



On Prime Hyperideals in ternary semihyperring

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Abstract:

Here in the research article we initialize and characterize the prime hyperideal and hyperideal, the semiprime hyperideal in ternary semihyperring. And some of the postulates of them were to be introduced.

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1. Introduction:

In 1934, Hyperstructure theory was born at the Scandinavian Mathematics 8th congress, where the hypergroup notion was introduced by Marty [11] as a generalization of groups. On ternary semi hyper groups and some of the postulates were studied by Davvaz and Leoreanu-Fotea [4]. The theme of this research article is to study prime, semiprime and irreducible hyperideals in ternary semihyperrings and introduced some basic properties on them. Now we investigate the some of the basic properties of hyperfilters and hypersemilattice congruence of ternary semihyperrings. We gave some of basic

properties of hyperfilters in ternary semihyperrings. Some of the differences between the hyperfilters and the prime hyperideals and hypersemilattice congruences in ternary semihyperrings are to be accepted.

2. Preliminaries:

Definition 2.1: Let us assume that $H-1$ be a non-empty set and $\circ : H-1 \times H-1 \rightarrow \wp^*(H-1)$ is the hyperoperation, where $\wp^*(H-1)$ was the set of all subsets of $H-1$. The couple $(H-1, \circ)$ is known as to be a hypergroupoid. If X and Y are any two non empty subsets of and $a \in H$ we have

$$X \circ Y = \bigcap_{x \in X, y \in Y} x \circ y,$$

$$X \circ \{a\} = X \circ a \text{ and } \{a\} \circ X = a \circ X$$

Definition 2.2: A non empty additive sub semi hyper group A of a ternary semi hyper ring $H-1$ is known as

1. a left hyper ideal of $H-1$ if $[H-1H-1A] \subseteq A$
2. a lateral hyper ideal of H if $[H-1IH-A] \subseteq A$



3. a right hyper ideal of H if $[AH-1H-1] \subseteq A$

If A is both left as well as right hyper ideal of H, then A is said to be a two sided hyper ideal of H-1. If A is a left, a lateral, a right hyper ideal of H-1 then A is said to be a hyper ideal of H-1.

Remark 2.3: The triplet $(H-1, \oplus, [\])$ which is a ternary semi hyper ring and for any element $h1 \in H-1$ then a left ideal generated by $h1$, a right ideal generated by $h1$, then the two sided ideal generated by $h1$, and the hyper ideal generated by $h1$ is to be represented as shown below

$$L(h1) = \langle h1 \rangle_l = \{ h1 \} \cup [H-1H-1h1]$$

$$M(h1) = \langle h1 \rangle_m = \{ h1 \} \cup [H-1h1H-1] \cup [H-1[H-1h1H-1]H]$$

$$R(h1) = \langle h1 \rangle_r = \{ h1 \} \cup [h1H-1H-1]$$

$$T(h1) = \langle h1 \rangle_t = \{ h1 \} \cup [H-1H-1h1] \cup [h1H-1H-1] \cup [H-1[H-1h1H-1]H-1]$$

$$J(h1) = \langle h1 \rangle = \{ h1 \} \cup [H-1H-1h1] \cup [H-1h1H-1] \cup [H-1[H-1h1H-1]H-1] \cup [h1H-1H-1]$$

Definition 2.4: Assume that H be a non empty ternary semi hyper ring and M non empty subset of H and also be a hyper ideal of H then M is said to be a maximal hyper ideal of H if $M \neq H$ and any proper ideal of I of H which does not exist so that $M \subseteq I$.

Lemma 2.5: Let X, Y and Z be any three hyperideals of a ternary semihyperring H, then $[XYZ]$ is a hyperideal of H.

For more preliminaries refer the references

3. On prime left hyperideals in ternary semihyperring:

This part of the article we will initiate the concept of prime hyper ideals in ternary semihyperring and characterised prime hyper ideals.

Definition 3.1: Assume that H be a ternary semihyperring and P a proper hyperideal of H is said to be a **prime hyperideal** of H if for any X, Y, Z be any three proper hyper ideals of H then the following condition is to be satisfied

\oplus	p	q	r	s	t	u
p	p	{p, q}	r	{r, s}	t	{t, u}
q	q	q	s	s	u	u
r	r	{r, s}	r	{r, s}	r	{r, s}
s	s	s	s	s	s	s
t	t	{t, u}	r	{r, s}	t	{t, u}
u	u	u	s	s	u	u

if $[XYZ] \subseteq P$ implies $X \subseteq P$ or $Y \subseteq P$ or $Z \subseteq P$

Definition 3.2: Assume that the ternary semihyperring H and P be a proper hyper ideal of H is said to be a **semiprime hyperideal** of H if $f \in X$ be any proper hyper ideal of H then the following condition is to be satisfied if $[XXX] \subseteq P$ implies $X \subseteq P$.

It is obviously known that every prime hyperideal of a ternary semihyperring H is also a semiprime hyperideal of H.

Definition 3.3: Assume that H be a ternary semihyperring and P-1 be a proper hyperideal of H is known as **irreducible**, if J and K are any hyper ideals of H, $J \cap K = P-1 \Rightarrow P-1 = J$ or $P-1 = K$ or .which is likely to be equal to , $J \cap K = P-1 \Rightarrow J \subseteq P-1$ or $K \subseteq P-1$.

Example 3.4: Let $H-1 = \{p, q, r, s, t, u\}$ and $[abc] = (a.b).c$ for all $a, b, c \in H-1$, the . will be defined as

.	p	q	r	s	t	u
p	p	q	r	r	t	t
q	q	q	s	s	u	u
r	r	s	r	s	r	r
s	s	s	s	s	s	s
t	t	u	r	r	t	u
u	u	u	s	s	u	u

Therefore, H-1 is a ternary semihyperring. Clearly, $I_1 = \{r, s\}$, $I_2 = \{r, s, t, u\}$ and H are left hyperideals of H-1. It can easily say that that I_2 and H-1 are prime left hyperideals. I_1 is irreducible left hyperideal.

Theorem 3.5: Assume that a ternary semi hyper ring H-1, and a semi prime ideal P of H-1 and $h1 \in H-1$. So that, $h1 \in P$ iff $[H-1H-1h1H-1] \subseteq P$.

Proof: suppose that a semi prime hyper ideal P of H-1.

Let us assume that $h1 \in P$, then, $[H-1H-1h1H-1] \subseteq [H-1H-1PH-1] \subseteq P$.

Conversely, assume that $[HHhHH] \subseteq P$ here the semi prime hyper ideal of H. So that



$\langle h1 \rangle = \{ h1 \} \cup [H-1H-1h1] \cup [H-1h1H-1] \cup [H-1[H-1h1H-1]H-1] \cup [h1H-1H-1] \cup [h1] \cup [H-1H-1h1] \cup [H-1h1H-1] \cup [H-1[H-1h1H-1]H-1] \cup [h1H-1H-1] \cup [h1] \cup [H-1H-1h1] \cup [H-1h1H-1] \cup [H-1[H-1h1H-1]H-1] \cup [h1H-1H-1] \subseteq [H-1H-1h1H-1] \subseteq P$. Here P is a semi prime hyper ideal of $H-1$, so that $\langle h1 \rangle \subseteq P$ and hence $h1 \in P$.

Theorem 3.6: Assume that a ternary semi hyper semi ring $H-1$ and a hyper ideal P of $H-1$ then the below conditions were equivalent:

- (1) P be a prime hyper ideal of $H-1$.
- (2) $[H-1H-1xH-1yH-1zH-1H-1] \subseteq P$ implies that $x \in P$ or $y \in P$ or $z \in P$.
- (3) $\langle x \rangle \langle y \rangle \langle z \rangle \subseteq P$ implies that $x \in P$ (or) $y \in P$ (or) $z \in P$.

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Proof: To show that (1) implies (2). Assume that a prime hyper ideal P of $H-1$ and $[H-1H-1xH-1yH-1zH-1H-1] \subseteq P$. Then, $[H-1H-1[xH-1H-1yH-1zH-1H-1]H-1H-1] \subseteq [H-1H-1PH-1H-1] \subseteq P$. This implies $[H-1H-1x][H-1H-1yH-1zH-1H-1] \subseteq P$. Since the prime hyper ideal P of $H-1$, so $[H-1H-1x] \subseteq P$ or $[H-1H-1yH-1zH-1H-1] \subseteq P$ or $[zH-1H-1] \subseteq P$. We know that for any prime hyper ideal which is to be a semi prime ideal. So that P is a semi prime hyper ideal

Hence, by the known result, $x \in P$ (or) $y \in P$ (or) $z \in P$.

To show that (2) \Rightarrow (3).

Let $\langle x \rangle \langle y \rangle \langle z \rangle \subseteq P$ for any $x, y, z \in P$

Then,

$[H-1H-1xH-1yH-1zH-1H-1] = [xH-1H-1][H-1H-1yH-1zH-1H-1] \subseteq \langle x \rangle \langle y \rangle \langle z \rangle \subseteq P$. Therefore, by (2) we will have $x \in P$ or $y \in P$ or $z \in P$.

To show that (3) \Rightarrow (1).

Assume that A, B, C are three hyper ideals of $H-1$ so that $[ABC] \subseteq P$.

Let us

assume that $B \not\subseteq P$ and $C \not\subseteq P$. Let $y \in B$ and $z \in C$ so that $y, z \notin P$

So that, for any $x \in A$ then we will have $\langle x \rangle \langle y \rangle \langle z \rangle \subseteq [ABC] \subseteq P$.

By our assumption (3) we will get $x \in P$, or $y \in P$, or $z \in P$, but $y, z \notin P$, so $x \in P$ and hence $A \subseteq P$. Therefore, P is a prime hyper ideal of $H-1$.

Corollary 3.7: A commutative ternary semi hyper ring $H-1$ and let P be a hyperideal of $H-1$ so that P is a prime hyperideal iff $\langle xyz \rangle \subseteq P$ which implies that $x \in P$ or $y \in P$ or $z \in P$ for all $x, y, z \in H-1$.

From the previous result of (3) \Rightarrow (1). The above result is also valid for (1,3) commutative ternary semi hyper ring.

Theorem 3.8: Let $H-1$ is a maximal hyper ideal of $H-1$, which is to be a prime ideal of $H-1$.

Proof: Assume that M be any maximal hyper ideal of $H-1$. Now we can prove that M is a prime ideal of $H-1$. And let $[H-1H-1aH-1bH-1cH-1H-1] \subseteq M$. Let us assume that $a, b \notin M$. $[H-1aH-1H-1bH-1]$ is a hyperideal of $H-1$ which contains a, b . But M is a hyper ideal, $M + [H-1aH-1H-1bH-1] = H-1$. As $1 \in H-1$, $1 = m + \sum_{i=1}^n s_i a_i x_i b y_i$, then $1c = (m + \sum_{i=1}^n s_i a_i x_i b y_i) c \subseteq M + [H-1aH-1H-1bH-1c] \subseteq M + [H-1H-1aH-1H-1bH-1cH-1H-1] \subseteq M$. Hence $c \in M$. therefore M is a prime hyper ideal.

Definition 3.9: A ternary semi hyper ring $H-1$ and A be a non empty subset of $H-1$ is known as to be a m -system if for any $x, y, z \in A$ then there exist the elements $a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8 \in H-1$ such that $[a_1 a_2 x a_3 a_4 y a_5 a_6 z a_7 a_8] \subseteq A$.

Theorem 3.10: Assume that $H-1$ is a ternary semihyperring and a proper hyper ideal of P of $H-1$ is a prime hyper ideal iff its complement $H-1 \setminus P$ has an m -system.

Proof: Assume that P be a prime hyper ideal of $H-1$. Let $x, y, z \notin P$. Therefore $x, y, z \in H \setminus P$. Let us suppose that $H \setminus P$ which is not an m -system. Therefore for every $a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8 \in H-1$, $[a_1 a_2 x a_3 a_4 y a_5 a_6 z a_7 a_8] \not\subseteq H \setminus P$. Thus, $[a_1 a_2 x a_3 a_4 y a_5 a_6 z a_7 a_8] \subseteq P$. Since P is a prime hyper of $H-1$, so by Theorem 3.6, we get $a \in P$ or $b \in P$ or $c \in P$. It is impossible. Hence, $H \setminus P$ is an m -system.

Conversely, suppose that $H-1 \setminus P$ is an m -system. Let $x, y, z \in H-1 \setminus P$. Then, there exist $a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8 \in H$ such that $[a_1 a_2 x a_3 a_4 y a_5 a_6 z a_7 a_8] \subseteq H-1 \setminus P$. Thus, $[a_1 a_2 x a_3 a_4 y a_5 a_6 z a_7 a_8] \not\subseteq P$. Hence, if



$x, y, z \notin P$, then $[H-1xH-1yH-1H-1yH-1H-1zH-1H-1] \notin P$. Thus, by known theorem, P is a prime hyper ideal of $H-1$.

Definition 3.11: A ternary semi hyper ring $H-1$ and a prime hyper ideal P of $H-1$ is said to be a **completely prime hyper ideal** of $H-1$ if $[xyz] \in P$ which implies that $x \in P$ or $y \in P$ or $z \in P$ for every element $x, y, z \in H-1$.

Theorem 3.12: If $H-1$ is a ternary semi hyper ring then every completely prime ideal is a prime hyperideal of $H-1$.

Proof: Let us assume that a completely prime hyper ideal P of a ternary semi hyper ring $H-1$

Assume that $x, y, z \in H-1$ and $[x < y > z] \subseteq P$. Therefore $[xyz] \in P$.

But P is a completely prime ideal either $x \in P$ or $y \in P$ or $z \in P$ hence P is a prime hyperideal of $H-1$.

Converse of the above theorem in general may not be true.

Theorem 3.13: Let $H-1$ be a commutative ternary semi hyper ring a hyperideal which is a completely prime ideal iff which is prime hyperideal.

Proof: Assume that Q is a prime hyperideal of a ternary semi hyper ring $H-1$.

Let $x, y, z \in H$, $[xyz] \subseteq Q$. Now $[xyz] \subseteq Q$, Q is a hyper ideal of $H-1$

$\Rightarrow [xyzH-1H-1H-1H-1H-1H-1H-1] \subseteq Q$,

Where $H-1$ is the ternary semihyperring with ternary multiplication identity.

Since $H-1$ is commutative, $[[H-1H-1a][H-1H-1bH-1H-1][cH-1H-1]] = [abcH-1H-1H-1H-1H-1H-1H-1] \subseteq Q$.

By theorem 3.6, either $a \in Q$ or $b \in Q$ or $c \in Q$.

Hence Q is a completely prime ideal of $H-1$.

In the reverse suppose that Q is a completely prime hyper ideal of $H-1$. By theorem 3.12, Q is a prime hyper ideal of $H-1$.

Lemma 3.14: Let H be a ternary semihyperring. A hyperideal P of a ternary semihyperring H is completely prime if and only if $H \setminus P$ is ternary sub-semihyperring of H .

Theorem 3.15: A hyperideal A of a ternary semi hyper ring $H-1$ which is completely prime iff $a_1, a_2, \dots, a_n \in H-1$, and n is a odd natural number, $[a_1 a_2 \dots a_n]$ belongs to $A \Rightarrow a_j \in A$, for any $j = 1, 2, 3, \dots, n$.

Proof: Let us assume that A is a completely prime hyper ideal of $H-1$

Let $a_1, a_2, \dots, a_n \in H-1$ here n is a odd natural number and $a_1 a_2 \dots a_n \in A$.

If $n=1$ clearly $a_1 \in A$.

If $n=3$ then clearly $[a_1 a_2 a_3] \in A \Rightarrow a_1 \in A$ or $a_2 \in A$ or $a_3 \in A$.

If $n=5$ then clearly $[[a_1 a_2 a_3] a_4 a_5] \in A \Rightarrow [a_1 a_2 a_3] \in A$ or $a_4 \in A$ or $a_5 \in A$

$\Rightarrow a_1 \in A$ or $a_2 \in A$ or $a_3 \in A$ or $a_4 \in A$ or $a_5 \in A$.

Hence by mathematical induction on n which is a odd natural number, then $a_1 a_2 \dots a_n \in A \Rightarrow a_j \in A$ for some $j = 1, 2, 3, \dots, n$. the converse of the above theorem is trivial case

Theorem 3.16 Let H is a ternary semihyperring with identity. Then, every maximal hyperideal of H , is prime hyperideal of H .

Proof: Let P be a maximal hyperideal of H . Let A, B, C be three hyperideals of H such that $[ABC] \subseteq P$. Suppose that $A, B \not\subseteq P$. Then, $A \cup P = H$ and $B \cup P = H$. Since $e \in H$, we have $e \in A \cup P$ and $e \in B \cup P$. Thus, $e \in A$ or $e \in P$ and $e \in B$ or $e \in P$. Since $e \notin P$, we have $e \in A$ and $e \in B$ implies $A = H$ and $B = H$. Now since $e \in H$, we have $C = [HHC] = [ABC] \subseteq P$ implies $C \subseteq P$. Therefore, P is a prime hyperideal of H .

Theorem 3.17: A ternary semi hyper semi ring $H-1$, and P is a m -system a hyper ideal Q of $H-1$ so that $P \cap Q = \emptyset$. Therefore their exist, a maximal hyper ideal M of $H-1$ containing Q and $M \cap P = \emptyset$ such that, M is also a prime hyper ideal of $H-1$.

Proof: Assume that $\tau_P = \{B : B \text{ is a prime hyper ideal of } H-1, \text{ and } Q \subseteq B, B \cap P = \emptyset\}$.



Clearly $Q \in \tau_P, \tau_P$ is non empty. τ_P is a partially ordered set by set inclusion. Assume that $\{M_i\}$ be any arbitrarily chain in τ_P . So the union of hyper ideals is a hyper ideal, $\bigcup_{i \in I} M_i$ is a hyper ideal of H-1.

Since $Q \subseteq M_i$ every $i \in I$, we have $Q \subseteq \bigcup_{i \in I} M_i$. Suppose that $(\bigcup_{i \in I} M_i) \cap P \neq \emptyset$. therefore there exist for some $a \in H-1$ so that $a \in (\bigcup_{i \in I} M_i) \cap P$. which gives us $a \in \bigcup_{i \in I} M_i$ and $a \in P$. Therefore, $a \in M_i$ for any $i \in I$ and also $a \in P$. Hence, $M_i \cap P \neq \emptyset$. A contradiction. Hence, $(\bigcup_{i \in I} M_i) \cap P = \emptyset$. Therefore $\bigcup_{i \in I} M_i$ which is

an upper bound of $\{M_i\}$. Since $\{M_i\}$ is an arbitrarily chain, we know that every chain is in τ_P and have an upper bound in τ_P . Therefore, by the known lemma zorn's lemma τ_P which contains a maximal element M . now we will prove that M is a prime hyper ideal of H-1. Assume that X, Y, Z be any three hyper ideals of H-1 so that $[XYZ] \subseteq M$. Let us say that $X \not\subseteq M, Y \not\subseteq M$ and $Z \not\subseteq M$. therefore there exist an elements $x \in X, y \in Y, z \in Z$ so that $x, y, z \notin M$. here $\langle x \rangle \cup M, \langle y \rangle \cup M$ and $\langle z \rangle \cup M$ are the hyper ideals of H-1 containing M . so $(\langle x \rangle \cup M) \cap P \neq \emptyset, (\langle y \rangle \cup M) \cap P \neq \emptyset$ and $(\langle z \rangle \cup M) \cap P \neq \emptyset$. Let $a \in (\langle x \rangle \cup M) \cap P, b \in (\langle y \rangle \cup M) \cap P$ and $c \in (\langle z \rangle \cup M) \cap P$. But $a, b, c \in P$ and P has m -system, we will have $[a_1 a_2 a a_3 a_4 b a_5 a_6 c a_7 a_8] \subseteq P$ for some $a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8 \in H-1$. Also $[a_1 a_2 a a_3 a_4 b a_5 a_6 c a_7 a_8] \subseteq [a_1 a_2 (\langle x \rangle \cup M) a_3 a_4 (\langle y \rangle \cup M) a_5 a_6 (\langle z \rangle \cup M) a_7 a_8]$. Here we will arise the following cases

Case-1 : If $a \in \langle x \rangle \subseteq X, b \in \langle y \rangle \subseteq Y, c \in \langle z \rangle \subseteq Z$, then $[a_1 a_2 a a_3 a_4 b a_5 a_6 c a_7 a_8] \subseteq [[H-1H-1X][H-1H-1YH-1H-1][ZH-1H-1]] \subseteq [XYZ] \subseteq M$.

Case-2 If $a \in M$, then $[a_1 a_2 a a_3 a_4 b h_5 a_6 c a_7 a_8] \subseteq [[H-1H-1M][H-1H-1MH-1H-1][MH-1H-1]] \subseteq [MMM] \subseteq M$. In that similar way if $b \in M$ or $c \in M$, then $[a_1 a_2 a a_3 a_4 b a_5 a_6 c a_7 a_8] \subseteq [H-1H-1[H-1MH-1]H-1H-1] \subseteq [HMH] \subseteq M$ and $[h_1 h_2 x h_3 h_4 y h_5 h_6 z h_7 h_8] \subseteq [H-1H-1H-1H-1[H-1H-1M]H-1H-1H-1] \subseteq [H-1H-1MH-1H-1] \subseteq M$. therefore, $P \cap M \neq \emptyset$. Which is a contradiction. Therefore, $X \subseteq M$ or $Y \subseteq M$ or $Z \subseteq M$. Hence M is prime hyper ideal of H-1.

4. Semiprime hyperideals:

In this part, we introduce the concepts of "semi prime hyper ideals in ternary semi hyper rings". We give some properties of "prime hyper ideals in ternary semi hyper rings". Some of the differences between "semi prime hyper ideal", "prime hyperideals and completely semiprime hyper ideals in ternary semi hyper rings" are considered.

$[H-1H-1[aH-1H-1aH-1H-1a]H-1H-1] \subseteq [H-1H-1PH-1H-1] \subseteq P$. This implies $[[H-1H-1a][H-1H-1aH-1H-1][aH-1H-1]] \subseteq P$. Since P is a semi prime hyper ideal of H-1, so $[H-1H-1a] \subseteq P$ or $[H-1H-1aH-1H-1] \subseteq P$ or $[aH-1H-1] \subseteq P$.

Hence, by Theorem 3.5, $a \in P$.

(2) \Rightarrow (3). $[\langle a \rangle \langle a \rangle \langle a \rangle] \subseteq P$ for some $a \in P$.

Then, $[H-1H-1aH-1H-1aH-1H-1aH-1H-1] = [[aH-1H-1][H-1H-1aH-1H-1][aH-1H-1]] \subseteq [\langle a \rangle \langle a \rangle \langle a \rangle] \subseteq P$.

Thus, by (2) we have $a \in P$.

(3) \Rightarrow (1). Assume that A is a hyper ideal of H-1 so that $[AAA] \subseteq P$. Let $A \not\subseteq P$. Let $a \in A$ such that $a \notin P$. Then, for any element $a \in A$ we will have $[\langle a \rangle \langle a \rangle \langle a \rangle] \subseteq [AAA] \subseteq P$.

By (3) we will get $a \in P$, but $a \notin P$ which is impossible and hence $A \subseteq P$.

Hence, P is a semi prime hyper ideal of H-1.

Theorem 4.1: A ternary semi hyper ring and a hyper ideal P of H-1. Then the given conditions are to be equivalent:

- (1) P which is to be a "semiprime hyperideal" of H-1.
- (2) $[H-1H-1aH-1H-1aH-1H-1aH-1H-1] \subseteq P$ implies $a \in P$.
- (3) $[\langle a \rangle \langle a \rangle \langle a \rangle] \subseteq P$ implies $a \in P$.

Proof: To show (1) \Rightarrow (2). Let us consider that P is a semiprime hyperideal of H-1 and $[H-1H-1aH-1H-1aH-1H-1aH-1H-1] \subseteq P$. Then,



Corollary 4.2: A commutative ternary semi hyper ring H-1 and a hyper ideal P of H-1 . such that , P is a semiprime hyper ideal iff $[aaa] \subseteq P$ implies $a \in P$ for all $a \in H$.

Proof: Assume that H-1 is a ternary semi hyper ring and P a semi prime hyperideal of H-1 and $[aaa] \subseteq P$ for all $a \in H$. Then we have $[H-1H-1[[[H-1H-1[aaa]]H-1H-1]H-1H-1] \subseteq [H-1H-1PH-1H-1] \subseteq P$. Since H-1 is a commutative ternary semihyper ring which implies that $[H-1H-1aH-1H-1aH-1H-1aH-1H-1] \subseteq P$. By the known result 4.1, P is a semi prime hyper ideal of H-1 implies that $a \in P$.

For the converse assume to be $[aaa] \subseteq P$ implies $a \in P$ for all $a \in H-1$. Let a hyper ideal A of H-1 so that $[AAA] \subseteq P$. Let $A \not\subseteq P$. Let $a \in A$ such that $a \notin P$. Then, for every $a \in A$ we have $[aaa] \subseteq [AAA] \subseteq P$ implies that $a \in P$ which is contradicts to our assumption. Therefore $A \subseteq P$ and therefore P is a semi hyper ideal of H-1.

Definition 4.3: A ternary semi hyper ring H-1 a non empty subset A of H-1 is known as p-system if for any $x \in A$ then there exist the elements $a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8 \in H-1$ so that $[a_1a_2aa_3a_4aa_5a_6aa_7a_8] \subseteq A$.

Note 4.4: Clearly we know that a ternary semi hyper ring H-1 “for each m-system is a p-system and union of p-systems is again a p-system too”.

Theorem 4.5: A ternary semi hyper ring H-1 and a proper hyper ideal P of H-1 is a semi prime hyper ideal iff its complement $H \setminus P$ has a p-system.

Proof: Assume that P is a prime hyper ideal of H-1. And also consider $a \notin P$. Then, $a \in H-1 \setminus P$. Let us consider that $H \setminus P$ is not a “p-system”. So that for each $a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8 \in H-1$, $[a_1a_2aa_3a_4aa_5a_6aa_7a_8] \not\subseteq H-1 \setminus P$. Thus, $[a_1a_2aa_3a_4aa_5a_6aa_7a_8] \subseteq P$. clearly P is semiprime hyperideal of H-1, so by the previous result 3.4, we will have $a \in P$. A contradiction will exist . Hence, $H \setminus P$ is a p-system.

In the sufficient part consider $H \setminus P$ has a p-system. Let $a \in H \setminus P$. Then, there exist $a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8 \in H-1$ such that $[a_1a_2aa_3a_4aa_5a_6aa_7a_8] \subseteq H-1 \setminus P$. therefore $[a_1a_2aa_3a_4aa_5a_6aa_7a_8] \not\subseteq P$. Hence, if $a \notin P$, then $[H-1 H-1aH-1 H-1aH-1 H-1aH-1 H-1] \not\subseteq P$. therefore by the known result 4.1, P is a semi prime hyper ideal of H-1.

Definition 4.6: A ternary semi hyper ring H-1 and a hyper ideal P of H-1 Then, P which is said to be a **completely semi prime hyper ideal** of H-1 if $[aaa] \in P$ implies $a \in P$ for every element $a \in H$.

Theorem 4.7: A hyperideal A of a ternary semi hyper ring H-1 which is to be a completely completely prime iff $x \in H, n$ is odd natural number, $[x^n] \in A \Rightarrow x \in A$.

Proof: Let us assume that A which is to be a completely semi prime hyper ideal of H-1.

Consider $x \in H-1$ where the odd natural number n and $[x^n] \in A$.

If $n = 1$ obviously $x \in A$.

If $n = 3$ then $[xxx] \in A \Rightarrow x \in A$.

If $n = 5$ then $[[xxx]xx] \in A \Rightarrow [xxx] \in A$ or $x \in A \Rightarrow x \in A$.

Hence by mathematical induction for any odd natural number n then $[x^n] \in A \Rightarrow x \in A$.

The converse part is trivial.

Theorem 4.8: Let H-1 be a ternary semi hyper ring then every completely prime hyper ideal of H-1 which is completely semi prime hyper ideal of H-1.

Proof: assume that A is a completely prime hyper ideal of a ternary semi hyper ring H-1.

Assume that $x \in H - 1$ and $[xxx] \in A$. Hence A is a completely prime hyper ideal of H-1, and $x \in A$. hence H-1 is a completely semi prime hyper ideal.

Theorem 4.9: A ternary semi hyper ring H-1, and P be p-system and a hyper ideal Q of H-1 so that $P \cap Q = \emptyset$. \exists a maximal hyper ideal M of H-1 containing Q and $M \cap P = \emptyset$. So that ,M is also a semi prime hyper ideal of H-1.

Proof: Assume that $\tau_p = \{B : B \text{ is a semiprime hyperideal of H-1, } Q \subseteq B, B \cap P = \emptyset\}$. We know that $Q \in \tau_p, \tau_p \neq \emptyset$. τ_p is partially ordered set by inclusion .Let $\{M_i\}$ which is an arbitrarily chain τ_p . Since the



union of hyper ideals is a hyper ideal, $\bigcup_{i \in I} M_i$ is a hyper ideal of H-1. Since $Q \subseteq M_i$ for all $i \in I$, we have

$Q \subseteq \bigcup_{i \in I} M_i$. Suppose that $(\bigcup_{i \in I} M_i) \cap P \neq \emptyset$. $\exists a \in H$ such that

$a \in (\bigcup_{i \in I} M_i) \cap P$. which implies $a \in \bigcup_{i \in I} M_i$ and $a \in P$. Thus, $a \in M_i$ for some $i \in I$ and $a \in P$. how ever $M_i \cap P$

$\neq \emptyset$. It is not possible. Hence, $(\bigcup_{i \in I} M_i) \cap P = \emptyset$. Thus, $\bigcup_{i \in I} M_i$ which is an upper bound of $\{M_i\}$. Since

$\{M_i\}$ is an arbitrarily chain, we will have that each chain τ_P has an upper bound in τ_P . Hence, by the known lemma Zorn's Lemma the family τ_P which contains the maximal element M . now to show that M is semi prime hyper ideal of H-1. Let A is a hyperideal of H-1 such that $[AAA] \subseteq M$. Assume that $A \not\subseteq M$. \exists an element $a \in A$ such that $a \notin M$. Now $\langle a \rangle \cup M$ is a hyper ideal of H-1 which contains M , so $(\langle a \rangle \cup M) \cap P \neq \emptyset$.

Let $x \in (\langle a \rangle \cup M) \cap P$. Since $x \in P$ and P is p -system, we have $[a_1 a_2 x a_3 a_4 x a_5 a_6 x a_7 a_8] \subseteq P$ for some $a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8 \in H-1$. Also $[a_1 a_2 a a_3 a_4 x a_5 a_6 x a_7 a_8] \subseteq [a_1 a_2 (\langle a \rangle \cup M) a_3 a_4 (\langle x \rangle \cup M) a_5 a_6 (\langle x \rangle \cup M) a_7 a_8]$. We have

Case I: If $x \in \langle a \rangle \subseteq A$, then $[a_1 a_2 x a_3 a_4 x a_5 a_6 x a_7 a_8] \subseteq [[H-1H-1A][H-1H-1AH-1H-1][AH-1H-1]] \subseteq [AAA] \subseteq M$.

Case II: If $x \in M$, then $[a_1 a_2 x a_3 a_4 x a_5 a_6 x a_7 a_8] \subseteq [[H-1H-1M][H-1H-1MH-1H-1][MH-1H-1]] \subseteq [MMM] \subseteq M$. Similarly, $[H-1MH-1] \subseteq M$ and $[H-1H-1MH-1H-1] \subseteq M$. Therefore, $P \cap M \neq \emptyset$, which is to be impossible. Thus, $A \subseteq M$. Therefore, M is a semiprime hyperideal of H-1.

Conclusion: The "ternary semihyperring" is a characterisation of the concepts of a "semiring", a "semihyperring". Since, the notions of "prime hyperideals", "semiprime hyperideals"; in a "ternary semihyperring" were initiated, and several illustrations given and characterized those hyperideals.

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