Effect of different crack sizes on the time varying gearmesh stiffness of gear pair

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Abstract

Gear tooth crack is a type of fault which need monitoring to prevent serious damage to the operating condition of the gear system. In this research work an analytical model is proposed to investigate the effect of gear tooth crack in the form of time varying gear mesh stiffness. In this case the tooth crack propagations is monitored along tooth width, crack depth and crack height are incorporated in this model for simulation of gear tooth root crack. Analysis is started with very small crack size to reflect in results the early stage of fault occurrence Analytical formulation is helpful in finding the mesh stiffness of a spur gear pair with different crack length, width and depth variation. The developed analytical model can predict the change of gear mesh stiffness with presence of a gear tooth crack and also compares the mesh stiffness data with one contact period of gear pair. The analysis is done with the gear pair having more than one tooth in contact during some part of the contact period. This gives single or double tooth contact in one period. Analytical results shows less stiffness when single tooth pair contact occurs for part of TVMS diagram and higher stiffness for remaining part due to double tooth contact.

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INTRODUCTION

Gearboxes are the most important mechanism in all automotive application, industries and some daily used system and works on high rotational speed to produce output. Spur gear is mostly used in general gearbox for power transmission. It works at different gear ratios to provide multiple torque ratios. Due to its wide application the monitoring of its health and its early fault detection is very important.

There are many type of fault may arise in gears due to high heat condition, excessive load, bad lubrications etc. like pitting, spalling , crack etc. There are lot of work carried out to investigate gear tooth stiffness with or without faults. Analytical models and Finite element methods are widely used approaches for calculations. Lin Han, Houjanet.al. (2017) determines the effect of crack, pitting, tooth breakage studied on helical gears. Firstly an analytical method is developed to incorporate the faults by combining slicing, integration and potential energy methods. Effect of various parameters on TVMS change was studies [1] Jungho Park et al. (2015) conducted experiment to find the mesh stiffness in faulty state of rotating gears. Transmission error (TE) was used as parameter to make difference between rotation of input and output gear [2]. Fakhar Chaari, Walid Baccar, et. al,(2007) proposed analytical method to quantify the reduction in stiffness due to common teeth faults. The effect of deflection due to bending, contact and fillet foundation was considered to calculate the stiffness. The combined effect of tooth spalling and breakage is observed by frequency and amplitude signal of the system [3]. Ankur Saxena et al. (2016) present computer simulation based analytical approach to quantify the TVMS reduction of gear pair due to various gear faults. The study mainly focused on fault like cracked tooth, chipped tooth and spalled tooth using potential energy method.[4] O D Mohammed et al used analytical approach to quantify the losses in time varying mesh stiffness due to presence of crack propagation in the teeth. The proposed crack propagation scenario can be applied for crack propagation modelling and monitoring simulation. [5] Zaigang Chen et. al. proposed the effect of tooth crack propagation along width and depth on the gear dynamics and simulated the corresponding change in statistical indicators [7]. Omar D Mohammed et al improved the method to find TVMS for large gears [9] Lewichki did many work on crack propagation and relates it's with backup ratio. The backup ratio is the ratio of rim thickness and the tooth height. He found that if the backup ratio is high then the crack propagates in gear teeth else in the rim [10]. So many work is done on the gear pair to find the crack behaviour of the gear teeth on mesh stiffness.

In this research work the crack is assumed in the lower part of the teeth and the size of the crack is varied. The variation of the crack is observed for crack depth variation, Crack width variation and crack height variation. A significant change the gear mesh stiffness are observed.

Time Varying gear mesh stiffness (TVMS) provides important information about health of a geared system. The crack propagation behaviour is measured in this work. The crack created on gear profile causes change in Time Varying gear mesh stiffness behaviour of the system. In this work the crack is assumed in mid of the gear tooth profile and then the effect of change in TVMS by variation in width, height and depth of crack is calculated. This work will help in fault finding of gear analytically when the crack is present.

II Methodology

The analytical calculation was done using potential energy method. The geometry of gear tooth is taken as shown in figure 1.

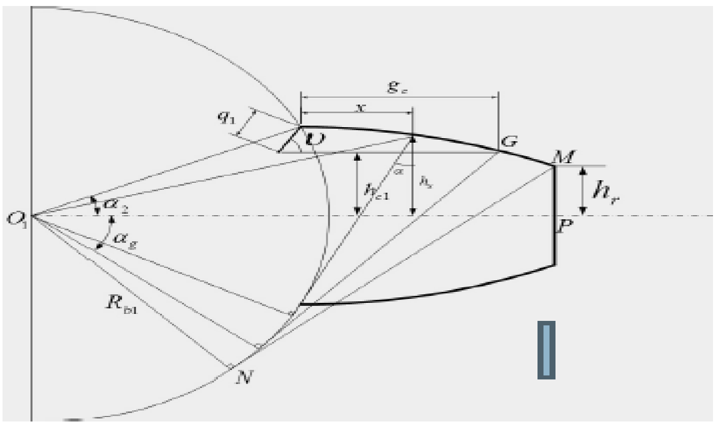

Fig. 1 The cross section of gear tooth [6]

Figure 1 shows the geometry of the gear. Here α_g is the pressure angle, R_b is the radius of base circle, h_r is the y value of tooth in calculation at tip, q_1 is the crack depth and some more geometrical parameters.

Analytical calculations. The Bending stiffness, Shear Stiffness, Axial Stiffness and Contact

stiffness were calculated by using the outcome of the following equations [4].
Axial stiffness is calculated by

$$\frac{1}{K_a}=\int_{-\alpha_1}^{\alpha_2}\frac{(\alpha_2-\alpha)cos\alpha sin^2\alpha_1}{2EL[sin\alpha+(\alpha_2-\alpha)cos\alpha]}d\alpha \qquad \text{-----------(1)}$$

Shear stiffness is calculated by

$$\frac{1}{K_s}=\int_{-\alpha_1}^{\alpha_2}\frac{2.4(1+v)(\alpha_2-\alpha)cos\alpha(cos\alpha_1)^2}{EL(sin\alpha_2-\frac{q1}{R_{b1}}sinu+sin\alpha(\alpha_2-\alpha)cos\alpha)}d \qquad \text{---------(2)}$$

Bending stiffness is calculated by

$$\frac{1}{K_b}=\int_{-\alpha_1}^{\alpha_2}\frac{12[1+cos\alpha_1\{(\alpha_2-\alpha)sin\alpha-cos\alpha\}]^2(\alpha_2-\alpha)cos\alpha}{EI_s[sin\alpha_2-\frac{q1}{Rb1}sinu+sin\alpha(\alpha_2-\alpha)cos\alpha]^2}d\alpha \qquad \text{--------(3)}$$

The Hertzian contact stiffness is a constant value along the line of action it is independent of contact position and expressed for healthy gear as-

$$K_h=\frac{\pi El}{4(1-\mu^2)} \qquad \text{-------------(4)}$$

The Hertzian Contact stiffness for faulty gear can be expressed as –

$$K_h=\frac{\pi E(l-l_s)}{4(1-\mu^2)} \qquad \text{----------------(5)}$$

Where E, L, μ represents young's modulus, tooth contact width and Poisson's ratio respectively.

Fillet Foundation Stifness: Modelling for finding Deflection due to fillet foundation: In this case the stiffness due to fillet foundation is calculated by first calculating deflection that can occur due to load. Sainsot et al derived the fillet foundation deflection of the gear tooth based on the theory of Mushkhelishvi and then they derived an analytical formula reflecting the gear body influenced teeth deflection by assuming linear and constant stress variation at root circle [3].

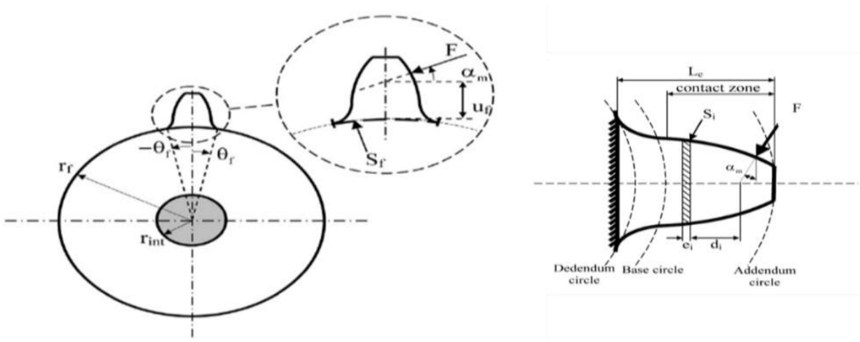

Figure-2: Spur gear showing different terminology [2]

Figure 2 shows various geometrical parameters that are used in the calculation of deflection due to banding for finding stiffness value[6].

$$\delta_f=\frac{Fcos^2\alpha_m}{WE}\left\{L^*\left(\frac{{u_f}^2}{{s^2}_f}\right)+M^*\left(\frac{u_f}{s_f}\right)+P^*(1+Q^*t_{g^2}\alpha_m)\right\} \qquad \text{-----------------(6)}$$

The coefficients L^*,M^*, P^*, Q^* can be calculated by polynomial functions

$$X_i(h_{f_i}, \theta_f) = \frac{A_i}{\theta^2{}_f} + B_i h^2{}_{f_i} + \frac{C_i h_{f_i}}{\theta_f} + \frac{D_i}{\theta_f} + E_i h_{fi} + F_i \text{-------}(7)$$

Table-1: Values of the coefficients of Eq. (7)

	Ai	Bi	Ci	Di	Ei	Fi
L* (hf i, θf)	−5.574×10−5	−1.9986× 10−3	−2.3015×10−4	4.77021 ×10−3	0.0271	6.8045
M * (hf i, θf)	60.111×10−5	28.100×10−3	−83.431×10−4	−9.9256× 10−3	0.1624	0.9086
P* (hf i, θf)	−50.952×10−5	185.50× 10−3	0.0538×10−4	53.300×10−3	0.2895	0.9236
Q* (hf i, θf)	−6.2042 ×10−5	9.0889× 10−3	−4.0964×10−4	7.8297× 10−3	−0.1472	0.6904

Fillet foundation stiffness can be given as

$$k_f = \frac{F}{\delta_f} \qquad \text{-------------------------}(5)$$

Equivalent Stiffness: The total equivalent mesh stiffness of single tooth pair in meshing can be expressed as-

$$K_{total} = \frac{1}{\frac{1}{K_h} + \frac{1}{K_{b_1}} + \frac{1}{K_{s_1}} + \frac{1}{K_{a_1}} + \frac{1}{K_{b_2}} + \frac{1}{K_{s_2}} + \frac{1}{K_{a_2}}} \qquad \text{-------------}(6)$$

Where subscripts 1and 2 denotes the driving gear and driven gear. The total equivalent mesh stiffness of double teeth pair in meshing can be expressed as

$$K_{total} = \sum_{i=1}^{2} \frac{1}{\frac{1}{K_{hi}} + \frac{1}{K_{b_{1i}}} + \frac{1}{K_{s_1}} + \frac{1}{K_{a_{1i}}} + \frac{1}{K_{b_{2i}}} + \frac{1}{K_{s_{2i}}} + \frac{1}{K_{a_{2i}}}} \qquad \text{---------------}(7)$$

Crack Propagation scenario.

In this case the crack is developed in the lower part of the pinion. Here the crack is symmetrical about the mid plane of the gear. The crack size then varied in three ways.

(a)The Width of crack varied by keeping depth & height of the crack constant

(b)The Depth of crack varied by keeping Height & Width of the crack constant

(c)The Height of crack varied by keeping depth & Width of the crack constant

III RESULTS:

Calculation of the mesh stiffness due to crack occur (developed intentionally here) a spur gear pair is used. The modelling of the gear pair was done using modelling software.

The specifications are as follows [9].

No. of teeth in gear	30
No. of teeth in pinion	25
Pressure angle	20^0
Width	20 mm
Module	2 mm

Modulus of elasticity	210 GPa
Modulus of Rigidity	79.8 GPa
Poisson's Ratio	0.3

For the calculation a torque 28.13 N-m corresponding to a force of 1000 N is applied on the gear in gear-Pinion pair. The crack is developed on the surface of pinion to find the effect on time varying gearmesh stiffness. The above data is used to find bending Stiffness, shear stiffness, axial stiffness, contact stiffness. The fillet foundation stiffness of individual gear and pinion is also incorporated. Then the combined /equivalent stiffness is found out using equation (7). Following results were Obtained Analytically.

3.1 Analytical results:

Case-1: The Width of crack varied by keeping depth & height of the crack constant : In this case the width of the crack varied from 2.5 mm to 10 mm in steps of 2.5 mm. Depth of crack remain constant to 1.51 mm and the height of the crack is 0.5 mm.

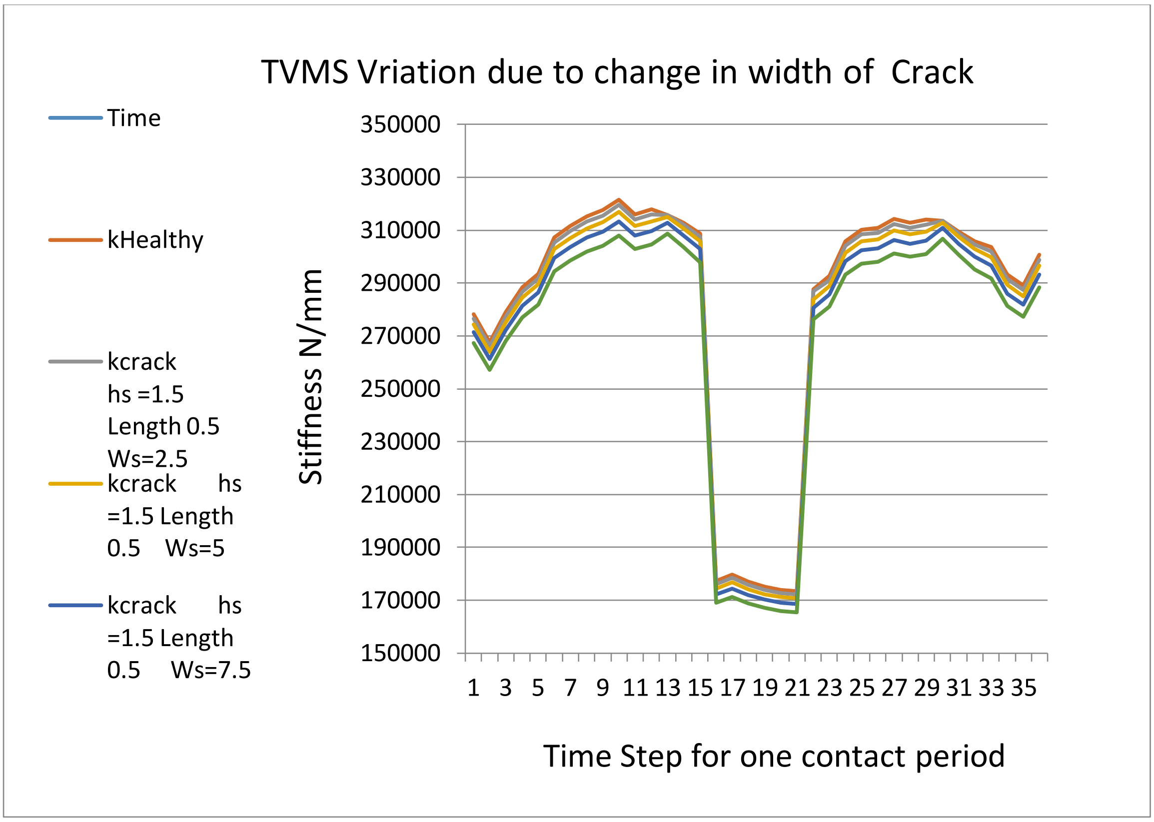

Figure-3: Effect of crack width variation on the time varying gear mesh stiffness of gear pair

Case-2: The Depth of crack varied by keeping Height & Width of the crack constant: In this case the Depth of the crack varied from 0.5 mm to 1.5 mm in four steps. Width of crack remain constant to 10 mm.

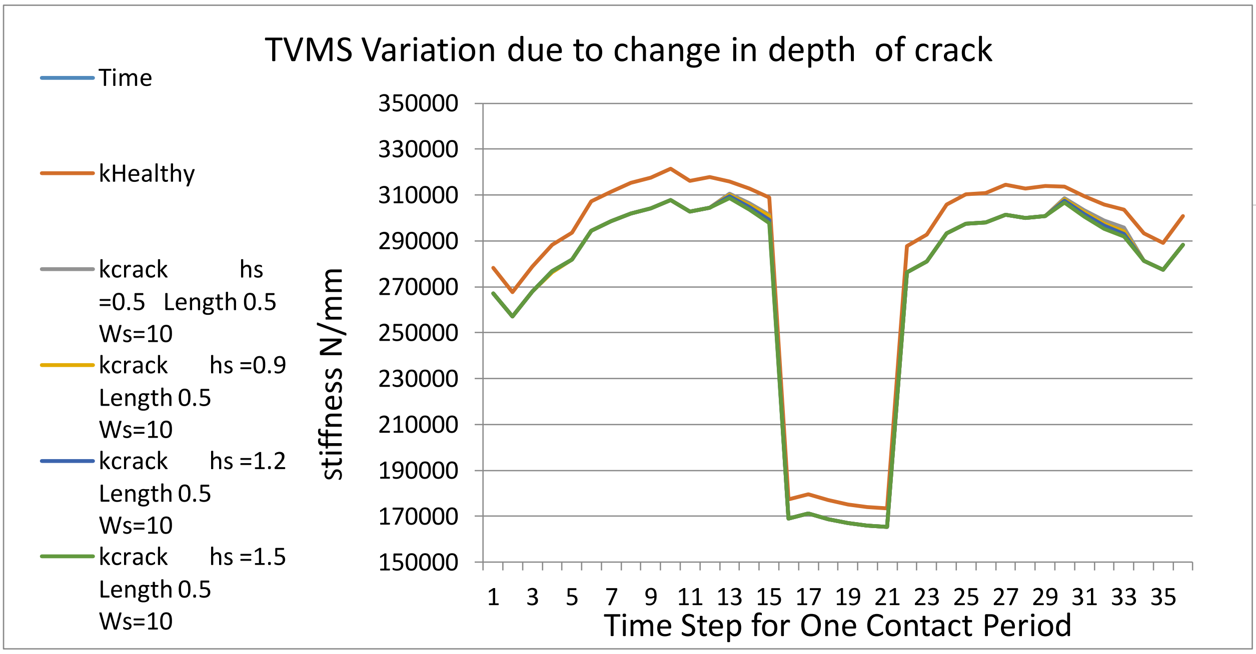

Figure 4: Effect of crack depth variation on the time varying gear mesh stiffness of gear pair

<u>Case-3: The Height of crack varied by keeping depth & Width of the crack constant:</u> In this case the Height of the crack varied from 0.5 mm to 1.5 mm in four steps. Width of crack remains constant to 10 mm.

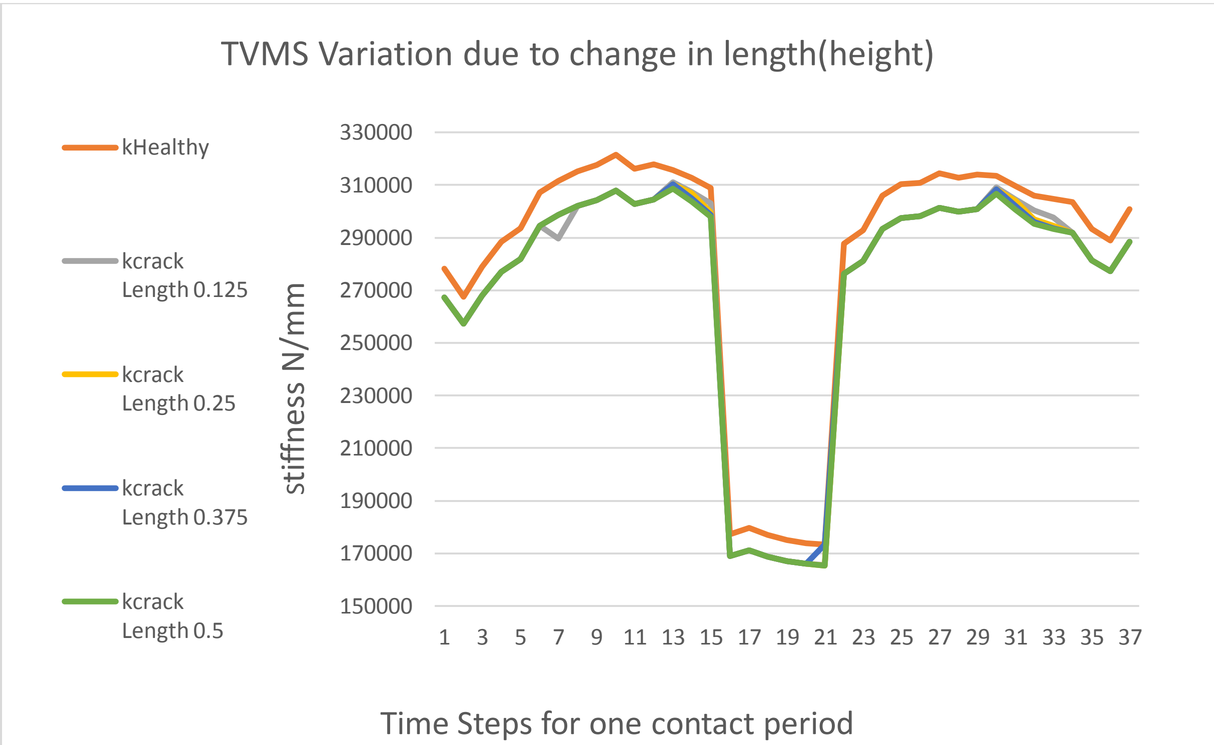

Figure-5: Effect of crack length/height variation on the time varying gear mesh stiffness of gear pair

IV CONCLUSION:

In this research work the crack depth scenario is discussed. The results are plotted for different crack variation like change in depth, width and change of crack size by changing one and keeping other two fixed. Following are the observations.

- By changing the width of the crack it is found that the gear mesh stiffness changes throughout the curve. For every point on the curve the TVMS values changes for each case. The maximum change occurs is about 3.5%.
- By changing the Depth of the crack it is found that the gear mesh stiffness changes throughout the curve. For every point on the curve the TVMS values changes for each case. The maximum change occurs is about 5.3%.
- By changing the length/height of the crack it is found that the gear mesh stiffness changes throughout the curve. For every point on the curve the TVMS values changes for each case. The maximum change occurs is about 4 %.

From above points it is concluded that the depth of crack is affecting more to the time varying mesh stiffness. So when the depth of crack will be more there will be more changes in the time varying mesh stiffness that ultimately a sign of more vibration in the gearbox.

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