Assessment of the Reliability and Performance of the Engineering System Using Exponentiated Weibull Distribution

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Abstract

In reliability analysis, the performance of the system can be improved by using a scale parameter of the lifetime model to obtain the reliability equivalence factor to design a system. In this paper, the performance of the system is improved through the modified shape parameter of the exponentiated Weibull model. The proposed approach has been employed to the system to determine its performance. Different methods are applied to the system, to enhance its reliability. The method of redundancy and reduction method have been used. The theoretical results are verified through illustration and we find that the cold duplication method outperforms the rest of the methods.

Keywords: Reliability Analysis, Parallel-Series system, Reduction method, Redundancy Method.

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1 Introduction

Reliability is the analysis of failures, their causes and consequences. According to IEEE, the reliability of the system or component is the capacity of the system or component to perform the required function under specific conditions for a specified period of time. Nowadays the reliability of the system is extremely important in maintaining its quality and performance. The maintenance of the system can be achieved by changing unreliable components, good quality components and arrangement of the components in the system. In reliability theory, one way to enhance system performance is to use the redundancy approach. These approaches are as, hot duplication method: in this case, it is assumed that some of the system components are duplicated in parallel. Cold duplication method: in this case, it is assumed that some of the system components are duplicated in parallel via a perfect switch.Unfortunately, for different reasons, the duplication of some or all components cannot be the optimal solution to enhance the system reliability because of high manufacturing costs, space limitation, expensive components etc. So that the engineer turns to the well-known method in reliability theory known as the reduction method. In this method, it is assumed that the failure rate of the components is reduced by multiplying the shape or scale parameter of the lifetime model by some factor, ρ, $0 < \rho < 1$. Once the reduction method has opted, the engineer has to decide how much amount the failure rate should be reduced, so that the system improved by the reduction method should be equivalent to the design of the system improved by one of the methods. The comparison of the designs produces the so-called reliability equivalence factor by Sarhan (2009). Rade (1993a) introduced the concept of the reliability equivalence factor through his researches in statistical quality control. The reliability equivalence factor is a factor by which a characteristic of system design has to be multiplied in order to reach equality of a characteristic for a different standard design. Rade (1993) applied this concept for the two-component parallel and series systems with independent and identical components whose lifetimes follow the exponential distribution. Maryam and Kannan (2021) studied the reliability of complex system by the method of redundancy using Alpha power transformed Rama distribution. Maryam and Kannan (2020) discussed and assessed the reliability of different systems using different methods. Beyond the assumptions of constant failure rates modeling by the exponential distribution, Xia and Zhang (2007) considered equivalence factors in Gamma distribution. In the previously mentioned studies, the hazard and the reliability functions are decreases or increases through the indexed scale parameter. In reliability general frame analysis, there exists other lifetime distributions for which the hazard and reliability functions are not affected by the scale parameter, and mainly affected by the shape parameter. El-Damcese and Khalifa (2008) obtained the reliability equivalence factors of series-parallel systems in the Weibull distribution.

The exponentiated Weibull family of distributions was introduced by Mudholkar and Srivastava (1993). The model consists of a scale parameter and a shape parameter. The exponentiated Weibull is a generalization of exponentiated exponential distribution as well as the Weibull family. The probability density function of exponentiated Weibull is uni-modal and approaches to a normal distribution with the increase in shape parameterα. The failure rate of the exponentiated Weibull distribution is a non-decreasing function of α for fixed parameter λ and γ. Rather and Subramanian (2020), discussed the exponentiated garima distribution with applications in engineering science. Rather et al. (2022), obtained Exponentiated Ailamujia distribution with statistical inference and applications of medical data.

The structure of this paper is organized as follows. In section 2, the reliability function of the exponentiated Weibull distribution is defined. In section 3, the reliability of the improved system by reduction method and standby redundancy method is obtained. In section 4, the reliability equivalence factor of the improved system like hot, cold and hybrid duplication method is derived. In section 5, numerical illustration is used to obtain the theoretical results of the system. In section 6, conclusion of the study is presented.

2. Exponentiated Weibull Model

The probability density function and cumulative distribution function of the random variable T having exponentiated Weibull distribution is given by

$$f_\alpha(t) = \alpha\gamma\lambda^\gamma t^{\gamma-1}\exp(-(\lambda t)^\gamma)\,[1-\exp(-(\lambda t)^\gamma)]^{\alpha-1} \quad t>0,\ \alpha,\ \lambda,\ \gamma>0 \tag{1}$$

$$F_\alpha(t) = [1-\exp(-(\lambda t)^\gamma)]^\alpha \tag{2}$$

The reliability function and failure rate of exponentiated Weibull distribution is defined as

$$R_\alpha(t) = \{1-[1-\exp(-(\lambda t)^\gamma)]^\alpha\} \tag{3}$$

$$h_\alpha(t) = \frac{\alpha\gamma\lambda^\gamma t^{\gamma-1}\exp(-(\lambda t)^\gamma)\,[1-\exp(-(\lambda t)^\gamma)]^{\alpha-1}}{\left\{1-[1-\exp(-(\lambda t)^\gamma)]^\alpha\right\}} \tag{4}$$

2.1 Parallel-Series System

The system considered here, consists of m subsystems connected in parallel with subsystem i consisting of n_i components connected in series for, $i = 1, 2, 3, \dots, m$.

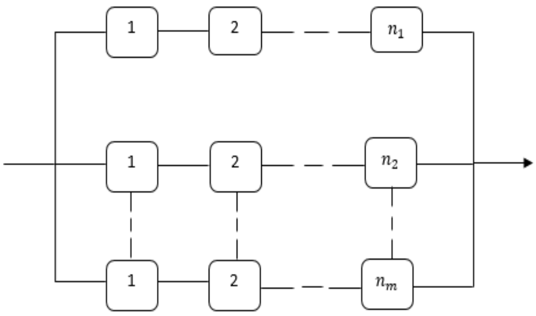

Fig. 1: Parallel-Series System

Let $R_i(t)$ be the reliability of the subsystem i and $r_{ij}(t)$ be the reliability of the component j in the subsystem, $j = 1, 2, 3, \dots, n_i,\ i = 1, 2, 3, \dots, m$, then

$$R_i(t) = \prod_{j=1}^{n_i} r_{ij}(t) \tag{5}$$

This implies that the reliability of the system is

$$R_s(t) = 1 - \prod_{i=1}^{m}\left(1 - R_i(t)\right) \tag{6}$$

Using equation (5) in equation (6), then the reliability of the system is

$$R_s(t) = 1 - \prod_{i=1}^{m}\left(1 - \prod_{j=1}^{n_i} r_{ij}(t)\right) \tag{7}$$

Assume that the system components are identically independent and follows the exponentiated Weibull lifetime model. This implies that reliability of each component is given by

$$r_{ij}(t) = 1 - \left[1 - \exp\left(-(\lambda t)^{\gamma}\right)\right]^{\alpha}\ , \quad t. > 0$$

Substituting for $r_{ij}(t)$ in equation (7), the reliability of the system becomes

$$R_s(t) = 1 - \prod_{i=1}^{m}\left(1 - \prod_{j=1}^{n_i}\left\{1 - \left[1 - \exp\left(-(\lambda t)\right)^{\gamma}\right]^{\alpha}\right\}\right) \tag{8}$$

$$R_s(t) = 1 - \prod_{i=1}^{m}\left(1 - \left\{1 - \left[1 - \exp\left(-(\lambda t)\right)^{\gamma}\right]^{\alpha}\right\}^{n_i}\right) \tag{9}$$

3 Design of Improved Systems

In this section, the reliability of the improved system according to the reduction method and standby redundancy of hot, cold, and hybrid method will be derived and discussed.

3.1 Reduction Method

In this method, it is assumed that the reliability of k_i components of the subsystem i, $i = 1, 2, 3, \ldots, m$, is improved by increasing their reliability function through multiplying the shape parameter γ by factor $0 < \omega < 1$. Hence the reliability of each component is defined by equation (10) as

$$r_{\omega}(t) = \left\{1 - \left[1 - \exp\left(-(\lambda t)^{\omega\gamma}\right)\right]^{\alpha}\right\} \tag{10}$$

This implies that the reliability of the system improved by reduction method is given by

$$R_s^{\omega}(t) = \left\{1 - \prod_{i=1}^{m}\left(1 - \left(r_{\omega}(t)\right)^{k_i}\left(r(t)\right)^{n_i - k_i}\right)\right\}$$

$$R_s^{\omega}(t) = \left\{1 - \prod_{i=1}^{m}\left(1 - \left(\left\{1 - \left[1 - \exp\left(-(\lambda t)^{\omega\gamma}\right)\right]^{\alpha}\right\}\right)^{k_i}\left(\left\{1 - \left[1 - \exp\left(-(\lambda t)^{\gamma}\right)\right]^{\alpha}\right\}\right)^{n_i - k_i}\right)\right\} \tag{11}$$

3.2 Hot Duplication Method

In this method, it is assumed that some of the system components are duplicated in parallel and always switched on with the original components. If the h_i components of the subsystem $i = 1, 2, 3, \ldots, m$ are hot duplicated, then the reliability function of each of these components is given by

$$r_h(t) = \left(2 - r(t)\right) r(t) \tag{12}$$

Substituting equation (3) in equation (12) we will obtain the reliability of the hot duplicated components.

$$r_h(t) = \left\{2 - \left(1 - \left[1 - \exp\left(-(\lambda t)^{\gamma}\right)\right]^{\alpha}\right)\right\}\left\{1 - \left[1 - \exp\left(-(\lambda t)^{\gamma}\right)\right]^{\alpha}\right\}$$

$$r_h(t) = \left\{1 - \left[1 - \exp\left(-(\lambda t)^{\gamma}\right)\right]^{\alpha}\right\}^{2} \tag{13}$$

This implies that the reliability function of the improved system by the hot duplication method is given by

$$R_s^h(t) = \left\{ 1 - \prod_{i=1}^{m} \left(1 - (r_h(t))^{h_{i}} \; (r(t))^{n_i - h_i} \right) \right\} \quad (14)$$

Using equation (13) in equation (14), we will get the following equation as defined below

$$R_s^h(t) = \left\{ 1 - \prod_{i=1}^{m} \left(\begin{matrix} 1 - \left(\left\{ 1 - \left[1 - \exp(-(\lambda t)^{\gamma}) \right]^{\alpha} \right\}^2 \right)^{h_i} \\ \left(\left\{ 1 - \left[1 - \exp(-(\lambda t)^{\gamma}) \right]^{\alpha} \right\} \right)^{n_i - h_i} \end{matrix} \right) \right\}$$

3.3 Cold Duplication Method

In this method, it is assumed that some of the system components are connected in parallel with the original components via perfect switch. The reliability function of each component improved by cold duplication via perfect switch is given by

$$r_c(t) = \left(1 + \ln\left(\frac{1}{r(t)} \right) \right) r(t) \;\; r_c(t) = (1 + \alpha (\lambda t)^{\gamma}) \; \left(1 - \left[1 - \exp(-(\lambda t)^{\gamma}) \right]^{\alpha} \right) \quad (15)$$

Substituting equation (15) in equation (16) and we obtain the reliability of the system improved by cold duplication method as

$$R_s^c(t) = \left\{ 1 - \prod_{i=1}^{m} \left(1 - (r_c(t))^{c_i} \; (r(t))^{n_i - c_i} \right) \right\} \quad (16)$$

$$R_s^c(t) = \left\{ \begin{matrix} 1 - \prod_{i=1}^{m} 1 - \left((1 + \alpha (\lambda t)^{\gamma}) \; \left(1 - \left[1 - \exp(-(\lambda t)^{\gamma}) \right]^{\alpha} \right) \right)^{c_i} \\ \left(\left(1 - \left[1 - \exp(-(\lambda t)^{\gamma}) \right]^{\alpha} \right)^{n_i - c_i} \right) \end{matrix} \right\}$$

3.4 Hybrid Duplication Method

In this method, we assume that some of the system components are hot duplicated and some are cold duplicated. Suppose h_i components of the subsystem i are hot duplicated and c_i are cold duplicated, $i = 1, 2, 3, \ldots, m$. Then the reliability function of the improved system by a hybrid of these components becomes

$$R_s^{hc} = \left\{ 1 - \prod_{i=1}^{m} \left(1 - (r_c(t))^{c_i} \, (r_h(t))^{h_i} \; (r(t))^{n_i - h_i - c_i} \right) \right\}$$

$$R_s^{hc}(t) = \left\{ 1 - \prod_{i=1}^{m} \left(\begin{matrix} 1 - \left[\left(1 - \left[1 - \exp(-(\lambda t)^{\gamma}) \right]^{\alpha} \right)^2 \right]^{h_i} \\ \left(1 - \left[1 - \left[1 - \exp(-(\lambda t)^{\gamma}) \right]^{\alpha} \right]^{c_i} \\ \left(1 - \left[1 - \left[1 - \exp(-(\lambda t)^{\gamma}) \right]^{\alpha} \right]^{n_i - h_{ii} - c_i} \right) \right) \end{matrix} \right) \right\} \quad (17)$$

4 Reliability Equivalence Factor

In this section, the reliability equivalence factor of the improved system will be derived. The reliability equivalence factor is denoted by $\omega\,(\alpha)$. The reliability equivalence factor is defined as the factor by which the shape parameter α should be multiplied in order to increase the reliability function of the original system by the reduction method equivalent of improving the system by the hot, cold and hybrid duplication.

For the hot duplication reliability equivalence factor, denoted by$\omega(\alpha)^h$ can be obtained by solving the following set of two equations.

$$R_s^{\omega}(t) = \alpha \tag{18a}$$

$$R_s^{h}(t) = \alpha \tag{18b}$$

Substituting equation (11) in equation (18a) and equation (14) in equation (18b) respectively, $\omega(\alpha)^h$ can be obtained by solving the following set of equations.

$$R_s^h = \left\{1 - \prod_{i=1}^{m}\left(1-\left(r_h(t)\right)^{h_i}\left(r(t)\right)^{n_i-h_i}\right)\right\} \tag{19}$$

$$R_s^{\omega} = \left\{1 - \prod_{i=1}^{m}\left(1-\left(r_{\omega}(t)\right)^{k_i}\left(r(t)\right)^{n_i-k_i}\right)\right\} \tag{20}$$

Similarly, for cold duplication method, $\omega(\alpha)^c$ can be obtained by solving the following set of two equations.

$$R_s^{\omega}(t) = \alpha$$

$$R_s^{c}(t) = \alpha \tag{18c}$$

Substituting the equation (11) in equation (18a) and equation (16) in equation (18c) respectively $\omega(\alpha)^c$ can be obtained by solving the following set of equations.

$$R_s^c = \left\{1 - \prod_{i=1}^{m}\left(1-\left(r_c(t)\right)^{c_i}\left(r(t)\right)^{n_i-c_i}\right)\right\} \tag{21}$$

$$R_s^{\omega} = \left\{1 - \prod_{i=1}^{m}\left(1-\left(r_{\omega}(t)\right)^{k_i}\left(r(t)\right)^{n_i-k_i}\right)\right\} \tag{22}$$

In case of hybrid duplication method, $\omega(\alpha)^{hc}$ can be obtained by solving the following set of equations.

$$R_s^{\omega}(t) = \alpha$$

$$R_s^{hc}(t) = \alpha \tag{18d}$$

Substituting equation (11) in equation (18a) and equation (17) in equation (18d) respectively $\omega(\alpha)^{hc}$ can be obtained by solving the following set of equations.

$$R_s^{hc} = \left\{1 - \prod_{i=1}^{m}\left(1-\left(r_c(t)\right)^{c_i}\left(r_h(t)\right)^{h_i}\left(r(t)\right)^{n_i-h_i-c_i}\right)\right\} \tag{23}$$

$$R_s^{\omega} = \left\{ 1 - \prod_{i=1}^{m} \left(1 - \left(r_{\omega}(t) \right)^{k_i} \left(r(t) \right)^{n_i - k_i} \right) \right\} \quad (24)$$

5 Numerical Results

To illustrate the theoretical results, we consider a system that consists of $n = 6$, components distributed in $m = 2$ subsystem connected in parallel with $n_1 = 3$ components in series in subsystem 1 and $n_2 = 3$ components in subsystem 2. Assume that the components are identically independent and follow exponentiated Weibull distribution with fixed scale parameter,$\lambda = 0.5$, and the shape parameter $\alpha = 4\gamma = 1$, to implement an increasing failure rate.

The improved system are designed by setting

(k_1, k_2), (h_1, h_2), (c_1, c_2) =
$(0, 1)$, $(1,1)$, $(2,0)$, $(2,1)$, $(2,2)$

Using the reduction, hot and cold duplication methods respectively

5.1 Reliability of the system based on different reducing factors

In this section, the reliability of the system is demonstrated with the help of different reducing factors. Here, we investigate the reliability of the original system and improved systems by using different reduction factors. Let's consider the value of the parameters as

$\lambda = 0.5,\ \ \alpha = 4,\ \ \gamma = 1, t = 1\ hr, n = 6,$

and $\ m = 2$

where, m is the number of subsystems in the system and n is the total number of components connected in series in the system. Substituting the value of the parameters in equation (25), we obtain the reliability of the original system as

$$R(t) = \left\{ 1 - \left(1 - (1 - (1 - \exp(-\lambda t)^{\gamma})^{\alpha})^{n} \right)^{m} \right\} \quad (25)$$

$$R(t) = \left\{ 1 - \left(1 - (1 - (1 - \exp(-\ 0.5))^{4})^{6} \right)^{2} \right\}$$

The reliability of the improved systems based on reduction method is obtained as

$$R_s^{\omega}(t) = \left\{ 1 - \left(1 - (1 - (1 - \exp(-\lambda t)^{\gamma\omega})^{\alpha})^{n} \right)^{m} \right\} \quad (26)$$

$$R_s^{\omega}(t) = \left\{ 1 - \left(1 - (1 - (1 - \exp(-0.5\ )^{\omega})^{4})^{6} \right)^{2} \right\}$$

Substituting the value for $\omega = 0.2$, 0.4, 0.6, 0.9, in equation (26), we obtain the reliability of improved system.

A. $\omega = 0.2$

$R_s^{\omega}(t) = 0.9999998$

B. $\omega = 0.4$

$R_s^{\omega}(t) = 0.9999583$

C. $\omega = 0.6$

$R_s^{\omega}(t) = 0.9992833$

D. $\omega = 0.9$

$$R_s^{\omega}(t) = 0.9901791$$

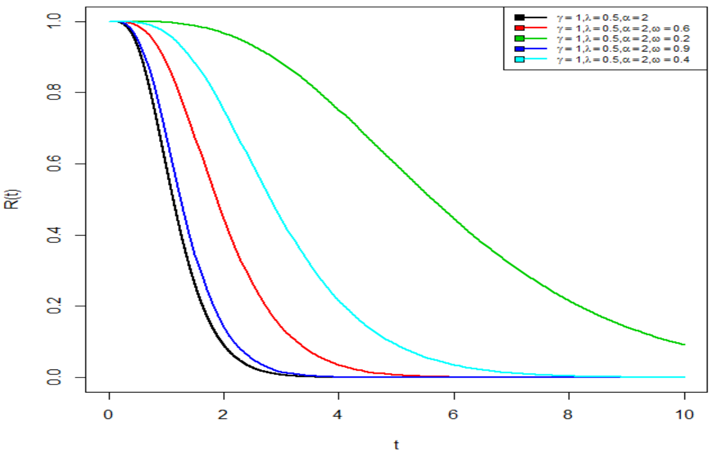

Figure.2: Reliability plot of exponentiated weibull distribution for different reduction factor

5.2 Determining the Reliability of the Original System and Improved Systems

In this subsection, the reliability of the original system, Standby Redundancy and Reduction Method is discussed.

The reliability of the system is obtained by reducing and duplicating all the components of the system as

$$R(t) = \left\{1 - \left(1 - (1 - (1 - \exp(-\lambda t)^{\gamma})^{\alpha})^{n}\right)^{m}\right\}$$

$$R(t) = \left\{1 - \left(1 - (1 - (1 - \exp(-0.5))^{4})^{6}\right)^{2}\right\}$$

$$R(t) = 0.9816492$$

The reliability of the improved systems are discussed as

5.2.1 Reduction Method

Here, we choose reduction factor $\omega = 0.5$ the reliability of the system improved by reduction method is obtained as

$$R_s^{\omega}(t) = \left\{1 - \left(1 - (1 - (1 - \exp(-\lambda t)^{\gamma\omega})^{\alpha})^{n}\right)^{m}\right\}$$

$$R_s^{\omega}(t) = \left\{1 - \left(1 - (1 - (1 - \exp(-0.5)^{\omega})^{4})^{6}\right)^{2}\right\} R_s^{\omega}(t) = 0.9997962$$

5.2.2 Hot Duplication Method

The reliability of system improved by hot duplicating of components is obtained as

$$R_s^{h}(t) = \left\{1 - \begin{pmatrix} 1 - ((1 - (1 - \exp(-\lambda t)^{\gamma})^{\alpha}) \\ (2 - (1 - (1 - \exp(-\lambda t)^{\gamma})^{\alpha})))^{n} \end{pmatrix}^{m}\right\}$$

$$R_s^{h}(t) = \left\{1 - \begin{pmatrix} 1 - ((1 - (1 - \exp(-0.5))^{4}) \\ (2 - (1 - (1 - \exp(-0.5))^{4})))^{6} \end{pmatrix}^{2}\right\}$$

$$R_s^{h}(t) = 0.9999881$$

5.2.3 Cold Duplication Method

The reliability of system improved by cold duplicating of components is obtained as

$$R_s^c(t)=\left\{1-\begin{pmatrix}1-((1-(1-\exp(-\lambda t)^{\gamma})^{\alpha})-\\(1-(1-\exp(-\lambda t)^{\gamma})^{\alpha})\\\ln(1-(1-\exp(-\lambda t)^{\gamma})^{\alpha}))^{n}\end{pmatrix}^{m}\right\}$$

$$R_s^c(t)=0.999996986$$

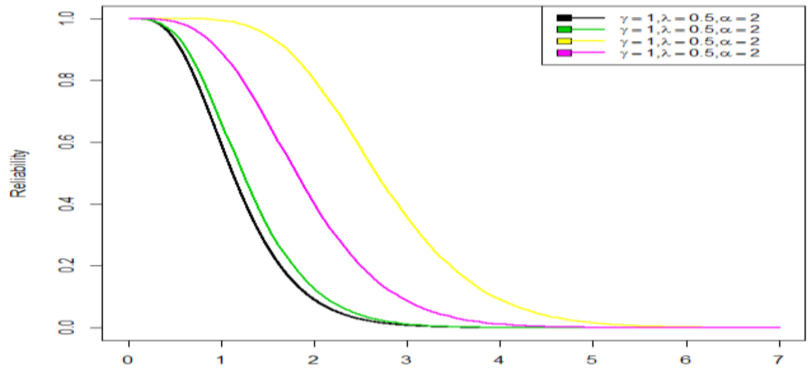

Figure.3: Reliability plot of original system and hot, cold, hybrid duplicated systems

5.3 Reliability of the Improved System by Duplicating the Components of the System

To illustrate the theoretical results, we consider a series-parallel that consists of $n = 6$, components distributed in $m = 2$ subsystem connected in parallel with $n_1 = 3$ components in series in subsystem 1 and $n_2 = 3$ components in series in subsystem 2. Assume that the components are identically independent and follow exponentiated Weibull distribution with fixed scale parameter, $\lambda = 0.5$, and the shape parameter $\alpha = 4, \gamma = 1$, to implement an increasing failure rate.

The improved system are designed by setting

(k_1, k_2), (h_1, h_2), (c_1, c_2) =
$(0, 1)$, $(1,1)$, $(2,0)$, $(2,1)$, $(2,2)$

5.3.1 Reduction Method

Here, we make combinations of components to know which combination is better for reduction that will increase the reliability of the system.

$$R_s^{\omega}(t)=\left\{1-\prod_{i=1}^{m}\left(1-\left(r_{\omega}(t)\right)^{k_i}\left(r\ (t)\right)^{n_i-k_i}\right)\right\}$$

For component (0, 1)

In this combination, none of the components from subsystem 1 is considered for reduction. We consider one component out of total number of components for duplication from subsystem 2.

$$R_s^{\omega}(t)=1-\left\{\left(1-(1-(1-\exp(-0.5))^4)^3\right)\begin{pmatrix}1-((1-(1-\exp(-0.5))^4)^2\\(1-(1-\exp(-0.5)^{0.5})^4)^1)\end{pmatrix}\right\}$$

$$R_s^{\omega}(t)=0.9965153$$

For component (1, 1)

In this combination, one component is taken from subsystem 1 and another one from subsystem 2, out of the total number of components present in the system.

$$R_s^{\omega}(t)=1-\left\{\begin{matrix}\left(\begin{matrix}1-(1-(1-\exp(-0.5))^4)^2\\(1-(1-\exp(-0.5)^{0.5})^4)^1\end{matrix}\right)\\\left(\begin{matrix}1-((1-(1-\exp(-0.5))^4)^2\\(1-(1-\exp(-0.5)^{0.5})^4)^1)\end{matrix}\right)\end{matrix}\right\}$$

$R_s^{\omega}(t)=0.9975355$

For components (2, 1)

In this combination, two components is taken from subsystem 1 and another one from subsystem 2, out of the total number of components in the system.

$$R_s^{\omega}(t)=1-\left\{\begin{matrix}\left(\begin{matrix}1-(1-(1-\exp(-0.5))^4)^1\\(1-(1-\exp(-0.5)^{0.5})^4)^2\end{matrix}\right)\\\left(\begin{matrix}1-((1-(1-\exp(-0.5))^4)^2\\(1-(1-\exp(-0.5)^{0.5})^4)^1)\end{matrix}\right)\end{matrix}\right\}$$

$R_s^{\omega}(t)=0.9985784$

For components (2,2)

In this combination, two components is taken from subsystem 1 and another two components from subsystem 2, out of the total number of components in the system.

$$R_s^{\omega}(t)=1-\left\{\begin{matrix}\left(\begin{matrix}1-(1-(1-\exp(-0.5))^4)^1\\(1-(1-\exp(-0.5)^{0.5})^4)^2\end{matrix}\right)\\\left(\begin{matrix}1-((1-(1-\exp(-0.5))^4)^1\\(1-(1-\exp(-0.5)^{0.5})^4)^2)\end{matrix}\right)\end{matrix}\right\}$$

$R_s^{\omega}(t)= 0.9991799$

For components (2, 0)

In this combination, two components are chosen/selected from subsystem 1 and from the subsystem 2, none of the component is selected for duplication out of the total number of components in the system.

$$R_s^{\omega}(t)=1-\left\{\begin{matrix}\left(\begin{matrix}1-(1-(1-\exp(-0.5))^4)^1\\(1-(1-\exp(-0.5)^{0.5})^4)^2\end{matrix}\right)\\\left(1-((1-(1-\exp(-0.5))^4)^3\right)\end{matrix}\right\}$$

$R_s^{\omega}(t) = 0.9979898$

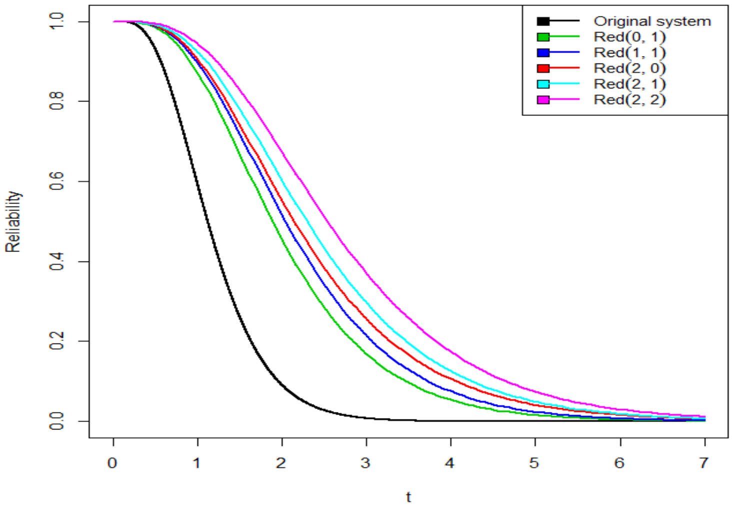

Figure.4: Reliability of original system and reduced components systems

5.3.2 Hot Duplication Method

Here, we make different combinations of components to know which combination is better for duplication that will increase the reliability of the system. One of the component is chosen from subsystem 1 and subsystem 2.

For components (0,1)

In this combination, none of the components from subsystem 1 is considered for duplication. We consider one component out of total number of components for duplication from subsystem 2.

$$R_s^h(t) = 1 - \left\{ \begin{pmatrix} \left(1-(1-(1-\exp(-\lambda t)^{\gamma})^{\alpha})^{n}\right) \\ \begin{pmatrix} 1-(1-(1-\exp(-\lambda t)^{\gamma})^{\alpha})^{n} \\ \begin{pmatrix} (2-(1-(1-\exp(-\lambda t)^{\gamma})^{\alpha})) \\ (1-(1-\exp(-\lambda t)^{\gamma})^{\alpha}) \end{pmatrix}^{n} \end{pmatrix} \end{pmatrix} \right\}$$

$R_s^h(t) = 0.996636693$

For components $(1,1)$

In this combination, one component is taken from subsystem 1 and another one component is selected from subsystem 2 for duplication, out of the total number of components in the system.

$R_s^h(t) = 0.99770435$

For components $(2,1)$

In this combination, two components are chosen from subsystem 1 and another one component has been selected for duplication from subsystem 2,out of the total number of components in the system.

$R_s^h(t) = 0.99879759$

For components (2, 2)

In this combination, two components are selected from subsystem 1 and another two components are taken from subsystem 2, out of the total number of components in the system.

$R_s^h(t) = 0.99937021$

For components (2, 0)

In this combination, two components are chosen from subsystem 1 and none of the component is taken from subsystem 2, out of the total number of components in the system.

$R_s^h(t) = 0.99823838$

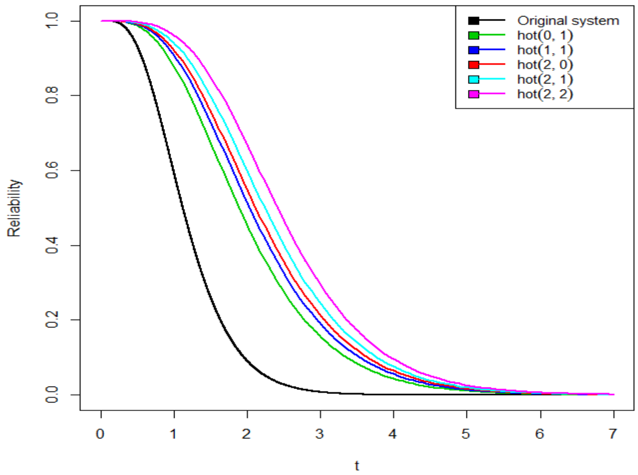

Figure.5: Reliability of original system and hot duplicated components

5.3.3 Cold Duplication Method

Here, we make different combinations of components to know which combination is better for duplication which/that will increase the reliability of the system.

For components (0, 1)

In this combination, none of the components from subsystem 1 is considered for duplication. We consider one component out of total number of components for duplication from subsystem 2.

$$R_s^c(t) = 0.996655944$$

For components (1, 1)

In this combination, one component is taken from subsystem 1 and another one component is selected for duplication from subsystem 2, out of the total number of components in the system.

$$R_s^c(t) = 1 - \left(\begin{array}{l} \left(1 - (1-(1-\exp(-\lambda t))^4)^2) \begin{pmatrix} (1-(1-\exp(-0.5))^4) - \\ (1-(1-\exp(-0.5))^4) \\ \ln(1-(1-\exp(-0.5))^4) \end{pmatrix}^1 \right) \\ 1 - \left((1-(1-\exp(-0.5))^4)^2) \begin{pmatrix} (1-(1-\exp(-0.5))^4) - \\ (1-(1-\exp(-0.5))^4) \\ \ln(1-(1-\exp(-0.5))^4) \end{pmatrix}^1 \right) \end{array} \right)$$

$$R_s^c(t) = 0.99773056$$

For components (2, 1)

In this combination, two components are chosen from subsystem 1 and another one component is selected from subsystem 2, out of the total number of components in the system.

$$R_s^c(t) = 0.998831241$$

For components (2, 2)

In this combination, two components are chosen from subsystem 1 and another two components are taken from subsystem 2, out of the total number of components in the system.

$$R_s^c(t) = 0.9999398086$$

For components (2, 0)

In this combination, two components are chosen from subsystem 1 and none of the component is taken from subsystem 2, out of the total number of components in the system.

$$R_s^c(t) = 0.99827782$$

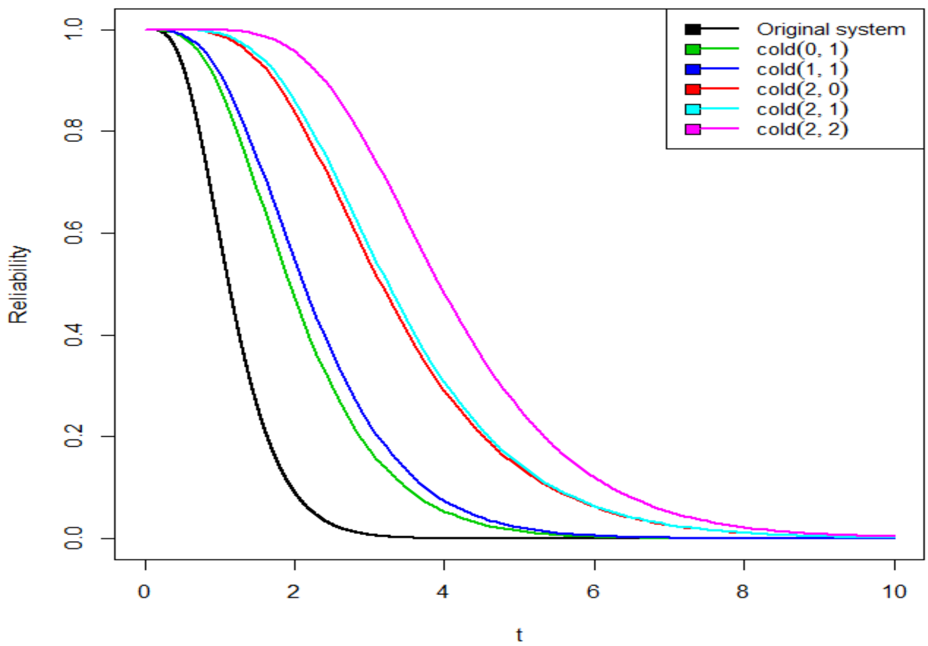

Figure.6: Reliability of original system and cold duplicated components

Plots Showing the Comparison of Reliability of Hot and Cold Duplicating Components

In this subsection, the comparison of reliability of the components is discussed on the bases of hot duplication and cold duplication method.

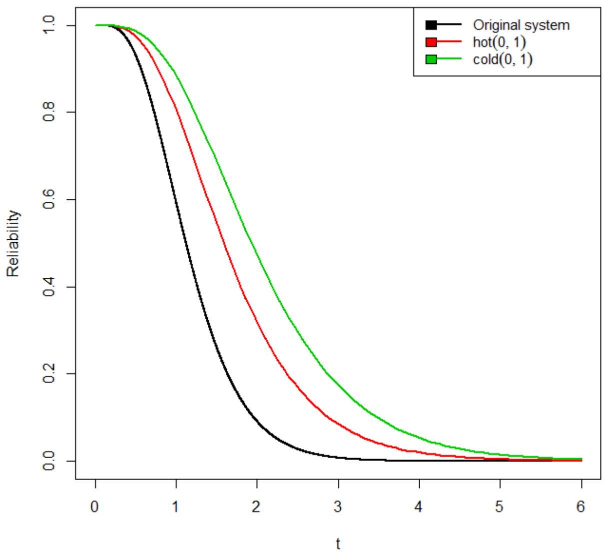

Figure.7: Reliability plot of original system and h(0,1), c(0,1) duplicated components

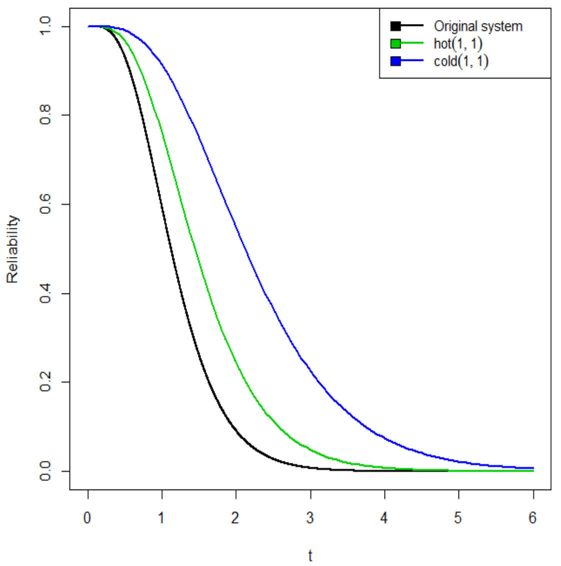

Figure.8: Reliability plot of original system and h(1,1), c(1,1) duplicated components

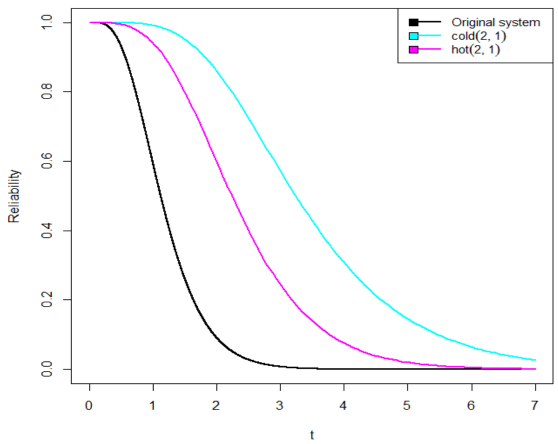

Figure.9: Reliability plot of original system and h(2,1), c(2,1)duplicated components

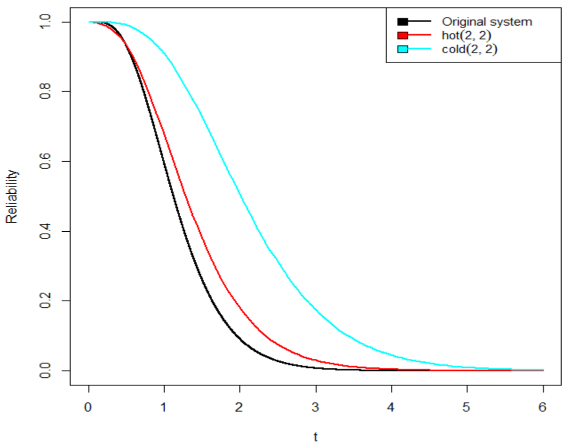

Figure.10: Reliability plot of original system and h(2,2), c(2,2) duplicated components

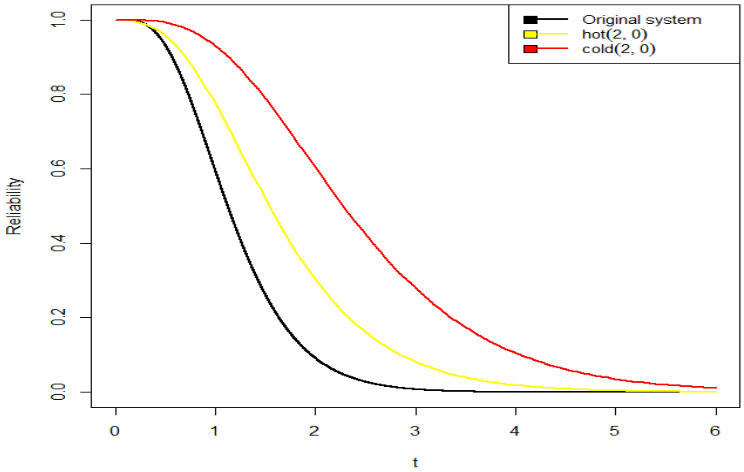

Figure.11: Reliability plot of original system and h(2,0), c(2,0) duplicated components

i. The figure 2, reveals that ason, the increase of the reducing factor the reliability of a system reduces from 0.999998 to 0.9901791.
ii. The reliability of the improved system by the method of reduction is maximum at $\omega = 0.2$.
iii. The figure 3, presents that the reliability of system is maximum in the order of

$$R_S^c(t) > R_S^{hc}(t) > R_S^h(t) > R(t)$$

iv. The figure 4, shows that the reliability of the components goes on increasing from original system to the improved system by reducing the components of the system. The reliability of the original system is 0.98164. The reliability of the system is maximum when the two components from the subsystem are reduced from subsystem 1, and two components from subsystem 2. i.e. 0.99917996.

$$R(0,1) < R(1,1) < R(2,0) < R(2,1) < R(2,2)$$

v. The figure 5, presents that the reliability of the components goes on increase when the system is improved by hot duplicating its components. The reliability of the original system is 0.98164. The reliability of the system is maximum, when the two components from subsystem are reduced from the subsystem 1, and two components from subsystem 2. i.e. 0.99937021.

$$R(0,1) < R(1,1) < R(2,0) < R(2,1) < R(2,2)$$

vi. The figure 6, demonstrates that the reliability of the components goes on the increase when the system is improved by cold duplicating its components. The reliability of the original system is 0.98164. The reliability of the system is maximum when the two components from subsystem are reduced from the subsystem 1, and two components from subsystem 2. i.e. 0.999398086.

$$R(0,1) < R(1,1) < R(2,0) < R(2,1) < R(2,2)$$

vii. The figures from figure 7-11, reveals that the components that are cold duplicated shows higher reliability of the system than the components improved by cold duplication and original system.

$$R(t) < Rh(0,1) < Rc(0,1)$$
$$R(t) < Rh(1,1) < Rc(1,1)$$
$$R(t) < Rh(2,0) < Rc(2,0)$$
$$R(t) < Rh(2,1) < Rc(2,1)$$
$$R(t) < Rh(2,2) < Rc(2,2)$$

6. Conclusion

In this paper, the reliability equivalence of parallel-series system is studied. The reliability of the original system and the improved system are obtained. The reliability of the system is improved by employing different methods. These are the reduction method and standby redundancy method. The redundancy method comprises hot duplication, cold duplication and hybrid duplication method. Among these methods, it has been demonstrated that the reliability of the system is at a higher-end by cold duplication method than hot duplication and reduction method. In the reduction method, the reliability of the system increases as the reduction factor tends to decrease. The reliability of cold duplicated components is higher than the components improved by hot duplication.

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