Alla[d], Zoalnoon A. A. A. Saad[e]

[a,b,c]Department of Physics, College of Science and Arts, Qassim University, Dariyah 58251, Saudi Arabia.

[a]Department of Physics & Math, Hantoub Faculty of Education, Al Gazira University, Hantoub, Sudan.

[d]Department of Physics, Faculty of Science, Sudan University of Science and Technology, Khartoum, Sudan

[e]Department of Physics, Faculty of Arts & Sciences, Dhahran Aljanoub, King Khalid University, KSA

**** Corresponding author email:n.ahamed@qu.edu.sa,zsaad@kku.edu.sa,https://orcid.org/0000-0003-0895-6683***

Abstract

Maxwell's equations and plank theory appear to give different results for the interaction of electromagnetic field with matter. In this work Maxwell's equations, plank hypotheses, beside green's function in addition to special relativity were used to join the two versions. The new model predicts a very small photon energy having an order of $\sim 10^{-50}$kg, agreeing with some previous models. It also shows that both versions give photon obeying the inverse square law and propagating as a travelling wave. Using Newton's kinetic energy, Plank and De Broglierelations for the photon, the ordinary mass energy relation was found.

Keywords: green's function, photon, rest mass, Maxwell's equations, energy, momentum, special relativity, plank hypothesis, travelling waves, inverse square law.

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1.

INTRODUCTION

According to Huygen principle, light behaves as waves. This wave nature was confirmed by the Maxwell's equations [2]. The wave nature of light succeeded in explaining the laws of reflection, refraction, and interference as well as diffraction phenomenon [3]. However, some physical phenomena like black body radiation, photoelectric effect, pair production and

Newton's laws are the oldest well-known physical laws. These laws successfully describe the behavior of large astronomical objects. They are also used to describe the wave motion of sound [1]. However, at that time it fails in describing the nature of light. This encourages Huygen to formulate his well-known principle.

the atomic optoelastic forces between the induced dipoles and the electromagnetic field [14]. They found that the photon has a mass drag proportional to the plank quantum energy and the refractive index.

Latterly,the vacuum zero-point potential for fields was utilized to determine the photon mass [15]. It was suggested that the photon has no rest mass but acquires mass when interacting with vacuum and fields. In addition, Klein – Gordon equation in spherical coordinate was used to present that when treating photons as strings they acquire rest mass [16]. The plank quantum energy was compared with the energy of classical oscillator and showed that electromagnetic waves result from a collections of photons having frequencies ranged from zero to w [17]. Maxwells equations, plank and De Broglie hypothesis beside the energy-momentum relativistic relations were applied to find the photon rest mass in terms of electron mass, plank constant, in addition to the electron charge and electric permittivity beside the speed of light in vacuum to find the photon mass$(39 \times 10^{-62}\text{kg})[18]$. Additionally, the space-time invariant length expression in a curved space, beside an expression for generalized momentum in the presence of electromagnetic field were applied to find a generalized momentum relation and the photon mass dependent on the electron charge and vector potential [19]. A new Dirac quantum equation based on generalized special relativity was derived [20]. It describes the behavior of photons propagating as electromagnetic waves, beside the behavior of the photons and quantum entanglement.

2. Photon rest mass from Maxwell's

Considering an electromagnetic wave with electric field intensity E propagates inside a medium having electric permittivityε, magnetic permeability and μ electric conductivityσ, Maxwell equation is given:

$$\nabla^2 E - \mu\varepsilon\frac{\partial^2 E}{\partial t^2} - \mu\sigma\frac{\partial E}{\partial t} = 0 \quad (1)$$

One can write the E in terms of the angular frequency w and the wave number K in the form:

$$E = e^{i(wt-kx)} \quad (2)$$

With $(w = 2\pi f)$ and $(K = 2\,\pi/\lambda)$ are related to the wave frequency f and wavelengthλ:

Compton effect cannot be described by the wave model [4]. This encourages Max Plank to suggest that the light and electromagnetic fields behave as a discrete quantum or a particle [2,3,4].This means that they have a dual nature. This dual nature leads to the formation of quantum mechanics[4,5].The most famous widely used quantum equation was made by Schrodinger [5].His quantum equation is mainly based on the classical Newton energy momentum relation. Later, Klein and Gordon beside Dirac formulated quantum equations based on the special relativity energy momentum relation [6].One of the most known issues that faced quantum laws are the interaction of electromagnetic radiation with bulk matter. This includes scattering, absorption, and emission processes [7].These processes can be described either by the wave model using Maxwell's equations or by the quantum model using quantum laws. The two models look quite different but recently some attempts tried to reconcile the two models [8,9].One of these attempts needed to understand the behavior and nature of photons.

The origin and nature of the photon was tackled by many researchers. The frequency dependent time delays in fast radio bursts (FRB) were applied to determine the upper limit of the photon mass. It found that the upper limit is about 3.2x 10^{-50}kg[12]. Likewise, the vortex gravitation theory was utilized to determine the density and mass of electromagnetic particles that arise atthe lowest orbit of the atomic torsion, by using the equality of centrifugal and nuclear forces to find photon density, Weinberg photon number per nucleon to determine the photon volume. and then, found the photon mass to be 2.5x 10^{-49}kg. [13]. Furthermore, the covariant theory of light in a medium was formulated by considering a light wave simultaneously with

$$\nabla^2 E = \mu\varepsilon \frac{\partial^2 E}{\partial x^2} = i^2 k^2 E = -k^2 E$$

$$\frac{\partial E}{\partial t} = iwE$$

$\frac{\partial^2 E}{\partial t^2} = -w^2 E$(3)

$-\mathrm{k}^2 E + \mathrm{w}^2 \mu\varepsilon E - iw\mu\sigma E = 0$ (4)

$-\mathrm{k}^2 + \mu\varepsilon \mathrm{w}^2 - iw\mu\sigma = 0$ (5)

Using plank and De Brogglie hypothesis, the energy E and the momentum P are given by:

$$E = \hbar w$$

$P = \hbar k$(6)

To relate equation (5) to the energy and momentum, multiply equation (5) by $\hbar^2$ to get:

$-\hbar^2 \mathrm{k}^2 + \mu\varepsilon\hbar^2 \mathrm{w}^2 - i\mu\sigma w\hbar^2 = 0$(7)

$-P^2 + \mu\varepsilon E^2 - i\mu w\sigma\hbar^2 = 0$(8)

Electromagnetic waves in free space are given by:

$\mu = \mu_0 \qquad \varepsilon = \varepsilon_0$ (9)

Where the speed of light in free space is given by:

$\mu_0 \varepsilon_0 = \frac{1}{c^2}$(10)

$E^2 - P^2 \mathrm{C}^2 - i\mu_0 w\sigma\hbar^2 c^2 = 0$(11)

$m_e \frac{dv}{dt} = eE$(12)

$$v = v_o e^{iwt}$$

$\frac{dv}{dt} = iwv$(13)

$iwm_e v = eE$(14)

$v = \frac{e}{im_e w} E = \frac{-ie}{m_e w} E$(15)

$J = nev = -\frac{ne^2}{m_e w} iE = (\sigma_1 + i\sigma_2)E = \sigma E$(16)

$$\sigma_1 = 0 \qquad \sigma_2 = -\frac{ne^2}{m_e w}$$

$\sigma = -\frac{ne^2 i}{m_e w}$(17)

$$E^2 - P^2 \mathrm{c}^2 - \frac{ne^2}{m_0}\mu_0 \hbar^2 c^2 = 0$$

$E^2 = P^2 \mathrm{c}^2 + \frac{ne^2}{m_e}\mu_0 \hbar^2 c^2$(18)

When the particle (photon) is at rest for an observer moving with the photon itself, the momentum vanishes and hence the mass m is equal to the rest mass m_0.

$P = 0 \qquad E = E_0$ (19)

$E_0^2 = \frac{he^2}{m_e}\mu_0 \hbar^2 c^2$(20)

Newtonian kinetic energy relation can be used to find the Einstein energy relation. According to Newtonian laws the kinetic energy is given by:

$E = \frac{1}{2} m v^2 = m\left(\frac{v}{\sqrt{2}}\right)^2$(21)

Assuming the photon as a particle moving in a wavy trajectory with maximum speed v, with c standing for the speed effective value in vacuum, thus:

$E = m\left(\frac{v}{\sqrt{2}}\right)^2 = mc^2$(22)

$c = \frac{v}{\sqrt{2}}$(23)

The Einstein energy relativistic expression can be derived also, by considering the case when the photon is in vacuum with zero conductivity, Equation (11) become:

$\sigma = 0$ (24)

$E = Pc$(25)

Since the photon momentum is:

$P = mc$(26)

Therefore, the energy E in equation (25) is given by:

$E = mc^2$(27)

The photon rest mass can result from a direct substitution of equations (22) and (19) in equation (18) to get:

$m^2c^4 = P^2c^2 + \frac{ne^2}{m_e}\mu_0\hbar^2c^2$(28)

$$m_0^2c^4 = \frac{ne^2\mu_0}{m_e}\hbar^2c^2$$

$m_0 = \frac{e\hbar}{c}\sqrt{\frac{\mu_0 n}{m_e}}$(29)

$$E = E_P(x,y,z)e^{-iwt}$$

$$\frac{\partial E}{\partial t} = iwE$$

$\frac{\partial^2 E}{\partial t^2} = i^2w^2E = -w^2E$(30)

Substituting the above relations in equation (1) yields:

$\nabla^2 E - w^2\mu\varepsilon E + i\mu\sigma wE = 0$(31)

$$\mathrm{w}^2\mu\varepsilon = \frac{\mathrm{w}^2}{v^2} = \mathrm{k}^2$$

$\nabla^2 E - K^2E = -i\mu\sigma wE$(32)

$\nabla^2 G + K^2G = \delta r r^1$(33)

$\nabla^2 \int (GA)dr^1 + K^2 \int (GA)dr^1 = A(r)$(34)

$$A = -i\mu\sigma wE$$

$$E = \int (GA)dr^1$$

$E = -i\mu\sigma w \int GAdr^1$(35)

However, when $r \to \infty$:

$G = -\frac{e^{ik.r}}{4\pi r}e^{-ik.r^1}$(36)

$E_P(x,y,z) = E_P(r,\epsilon,\emptyset) = E_0e^{ik.r}e^{-iwt}$(37)

$$E = +\frac{i\mu\sigma w}{4\pi r}e^{ik.r}e^{-iwt}E_0\int_0^{r_0} e^{-ik.r^1}e^{ik.r^1}\,dr^1$$

$E = -\frac{i\mu\sigma w r_0 e^{i(k.r-wt)}}{4\pi r}(E_0)$(38)

$J = \frac{\partial D}{\partial t} = \epsilon\frac{\partial E}{\partial t} = -iw\epsilon E = \sigma E$(39)

$$\sigma = -iw\epsilon$$

$$E = \frac{-i^2\mu\varepsilon w^2 r_0 E_0 e^{i(k.r-wt)}}{4\pi r}$$

$E = \frac{w^2 r_0 E_0 e^{i(k.r-wt)}}{4\pi c^2 r}$(40)

For harmonic oscillator, the classical energy is given by:

$E_n = \frac{1}{2}mw^2A^2 = \frac{1}{2}mw^2r_0^2 = \frac{1}{2}mw^2r^2$(41)

Also, from classical relativistic relations:

$E_n = mc^2$ (42)

$E = \frac{\frac{1}{2}mw^2r^2E_0}{2\pi mc^2r^2}e^{i(k.r-wt)}$(43)

$$E = \frac{E_0}{2\pi\ r^2}e^{i(k.r-wt)}$$

Hence:

$E_r = |E| = \frac{E_0}{2\pi\ r^2}$(44)

But for n photons, the energy is given by:

$E_r = n\hbar w$(45)

$n = \frac{n_0}{r^2}$(46)

$E_r = \frac{n_0\hbar w}{r^2}$(47)

Thus comparing (44) and (47) gives:

$n_0\hbar w = \frac{E_0}{2\pi}E_0 = n_0hw$(48)

by previous investigations, which redefines momentum and energy in terms of vector potential [14,18,19]. The fact that both Maxwell and plank versions show that the photon propagate as an electromagnetic wave agrees with previous work that treats the photon as a single travelling wave[17].

4. CONCLUSION

Using Maxwell's equations, plank and De Brogliehypothesis beside special relativistic energy-momentum relation, the photon rest mass was found as having the order 10^{-50}kg. This estimation agrees with some previous studies. The ordinary energy-mass relativistic relation was found using newton energy relation for a photon moving in free space. Apply in ggreen's function, the Maxwell and plank expressions for photons obeys inverse square law and also shows that the photons propagate as travelling waves.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have influenced the work reported in this study.

Data Availability Statement

This manuscript has no associated data or the data will not be deposited. [Authors' comment: All the used data are presented in this work or published in the references.]

5. REFERENCES

This means that when one considers photons as strings and harmonic oscillators the coulomb law and scattering expression for electric field intensity conform with each other.

3. DISCUSSION

Maxwell's equation (1) beside the travelling wave solution (2) in addition to plank and De Broglie hypothesis in equation (6) to find a special relativistic energy-momentum relation (11). The photon rest mass energy was found in equation (20). Considering the photon in free space the Newtonian energy relation gives the famous relativistic energy mass relation (22). Using the expression (11) and (20) compared with special relativity mass relation, the photon mass was found in equation (29). Using Maxwell equation (31) and green's equation (33), beside green's function (36), the electric field in equation (43) propagates as travelling wave obeying inverse square law as shown by equation (44). Equation (46) shows that the photons density obeys inverse square law where no is proportional to the total flux. The estimation of the photon mass according to equation (29) gives:

$m_0 \sim 10^{-19} \times 10^{-34} \times 10^{-8} \times 10^{11} \sim 10^{-50}$kg

The resulted photon mass conforms with that obtained by previous studies [12-13].Hence,the result obtained by current work using Maxwells equations for photons interacting with matter,in agreements with the results obtained

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