A Mathematical Model of the Impact of Deforestation on the Growth of Forest Resources

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Abstract:

A non-linear mathematical model is proposed and analyzed to study the impact of deforestation on growth of forest resources. A general model is taken to obtain the consistency of result in normal conditions. The best model will yield outcomes that are comparable to those found in typical circumstances. Plotting of graph indicates that, the density of forest resources decreases to a lower equilibrium as deforestation increases continuously. Furthermore, it has been noted that the parameters x, y, and z influence the growth rate of forests.

Keywords: Afforestation, Deforestation, Growth rate of forest etc.

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I. Introduction

In 1798, Thomas R. Malthus published the first version of the mathematical model. He put forward a mathematical theory of population growth that states that population increase are proportionate to the size of the population. A mathematical model is a language and mathematical concept-based description of a system. Mathematical modelling is the term used to describe the process of creating a mathematical model.

In terms of the ecology and socioeconomics, forests are extremely important. In addition to being beneficial to the economy, forests prevent landslides and reduce soil erosion. Rainfall is also drawn to forests. The protection that forests provide against strong, chilly or hot, dry winds is a key environmental problem. The largest, most intricate, most valuable natural resource is the forest. On average, forests cover one-third of the planet's land surface. India is a sizable nation with a diverse range of climates.

The degree of development and forest exploitation varies across India's diverse regions. However, in terms of the scientific perspective on the development of forests, today's understanding of a forest includes any area of land utilised for a variety of purposes and whether or not it is covered in trees, shrubs, climbers, etc. One of the most remarkable characteristics of the land surface is the presence of forests. They differ significantly from meadows and pastures in terms of content and density.

We are well aware that forests are an essential component of our biosphere and provide valuable ecological functions. In addition to providing habitat for many animal species, they contribute to the global cycling of water, oxygen, carbon, and nitrogen. Forests provide the wood required to make homes, paper, and furniture in connection to human requirements. Unfortunately, humans frequently overuse forests. A forest can become a desert if trees are cut down without being replaced. Such a hazard exists for the Doon Valley in Uttaranchal, India, where industrialisation has severely impacted the region's forest [1].

Human activities such as habitat loss, fragmentation and making excessive use of a

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resources as well as indirect effect of human activities such as climate change are the main reason for depletion of forest population. If industries will establish on a large scale, the demand for raw material also increased. So, forest had to clear for fulfilling of requirements. Through an increase in agricultural land, a 1% rise in population growth results in a 2.7 % increase in the rate of deforestation. Forests are home to about 80% of all known species on Earth as well as almost 50% of all known species. Another way that trees combat the heat is by absorbing the greenhouse gases that cause it. Industrialists cause maximum damage to the forest as they overexploit the forest and its products like woods for obtaining more benefit. Expansion of agriculture is usually seen as a main tool of forest loss. The urbanization can cause indirect loss of forest, by encouraging agriculture expansion in forested areas. Nowadays, regulation and adjustment of natural resources like wildlife population, forest population, agriculture land and fish's population are one of the authenticate topics in research. The effect of forests on the climate was examined by Sharma and Kesarkar [15]. According to Yadav A.K. and Chaudhary R. [19], the use of the forest affects how dense an area is expressed. Chaudhary et al. [20] proposed a mathematical model for forest growth in which they considered afforestation is more than deforestation. So we consider a general mathematical model where deforestation is more than the afforestation. Other researchers have also used the mathematical modelling in other sectors also [21-30]. The purpose of this paper is to show how deforestation affects the growth of forest if deforestation is more than the afforestation.

Notations

x = Population of forest
t = The rate at which population of forest increases
α = Proportionality constant
a = Afforestation
b = Deforestation
β= Difference between afforestation and deforestation x_0

II. Formulation of the Problem

Consider the proposed model to be the forest growth model. The rate of change, if afforestation and deforestation occur at constant rates a & b, respectively.

$$\frac{dx}{dt} = \alpha x - a + b \quad (1)$$

$$\frac{dx}{dt} = \alpha x - (a - b) \quad (2)$$

$$\frac{dx}{dt} = \alpha x - \beta$$

Where, $a - b = \beta$ (3)

$$\frac{dx}{\alpha x - \beta} = dt \quad (4)$$

The solution of the equation subject to the boundary conditions

$x = x_0$ at t = 0 (5)

III. Solution of the Problem

Integrating equation (4)

$$log(\alpha x - \beta) = \alpha t + \alpha c_0 \quad (6)$$

Now applying the boundary conditions (5)

$$log(\alpha x_0 - \beta) = \alpha c_0$$

$$c_0 = \frac{\log(\alpha x_0 - \beta)}{\alpha}$$
(7)

Now, $log(\alpha x - \beta) = \alpha t + \frac{log(\alpha x_0 - \beta)}{\alpha}$ (8)

$$log\left(\frac{\alpha x - \beta}{\alpha x_0 - \beta}\right) = \alpha t$$

$$\alpha x - \beta = (\alpha x_0 - \beta)\, e^{\alpha t}$$

$$x(t) = \left(x_0 - \frac{\beta}{\alpha}\right) e^{\alpha t} + \frac{\beta}{\alpha} \quad (9)$$

IV. Results and Conclusions

The model presented in this research is more grounded in reality. The result shows the growth of a forest plantation at a time when afforestation and deforestation are occurring. We demonstrated that the two diminishing parameters $x_0 e^{\alpha t}$ and $\frac{\beta}{\alpha}(1 - e^{\alpha t})$ are required for forest plant growth. It has been noted that afforestation and deforestation affect the growth of forests.

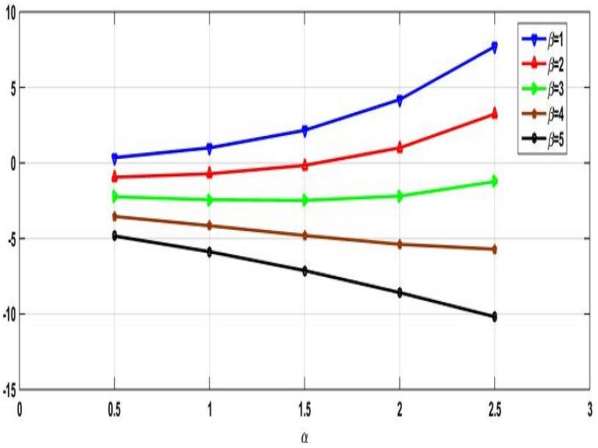

Fig.1: Variation of forest growth at t = 1 and x_0 = 1, graph between x(t) and α

According to the graph discussed above in Fig.1, it is clear that if β is increases by one unit continuously, then curve of rate of forest growth x(t) firstly increases towards the positive direction of y-axis then slightly decreases and then at the value of β = 3, it becomes zero which is a trivial case. After that, the curve of rate of forest growth turns downward towards the negative direction of y-axis. Therefore, for forest growth we have to minimize the value of β.

According to the graph discussed above in Fig.2, it is clear that if β is increases by one unit continuously, then curve of rate of forest growth x(t) firstly increases towards the positive direction of y-axis then slightly decreases and in finally, the curve of forest growth turns downward towards the negative direction of y-axis. Therefore, for forest growth we have to minimize the value of β.

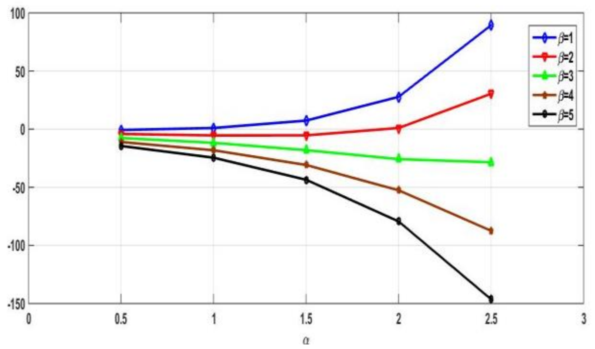

Fig.2: Variation of forest growth at t = 2 and x_0 = 1, graph between x(t) and α.

There is a slightly difference in the graphs, we have discussed above, if we change the value of t, the curve becomes steeper.

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