Consequence of rotation on a radiative fluid flow past a porous plate in presence of Dufour number

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ABSTRACT

An accurate solution to the problem of a three dimensional MHD time dependent flow of a viscous incompressible fluid induced by a suddenly started infinite vertical porous plate with uniform mass diffusion in a rotating fluid under the influence of an applied transverse magnetic field and diffusion thermo is presented. The equations governing the flow are solved analytically by adopting Laplace transform technique in closed form. The profiles for the velocity, temperature, concentration fields, and skin-friction, Nusselt number and Sherwood number at the plate are demonstrated graphically for various values of the parameters involved in the problem and the results are physically interpreted.

Keywords: MHD, Dufour effect, mass transfer, Nusselt number

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INTRODUCTION

Magnetohydrodynamics *(MHD)* concerns with the study of fluids under electro-magnetic effects. Now-a-days applications of MHD principles obtain great importance because of its wide ranging utilities in various fields such as geophysics, astronomical science, space science etc. MHD principles have got applications in biomedical engineering like magnetic drug targeting, magnetic devices for cell separation, cancer tumors treatment, magnetic endoscopy, cell death by hyperthermia etc. Because of importance of MHD principle in different field, many researchers give their attentions to do work in the field of MHD. Significant contribution in the MHD field was given by Alfven (1942). After works of Alfven several researchers were doing many good works in MHD field. The name of some of them are Sarada and Shankar (2013), Ahmed and Sinha (2014), Abbas et al. (2020), Bera (2020), Manzoor et al. (2021) etc.

Dufour effect is the energy flux caused by both the gradient of temperature and composition. In general, this effect is smaller in magnitude as compared to the effect defined by Fourier and Fick Law and was frequently ignored when working on mass and heat transfer. The research works where the above-mentioned studies are implemented by the various researcher, few of these are Reddy *et al*. (2016), Alam and Ahmmad (2011), Anghel *et al*. (2000).

Because of the importance of finding out MHD flow issues in rotating fluid with Dufour effect, we've got planned to investigate an unsteady MHD transient heat and mass transfer flow with Dufour effect past a uniformly accelerated porous plate in presence of magnetic field. This work is a generalization to the work done by Sinha S. and Sarma M.K. (2019) to consider the effect of diffusion thermo. In the process of generalization, almost exact results are drawn which is shown by virtue of comparison graph with the work of Sinha S. and Sarma M.K. (2019)

MATHEMATICAL FORMULATION

A three dimensional MHD time dependent flow of a viscous incompressible fluid induced by a suddenly started infinite vertical porous plate with uniform mass diffusion in a rotating fluid under the influence of an applied transverse magnetic field and diffusion thermo is considered. To idealize the present fluid flow problem, the following assumptions are taken to be account:

- The variations of all fluid properties other than the variations of density except in so far as they give rise to a body force are ignored completely.
- All the physical variables are functions of z' and t' only as the plate are infinite in x', y'.
- The magnetic Reynolds number is so small that the induced magnetic field can be neglected.
- The polarization effects are assumed to be negligible and hence the induced electric field is also negligible.
- The viscous and magnetic energy dissipation in the energy equation is neglected.

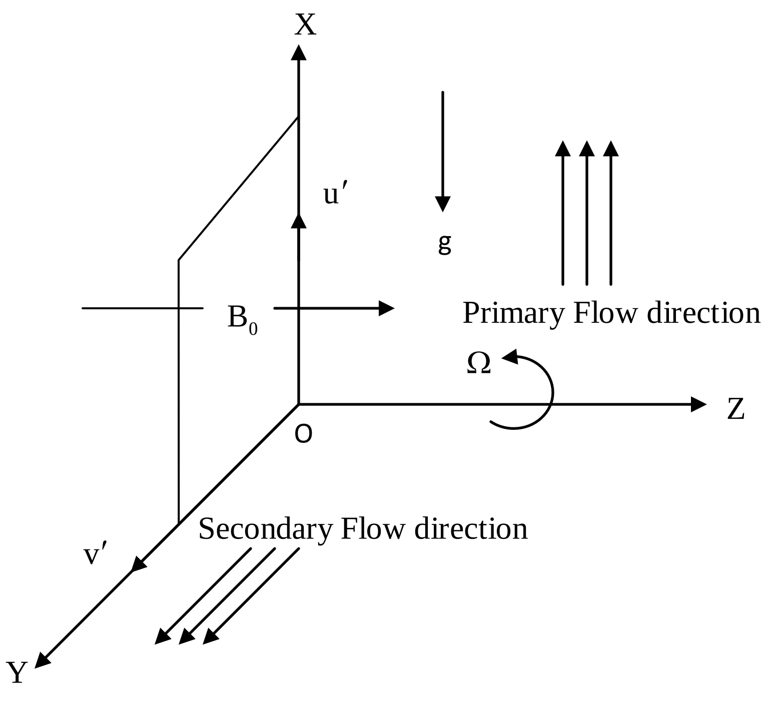

Figure 1: Flow Configuration

A co-ordinate system (x', y', z') with X - axis vertically upwards along the plate, Y- axis perpendicular to the plate and directed into the fluid region and Z- axis along the width of the plate is considered. Let the components of velocity along with X and Y axes should be u' and v'. Let these velocity components are chosen in the upward direction along the plate and normal to the plate respectively. Both the plate and the fluid are in a state of rigid rotation with uniform angular velocity Ω' about the Z- axis.

Under the usual boundary layer approximation and closely following Vijayalakshmi and Kamalm (2013) and Ahmed *et. al*. (2011), the equations that describe the physical situation are given by

$$\frac{\partial u'}{\partial t'} - w'_0 \frac{\partial u'}{\partial z'} - 2\Omega' v' = \upsilon \frac{\partial^2 u'}{\partial z'^2} + \frac{\sigma B_0^{\,2}\left(mv' - u'\right)}{\rho\left(1+m^2\right)} \qquad \rightarrow (1)$$

$$\frac{\partial v'}{\partial t'} - w'_0 \frac{\partial v'}{\partial z'} - 2\Omega' u' = \upsilon \frac{\partial^2 v'}{\partial z'^2} + \frac{\sigma B_0^{\,2}\left(mu' + v'\right)}{\rho\left(1+m^2\right)} \qquad \rightarrow (2)$$

$$\rho C_p \frac{\partial T'}{\partial t'} = K_T \frac{\partial^2 T'}{\partial z'^2} - \frac{\partial q_r}{\partial z'} + \frac{D K_T \rho}{C_s} \qquad \rightarrow (3)$$

$$\frac{\partial C'}{\partial t'} = D \frac{\partial^2 C'}{\partial z'^2} \qquad \rightarrow (4)$$

Where w'_0 is the constant suction velocity and υ is the Kinematic viscosity, the term $\frac{\partial q_r}{\partial z'}$ represents the radiative flux with distance normal to the plate with the following boundary conditions:

$$\left.\begin{array}{ll} u'=0, v'=0, T'=T'_\infty, C'=C'_\infty & for\ t' \le 0, \forall z' \\ u'=a't', v'=0, T'=T'_w, C'=C'_w & for\ z'=0 \\ u'=0, v'=0, T'=T'_\infty, C'=C'_\infty & for\ z' \to \infty \end{array}\right\} \qquad \rightarrow (5)$$

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By Rosselend approximation (Singh and Singh (2000)), the radiative heat flux of an optically thin gray gas can be expressed by the following result:

$$\frac{\partial q_r}{\partial z'} = -4K\sigma'\left(T_\infty'^4 - T'^4\right) \qquad \rightarrow (6)$$

It is assumed that the temperature differences within the flow are sufficiently small that T'^4 may be expressed as a linear function of the temperature. This is accomplished by expanding T'^4 as a Taylor series about T'_∞ and neglecting the higher order terms, we obtain

$$T'^4 \cong 4T_\infty'^3 T' - 3T_\infty'^4 \qquad \rightarrow (7)$$

By using (6) and (7) the equation (3) reduces to

$$\rho C_p \frac{\partial T'}{\partial t'} = K_T \frac{\partial^2 T'}{\partial z'^2} + 16K\sigma' T_\infty'^3\left(T'_\infty - T'\right) \qquad \rightarrow (8)$$

To normalize the given flow model, the following non-dimensional quantities are incorporated:

$$\left.\begin{aligned} & z = \frac{w'_0 z'}{\upsilon},\ t = \frac{w_0'^2 t'}{\upsilon},\ u = \frac{u'}{u'_0},\ v = \frac{v'}{u'_0},\ M = \frac{\sigma B_0{}^2 \upsilon}{\rho w_0'^2}, \\ & \Omega = \frac{2\Omega' \upsilon}{w'_0},\ a = \frac{a' v}{w_0{}^3},\ \theta = \frac{T' - T'_\infty}{T'_\omega - T'_\infty},\ R = \frac{16 K \upsilon^2 \sigma' T_\infty'^3}{K_T w_0{}^2}, \\ & \Pr = \frac{\upsilon}{\alpha},\ \phi = \frac{C' - C'_\infty}{C'_\omega - C'_\infty},\ Sc = \frac{\upsilon}{D}, \Pr = \frac{K_T}{\upsilon \rho C_p}, Du = \frac{D K_T\left(C'_w - C'_\infty\right)}{\rho C_s C_p \upsilon\left(T'_\omega - T'_\infty\right)} \end{aligned}\right\} \rightarrow (9)$$

In view of the equations (6) – (9), the equations (1) – (4) reduce to the following dimensional form :

$$\frac{\partial u}{\partial t} - \frac{\partial u}{\partial z} - v\Omega = \frac{\partial^2 u}{\partial z^2} + M.\frac{mv - u}{1 + m^2} \qquad \rightarrow (10)$$

$$\frac{\partial v}{\partial t}-\frac{\partial v}{\partial z}+u\Omega=\frac{\partial^2 v}{\partial z^2}-M.\frac{mu+v}{1+m^2} \rightarrow (11)$$

$$\frac{\partial \theta}{\partial t}=\frac{1}{\Pr}\frac{\partial^2\theta}{\partial z^2}-\frac{1}{\Pr}R\theta+Du\frac{\partial^2\phi}{\partial z^2} \rightarrow (12)$$

$$\frac{\partial \phi}{\partial t}=\frac{1}{Sc}\frac{\partial^2\phi}{\partial z^2} \rightarrow (13)$$

From (10) and (11) using $q=u+iv,\ A=i\Omega+\frac{M(1+mi)}{1+m^2}$, the following dimensionless form of fluid velocity is obtained:

$$\frac{\partial q}{dt}=\frac{\partial q}{dz}+\frac{\partial^2 q}{dz^2}-q \rightarrow (14)$$

The corresponding boundary conditions are :

$$\left.\begin{array}{l} t\le 0:\ q=0,\ \theta=0,\ \phi=0\ \ for\ all\ values\ of\ z \\ t>0:\ q=at,\ \theta=1,\ \phi=1\ at\ z=0. \\ \qquad and\ q\to 0,\ \theta\to 0,\ \phi\to 0\ as\ z\to\infty \end{array}\right\} \rightarrow (15)$$

SOLUTION OF THE PROBLEM

On taking Laplace Transform of equations (14), (12), and (13), the combined initial and boundary valu problem reduces to a boundary value problem governed by the equations

$$\frac{d^2\bar{q}}{dz^2}+\frac{d\bar{q}}{dz}-(A+S)\bar{q}=0 \rightarrow (16)$$

$$\frac{d^2\bar{\theta}}{d z^2}-\Pr(s+C)\bar{\theta}=-Du\frac{d^2\bar{\phi}}{dz^2} \quad ,where\ C=\frac{R}{\Pr} \rightarrow (17)$$

$$\frac{d^2\bar{\phi}}{d z^2}-sSc\bar{\phi}=0 \rightarrow (18)$$

The boundary conditions (15) under Laplace Transform reduced to

$$\left.\begin{array}{l}\bar{q}=\frac{a}{s^2},\ \bar{\theta}=\frac{1}{s},\ \bar{\phi}=\frac{1}{s} \quad at \quad z=0 \\ \bar{q}=0,\ \bar{\theta}=0,\ \bar{\phi}=0 \quad at \quad z\to\infty\end{array}\right\} \qquad \to (19)$$

The equations (16) –(18) are ordinary coupled second order differential equations. These equations are solved by inverse laplace transform to get the expressions for velocity, temperature and concentration equatios as:

$$q=\psi_6 \qquad \to (20)$$

$$\theta=\psi_1 - De^{-Gt}\psi_4 \qquad \to (21)$$

$$\phi=\psi_5 \qquad \to (22)$$

Skin friction :

The non dimensional form of skin friction at the plate is given by :

$$\tau=\left(\frac{\partial q}{\partial z}\right)_{z=0}=\xi_6$$

Nusselt number :

The non dimensional form of the rate of heat transfer in terms of Nusselt number at the plate is given by :

$$Nu=-\left(\frac{\partial \theta}{\partial z}\right)_{z=0}=\xi_1+De^{-Gt}\xi_4$$

Sherwood number :

The non-dimensional form of the rate of mass transfer in terms of Sherwood number at the plate is given by :

$$Sh=-\left(\frac{\partial \phi}{\partial z}\right)_{z=0}=\xi_5$$

RESULTS AND DISCUSSION

Mathematical computations from the investigative solutions for the representative velocity field (Primary and Secondary), temperature field, concentration field, the co-efficient of skin friction (Primary and Secondary), the rate of heat transfer in terms of Nusselt number and the rate of mass transfer in terms of Sherwood number are made to get the substantial approaching in to the problem. Various graphs of temperature field and co-efficient of rate of heat transfer in terms of Nusselt number bhave been carried out against different physical parameters viz Dufour number (Du), Prandtl number (Pr), Radiation parameter (R) and Schmdit number (Sc) involved in the problem.

The temperature profile under the influence of Prandtl number, Dufour number and Radiation parameter against the normal co-ordinate z are demonstrated in figures 2-4. It is depicted from figure 2 and 4 that rising of Pr or R produces a divergent control on θ which indicates the fact that the fluid temperature falls down gradually for increasing values of momentum diffusivity and thermal radiation. Furthermore, the fluid temperature as expected asymptotically falls from its utmost value at $z = 0$ to its least value at $z \to \infty$.

Figure 3 reveals the behavior of diffusion thermo on temperature profile. The energy flux caused by the gradient of both composition and temperature reduces θ initially near the plate and then accelerates far away from the plate.

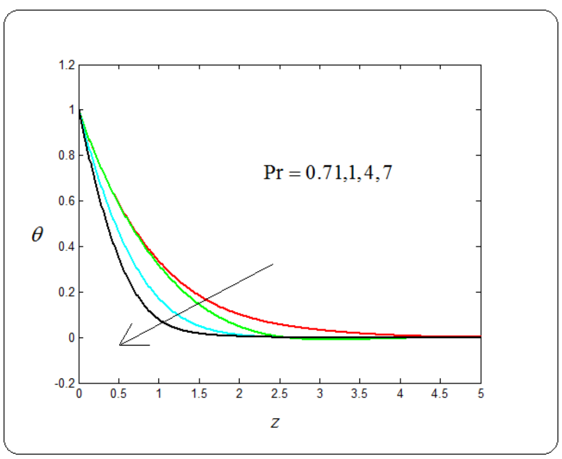

Figure 2: Température versus z for R=1, Du=1, Sc=0.60, t=1

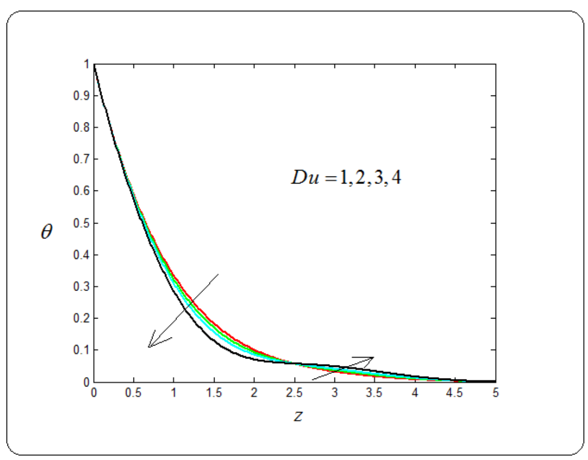

Figure 3: Température versus z for Pr=0.71, R=1, Sc=0.60, t=1

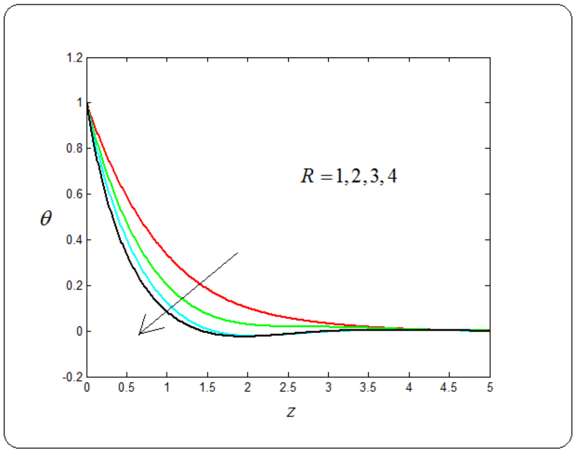

Figure 4: Température versus z for Pr=0.71, Du=1, Sc=0.60, t=1

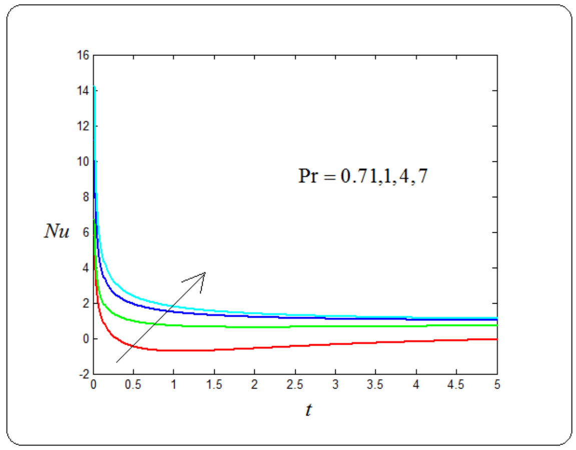

Figure 5: Nusselt number versus t for R=1, Du=1, Sc=0.60

The consequences of Prandtl number, Dufour number and Radiation parameter on the co-efficient of rate of heat transfer in terms of Nusselt number have been displayed in figures 5-7. It is found from figure 5 that the magnitude of the rate of heat transfer increases with higher values of Pr. This simulates that low thermal diffusivity leads the substantial rise in the heat transfer rate. Figure 7 demonstrates that the co-efficient of rate of heat transfer is enhanced with the rise of radiation parameter R. ie the energy flux is raised due to low thermal conductivity.

The effect of Dufour number on Nu from the plate to the fluid has been displayed in figure 6. It is inferred from this figure that the co-efficient of rate of heat transfer moves down due to the influence of diffusion thermo.

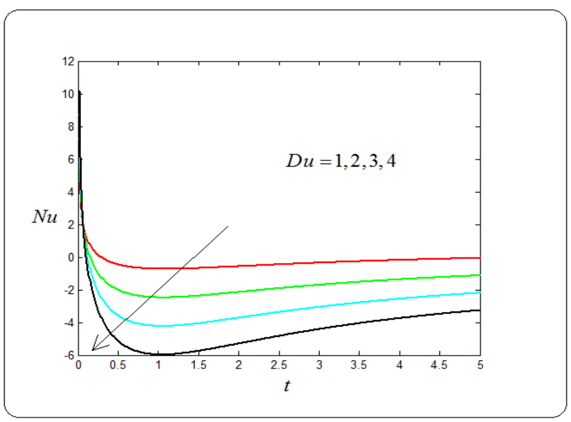

Figure 6: Nusselt number versus t for R=1, Pr=0.71, Sc=0.60

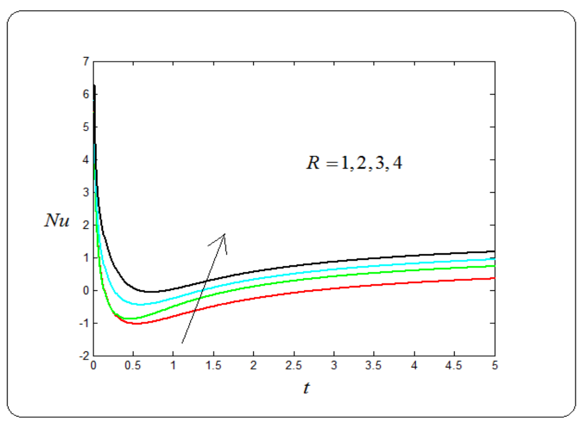

Figure 7: Nusselt number versus t for Du=1, Pr=0.71, Sc=0.60

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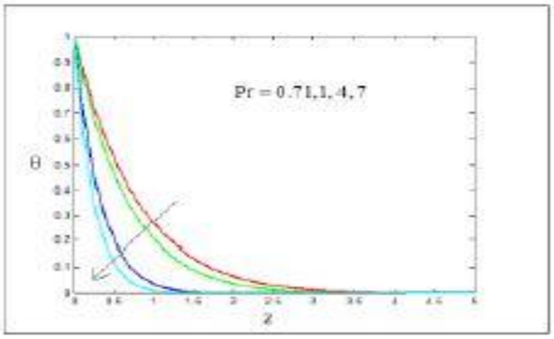
COMPARISON OF RESULTS



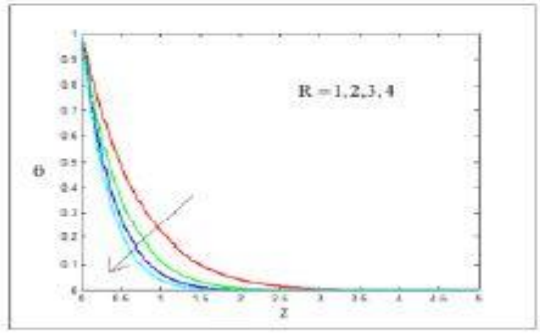


Figure 5: Temperature against z for R=1, t=1

Figure 6: Température against z for Pr=0.71, t=1

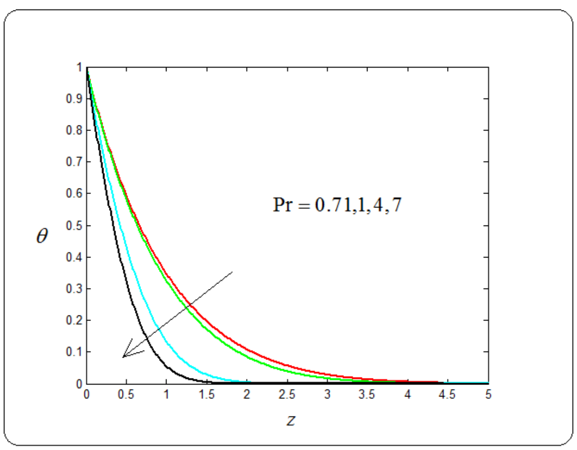


Figure 8: Température versus z for R=1, Du=0, t=1

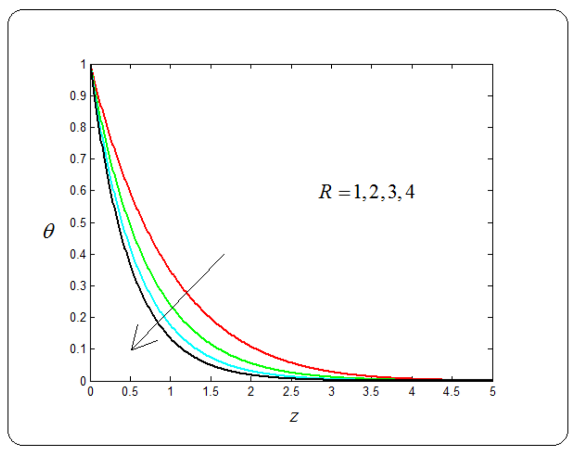


Figure 9: Température versus z for Pr=0.71, Du=0, t=1

For comparing outcomes of the present research, the results of Sinha S. and Sarma M.K. (2019) are used. Comparing figures 8 and 9 with figures 5 and 6 (Sinha S. and Sarma M.K. (2019)), it is observed that the same kind of behaviour due to the implementation of Dufour Effect. With the imposition of Dufour Effect, the temperature profile is almost similar making an admirable fact with the findings investigated by Sinha S. and Sarma M.K. (2019) and the present authors.

CONCLUSION

The following conclusions can be drawn from the present investigation of the problem :

1. The temperature falls down gradually for increasing values of momentum diffusivity and thermal radiation.

2. The energy flux caused by the gradient of both composition and temperature reduces θ initially near the plate and then accelerates far away from the plate.

3. The co-efficient of rate of heat transfer is enhanced with the rise of Prandtl number and radiation parameter.

4. Nusselt number moves down due to the influence of diffusion thermo.

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APPENDIX

$$\psi_1 = \psi\left(\Pr, C, z, t\right) = \frac{1}{2}\left[e^{\sqrt{\Pr}\sqrt{C}z} erfc\left(\frac{\sqrt{\Pr}z}{2\sqrt{t}} + \sqrt{Ct}\right) + e^{-\sqrt{\Pr}\sqrt{C}z} erfc\left(\frac{\sqrt{\Pr}z}{2\sqrt{t}} - \sqrt{Ct}\right)\right]$$

$$\psi_2 = \psi\left(\Pr, C-G, z, t\right) = \frac{1}{2}\left[\begin{array}{l} e^{\sqrt{\Pr}\sqrt{C-G}z} erfc\left(\frac{\sqrt{\Pr}z}{2\sqrt{t}} + \sqrt{(C-G)t}\right) \\ + e^{-\sqrt{\Pr}\sqrt{C-G}z} erfc\left(\frac{\sqrt{\Pr}z}{2\sqrt{t}} - \sqrt{(C-G)t}\right)\end{array}\right]$$

$$\psi_3 = \psi\left(Sc, -G, z, t\right) = \frac{1}{2}\left[\begin{array}{l} e^{\sqrt{Sc}\sqrt{-G}z} erfc\left(\frac{\sqrt{Sc}z}{2\sqrt{t}} + \sqrt{(-G)t}\right) \\ + e^{-\sqrt{Sc}\sqrt{-G}z} erfc\left(\frac{\sqrt{Sc}z}{2\sqrt{t}} - \sqrt{(-G)t}\right)\end{array}\right]$$

$$\psi_4 = \psi_2 - \psi_3$$

$$\psi_5 = \omega(Sc, z, t) = erfc\left(\frac{\sqrt{Sc}\, z}{2\sqrt{t}}\right)$$

$$\psi_6 = \varphi(b, z, t) = \frac{1}{2}\, a^{-\frac{1}{2}z}\left[\left(t + \frac{z}{2\sqrt{b}}\right)e^{\sqrt{b}\,z}\, erfc\left(\frac{z}{2\sqrt{t}} + \sqrt{bt}\right) + \left(t - \frac{z}{2\sqrt{b}}\right)e^{-\sqrt{b}\,z}\, erfc\left(\frac{z}{2\sqrt{t}} - \sqrt{bt}\right)\right]$$

$$\xi_1 = \xi(\Pr, C, t) = \frac{\sqrt{\Pr}}{\sqrt{\pi}\sqrt{t}}\, e^{-Ct} + \sqrt{\Pr}\sqrt{C}\, erf\left(\sqrt{Ct}\right)$$

$$\xi_2 = \xi(\Pr, C - G, t) = \frac{\sqrt{\Pr}}{\sqrt{\pi}\sqrt{t}}\, e^{-(C-G)t} + \sqrt{\Pr}\sqrt{C - G}\, erf\left(\sqrt{(C-G)t}\right)$$

$$\xi_3 = \xi(Sc, -G, t) = \frac{\sqrt{Sc}}{\sqrt{\pi}\sqrt{t}}\, e^{-(-G)t} + \sqrt{Sc}\sqrt{-G}\, erf\left(\sqrt{(-G)t}\right)$$

$$\xi_4 = \xi_2 - \xi_3$$

$$\xi_5 = \zeta(Sc, t) = -\sqrt{\frac{Sc}{\pi . t}}$$

$$\xi_6 = \varsigma(b,\, t) = a\left[-\sqrt{\frac{t}{\pi}}\; e^{-bt} - \left(\sqrt{b}\, t + \frac{1}{2\sqrt{b}}\right) erf\left(\sqrt{bt}\right) - \frac{t}{2}\right]$$

$$A = i\Omega + \frac{M(1+mi)}{1+m^2},\;\; b = \frac{1+4A}{4},\;\; C = \frac{R}{\Pr},\; A^* = Sc - \Pr,\; B = -\Pr C,\; D^* = \frac{-Du\, Sc}{A^*}, G = \frac{B^*}{A^*}$$

$$\mu = \upsilon\rho,\; \Pr = \frac{\upsilon}{\alpha},\; \frac{1}{\Pr} = \frac{K_T}{\upsilon\rho Cp},\; Sc = \frac{\upsilon}{D}, q = u + iv$$