Optimal Finite Impulse Response Fractional-order Digital Differentiator using Jaya Algorithm

Puneet Bansal[1*] and Sandeep Singh Gill[2]

[1] (Research Scholar) Department of Electronics & Communication Engineering,
I. K. G. Punjab Technical University, Kapurthala, Jalandhar, Punjab-144603, India
[1]Department of Electronics & Communication Engineering,
University Institute of Engineering & Technology, Kurukshetra University, Kurukshetra, Haryana-136119, India
[2]Department of Electronics & Communication Engineering,
National Institute of Technical Teachers Training & Research, Chandigarh-160019, India
E-mail: puneet4u@gmail.com[1], ssg@nitttrchd@ac.in[2]

Abstract: -
The JAYA method, a population-based metaheuristic optimization approach, is used in this study to present the design of a digital finite impulse response fractional order differentiator (FIR-FOD). The design seeks to identify such ideal differentiator coefficients that minimize their absolute errors with the desired ideal differentiator response. The extensive simulation results and Wilcoxon rank-sum test at 99% level of confidence show how well JAYA algorithm performs better in finding the best coefficients of digital FIR-FOD in comparison with Cuckoo Search Algorithm (CSA) and Particle Swarm Optimization (PSO). Because it has no dependency on method-specific parameters, the JAYA algorithm achieves faster convergence than the CSA and PSO algorithms for this design problem.

Keywords: - Fractional calculus, Finite Impulse response, Differentiator, Fitness, Metaheuristics
DOINumber: 10.14704/NQ.2022.20.12.NQ77258 **NeuroQuantology2022;20(12): 2654-2660**

Introduction

The unique topic of mathematical modelling is one in which researchers are becoming increasingly interested. Fractional order operators simulate the dynamics of physical systems more effectively than integer order operators. A branch of mathematics called fractional calculus investigates the various ways that differentiation and integration may be used to express the powers of real or complex numbers. Digital FOD architecture has been a significant subject of study in the optical process control systems, image recognition, automated regulation, fluid mechanics, electromagnetic theory and electrical networks [1], [2], [3], [4].

Significant study on continuous-time FODs has been done by a number of researchers. A technique for realizing immittance with a fractional operator is provided in [5]. Using the least squares approach, approximate values of the FOD and integrator have been obtained [6]. Due to a rise in digital applications, discrete-time FODs have recently become more prevalent. The fractional derivative is generated in a FOD that is based on Taylor series expansion [7]. The Newton series expansion-based FOD for polynomial signals is provided in [8].

The unimodal fitness function is used in these traditional methods to simulate the ideal FOD. Traditional methods don't work well for high-dimensional multimodal optimization problems like FOD and are only viable in case of unimodal fitness functions. To find a global optimal solution in this situation, metaheuristic optimization methods like PSO [9], CSA [10], and Jaya algorithm [11] are employed. In case of

filter design, optimization methods like ant colony optimization [12], PSO [13], and the cuckoo search algorithm [14] are a few of the often employed. In [15], theory of operation, application and implementation of digital differentiator is presented. The cascade structure require higher order filter and thereby more delay as compared with others [16]. The digital Riesz fractional order differentiator is used for image sharpening application [17]. The design of half-band differentiator is discussed in [18].
The frequency response of ideal digital FOD is written as

$$H_{id}(\omega) = (j\omega)^{\alpha} \quad (1)$$

Here 'α' is a fractional number. ω is a normalised frequency which vary between [0,1], and $j = \sqrt{-1}$. There are four steps include for designing FOD as shown in Figure 1.

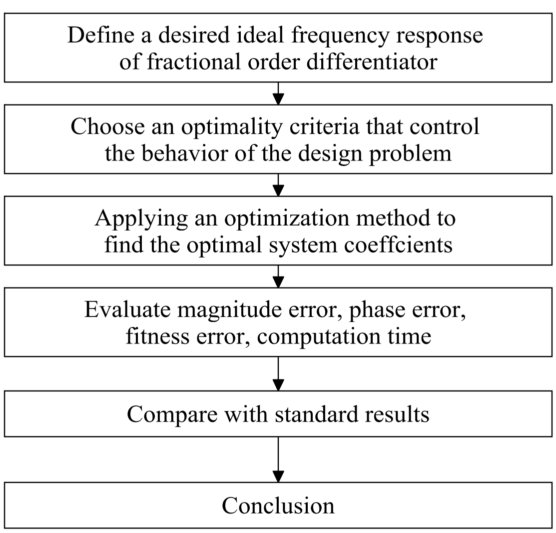

Figure 1. The design procedure of FIR-FOD

The remaining paper is ordered as: The formulation of the design issue for the FIR-FOD based on weighted least square based fitness response is illustrated in Section 2. Additionally, a generic definition of the integral-differential operator, i.e., Grünwald-Letnikov, as well as the explanation of fractional derivatives are provided. In Section 3, the basics of PSO algorithm, CSA and JAYA algorithm are discussed. In Section 4, obtained simulations are interpreted along with statistical "Wilcoxon rank-sum test". Finally, in Section 5, the paper conclusion is made.

1. Problem Formulation

The fractional derivative is computed using definition in [19]. An ideal response of the FIR-FOD is written as:

$$H_{ideal}(\omega) = (j\omega)^{\alpha} \quad (2)$$

where 'α' is a fractional number, $H_{ideal}(\omega)$ is an ideal FOD frequency response, ω is a normalized frequency which varies between [0 1]. For length N, Z-transform of FIR-FOD function is used as:

$$H(z) = \sum_{n=0}^{N-1} \mathrm{h(n)} z^{-n} \quad (3)$$

Here, weighted least square fitness function, is employed to obtain best design and is expressed by:

$$J_f = \int_0^1 W_1(\omega) \left|abs\left(H_{ideal}(\omega)\right) - abs\left(H(\omega)\right)\right|^2 + W_2(\omega)\left|phase\left(H_{ideal}(\omega)\right) - phase\left(H(\omega)\right)\right|^2 d\omega \quad (4)$$

Here $W_1(\omega) = 0.8$ and $W_2(\omega) = 0.2$ are non-negative weighting functions. The specific fitness function given in (4) is minimized in this study utilizing optimization methods like PSO, CSA and Jaya algorithm.

2. Jaya Algorithm

In this section, Jaya algorithm is discussed. Other optimization algorithms like PSO and CSA are discussed in [9] and [10] in detail. The JAYA term is Sanskrit in origin, it means "victory"[20] . This is a population-based metaheuristic algorithm. There are no algorithm-specific control parameters needed for this method; just the standard control parameters are needed [11]. During the ith iteration, $X_{j,k,i}$ represents the j^{th} variable for the k^{th} candidate, then this value is updated in accordance with (5).

$$X'_{j,k,i} = X_{j,k,i} + r_{1,j,i}(X_{j,best,i} - |X_{j,k,i}| - r_{2,j,i}(X_{j,worst,i} - |X_{j,k,i}|) \quad (5)$$

The values of the variables $X_{j,best,i}$ and $X_{j,worst,i}$ represent the best and worst candidates, respectively. "In the range [0, 1], $r_{1,j,i}$and $r_{2,j,i}$ are the two random numbers chosen for the j^{th} variable during the i^{th} iteration, and $X'_{j,k,i}$ is the updated value of $X_{j,k,i}$" [11]. The important terms "$r_{1,j,i}(X_{j,best,i} - |X_{j,k,i}|$" and "$r_{2,j,i}(X_{j,worst,i} - |X_{j,k,i}|$" represent the solution's propensity to shift toward best solution and away from worst solution, respectively. If $X'_{j,k,i}$ provides a better function value, it is accepted. The final iteration's acceptable function values are all kept, and they serve as the input for the next iteration.

3. Simulation Results and Discussions

These optimal results are obtained after performing around 100 simulation trials in MATLAB 2015a version on Intel® Core™ i5, 1.60 GHz with 6GB RAM with random changes in these control parameters shown in Table 1.

Table 1: FIR-FOD Control parameters

Parameters	Symbol	PSO	CSA	JAYA
Population size	Popsize	25	25	25
Max. iteration cycle	Epoch	1000	1000	1000
Inertia weight	w	0.9-0.4	-	-
Learning parameters	C_1, C_2	2,2	-	-
Particle velocity	v_{min}, v_{max}	0.01, 1	-	-
Discovering rate of alien eggs	P_a	-	0.25	-
Number of nests	n	-	25	-
No. of design variables	-	-	-	N+1
Limits of filter coefficients		-1, +1	-1, +1	-1, +1

The digital FIR FOD presented has normalized frequency range of $0 \leq \omega \leq 1$, filter order= 10, fractional-order α=0.5 and weighting function W_1 (ω)= 0.8 & W_2 (ω) =0.2. Optimal coefficients of digital FIR-FOD with the order of 10 are shown in Table 2.

Table 2: Optimized filter coefficients

Order	**Technique**	**Optimized coefficients (h_k)**
10	PSO	1, -0.9679, 0.955, -0.99, -0.1657, 0.7643, -0.6373, 0.2753, -0.1384
	CSA	0.99, -0.1545, -0.8584, -0.0244, 0.7504, -0.2968, -0.8605,0.9872, -0.4167
	JAYA	1, -0.1466, -0.9406, 0.0897, 0.8534, -1, 0.3295, 0.0536, -0.1145

In terms of magnitude response, phase response, absolute magnitude error, and absolute phase error, Figures 2-3 provide a graphical comparison achieved by the PSO, CSA, and JAYA algorithms with the ideal FIR -FOD.

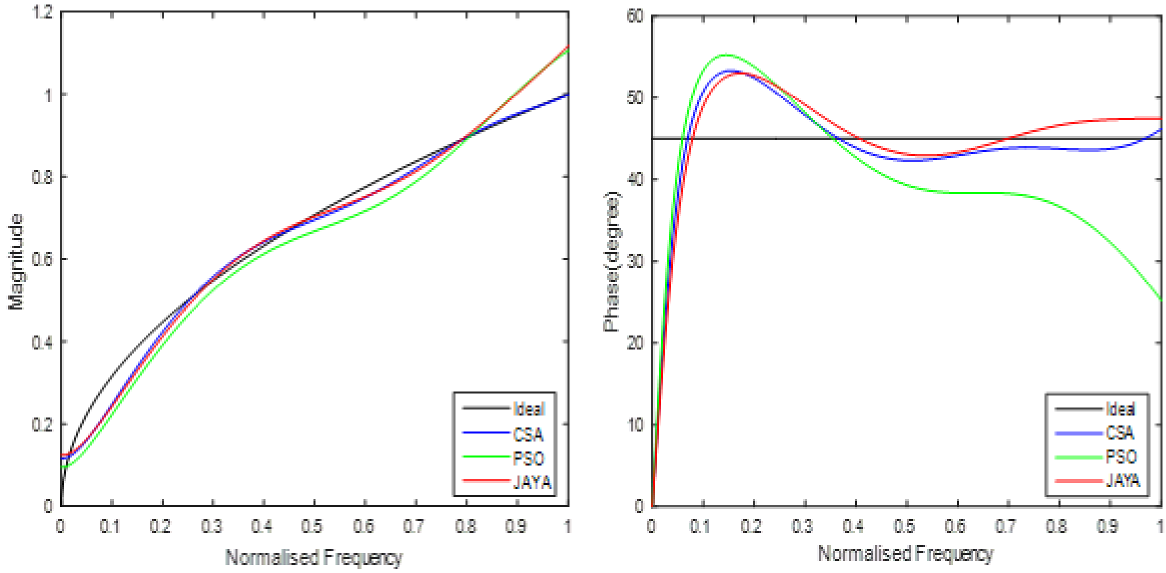

Fig. 2. 10th order FIR-Fractional order digital differentiator magnitude response and phase response

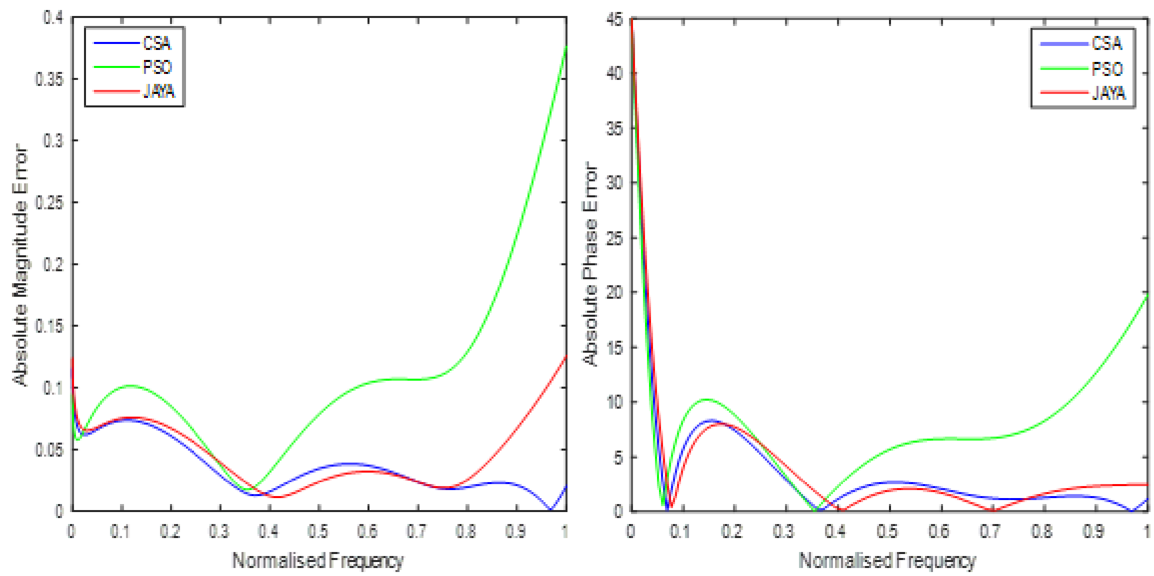

Fig. 3. 10th order FIR-Fractional order digital differentiator absolute magnitude error and absolute phase error

4.1 Performance Analysis

In Table 3, performance analysis for the proposed JAYA based FIR-FOD design are reported and compared with PSO and CSA based designs. It is concluded here that FIR-FOD designs using JAYA are better than PSO and CSA based designs in terms of these performance paramenters for 10th order FIR-FOD design.

Table 3: Performance analysis of absolute magnitude error

Algorit	**Mini**	**Maxi**	**Me**	**Varia**	**Stand**

hms	mum	mum	an	nce	ard deviation
PSO	3.2154E-04	0.0554	0.0141	1.5247E-04	0.0123
CSA [21]	1.1402E-05	0.0184	0.0051	1.7542E-05	0.0042
JAYA	3.1443E-05	0.0172	0.0050	1.3565E-05	0.0037

In Table 4, performance analysis of absolute phase error of 10^{th} order FIR-FOD is accounted. The calculated minimum, maximum, mean, variance and standard deviation for absolute phase error of the proposed JAYA based FIR-FOD design are reported and compared with PSO and CSA based designs.

Table 4: Performance analysis of absolute phase error

Algorithms	Minimum	Maximum	Mean	Variance	Standard deviation
PSO	5.0145E-04	0.6684	0.0841	0.0125	0.1118
CSA [21]	4.6481E-04	0.1614	0.0412	9.4574E-04	0.0308
JAYA	3.2546E-05	0.1345	0.0304	9.2451E-04	0.0304

It is concluded here that FIR-FOD designs using JAYA are better than PSO and CSA based designs in terms of these performance paramenters for 10^{th} order FIR-FOD design.

The computation time of JAYA based 10^{th} order FIR-FOD design is 52.47 sec whereas it is 49.25 sec and 57.15 sec for CSA and PSO based designs. It is concluded here that JAYA algorithm is competitive in speed with the PSO and CSA algorithms for obtaining optimized FIR-FOD designs.

Wilcoxon rank-sum test at 99% level of confidence is used in Table 5 to compare the performance consistency of PSO, CSA, and JAYA algorithms based digital FIR-FOD. It is shown that JAYA performs better than PSO and CSA as the p-value is less than 0.01.

Table 5: Statistical analysis using Wilcoxon rank-sum test

Comparison	Absolute magnitude error *p-value*	Absolute phase error *p-value*
JAYA versus PSO	1.25468534182645e-08	4.25468536849442e-7
JAYA versus CSA	0.008874512455214	0.006785642874568

The convergence curve of 10^{th} order FIR-Fractional order digital differentiator using PSO, CSA and JAYA algorithms is shown in Figure 4.

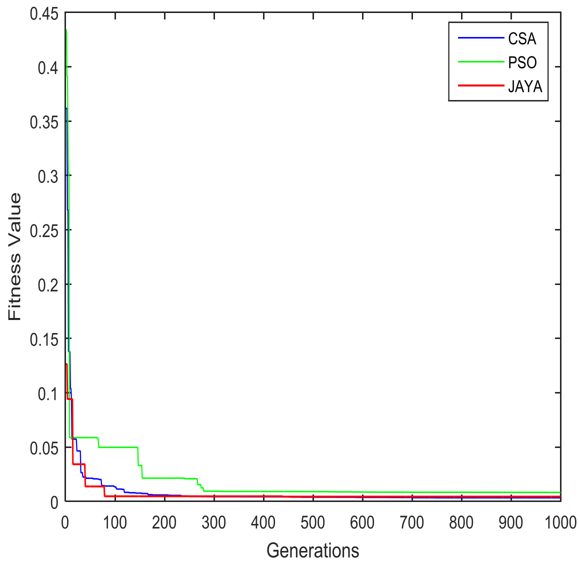

Fig. 4. Convergence curve of 10^{th} order FIR-Fractional Order Digital Differentiator

Here, FIR-Fractional order digital differentiator design using JAYA algorithm achieved convergence fast as compared to PSO and CSA algorithms. Also, JAYA based design is able to achieve both exploitation and exploration better as compared to that with PSO and CSA algorithms.

5. Conclusions and Future Scope

In this article, FIR Fractional order digital differentiators with 10^{th} order is designed using PSO, CSA, and JAYA algorithms. Optimal coefficients are calculated to obtain minimum absolute magnitude and phase errors. Wilcoxon rank-sum test at 99% level of confidence is performed to show the robustness of the results. JAYA based FOD design achieved better exploration as well as exploitation capabilities and fast convergence as compared to PSO and CSA techniques.

Acknowledgements: This work is supported by the I. K. G. Punjab Technical University, Kapurthala, Jalandhar, Punjab-144603, India. We are very thankful to the University for all support.

References

[1] S. Das, *Functional fractional calculus for system identification and controls*. 2008.

[2] K. S. Miller and B. Ross, *An introduction to the fractional calculus and fractional differential equations*, 1st ed. New York: Wiley, 1993.

[3] N. Engheta, “Fractional derivatives, fractional integrals and electromagnetic theory,” *1999 Int. Conf. Comput. Electromagn. its Appl. ICCEA 1999 - Proc.*, pp. 1–4, 1999, doi: 10.1109/ICCEA.1999.825051.

[4] A. Aggarwal and M. Kumar, “Design of Digital Differentiator Using the L 1 -Method and Swarm Intelligence-Based Optimization Algorithms,” *Arab. J. Sci. Eng.*, 2018, doi: 10.1007/s13369-018-3188-0.

[5] S. C. Dutta Roy, “On the Realization of a Constant-Argument Immittance or Fractional Operator,” *IEEE Trans. Circuit Theory*, vol. 14, no. 3, pp. 264–274, 1967, doi: 10.1109/TCT.1967.1082706.

[6] R. S. Barbosa, J. A. Tenreiro Machado, and M. F. Silva, “Time domain design of fractional differintegrators using least-squares,” *Signal Processing*, vol. 86, no. 10, pp. 2567–2581, 2006, doi: 10.1016/j.sigpro.2006.02.005.

[7] C.-C. Chien-Cheng Tseng, “Design of fractional order digital FIR differentiators,” *IEEE Signal Process. Lett.*, vol. 8, no. 3, pp. 77–79, 2001, doi: 10.1109/97.905945.

[8] S. Samadi, M. O. Ahmad, M. N. S. Swamy, and A. Abstract, “Exact Fractional-Order Differentiators for Polynomial Signals,” vol. 11, no. 6, pp. 529–532, 2004.

[9] R. Eberhart and J. Kennedy, “A new

optimizer using particle swarm theory," *MHS'95. Proc. Sixth Int. Symp. Micro Mach. Hum. Sci.*, pp. 39–43, doi: 10.1109/MHS.1995.494215.

[10] X.-S. Yang and S. Deb, "Cuckoo Search via Levey Flights," *2009 World Congr. Nat. Biol. Inspired Comput. (NaBIC 2009)*, pp. 210–214, 2009, doi: 10.1109/NABIC.2009.5393690.

[11] R. Venkata Rao, "Jaya: A simple and new optimization algorithm for solving constrained and unconstrained optimization problems," *Int. J. Ind. Eng. Comput.*, vol. 7, no. 1, pp. 19–34, 2016, doi: 10.5267/j.ijiec.2015.8.004.

[12] D. Hai-bin, W. Dao-bo, and Y. Xiu-fen, "Novel Approach to Nonlinear PID Parameter Optimization Using Ant Colony Optimization Algorithm," vol. 3, pp. 73–78, 2006.

[13] D. Sun, J. Zhao, and X. Zhao, "2009 International Conference on Artificial Intelligence and Computational Intelligence A Recursive Variable Fractional Delay FIR Filters Design Scheme Using Particle Swarm Optimization Algorithm," no. 3, 2009, doi: 10.1109/AICI.2009.288.

[14] S. Dhabal, N. Chakraborty, A. Mukherjee, and J. Biswas, "Design of higher order FIR low pass filter using Cuckoo search algorithm," in *2016 International Conference on Communication and Signal Processing (ICCSP)*, 2016, pp. 936–941, doi: 10.1109/ICCSP.2016.7754285.

[15] B. T. Krishna and S. S. Rao, "On design and applications of digital differentiators," *4th Int. Conf. Adv. Comput. ICoAC 2012*, no. 2, pp. 1–7, 2012, doi: 10.1109/ICoAC.2012.6416802.

[16] H. Johansson and S. Member, "On FIR Filter Approximation of Fractional-Order," vol. 3, no. 3, pp. 404–415, 2013, doi: 10.1109/JETCAS.2013.2273853.

[17] C. C. Tseng and S. L. Lee, "Design of digital Riesz fractional order differentiator," *Signal Processing*, vol. 102, pp. 32–45, 2014, doi: 10.1016/j.sigpro.2014.02.017.

[18] A. Aggarwal, T. K. Rawat, M. Kumar, and D. K. Upadhyay, "Efficient design of digital fir differentiator using L1-method," *Radioengineering*, vol. 25, no. 2, pp. 383–389, 2016, doi: 10.13164/re.2016.0383.

[19] R. Scherer, S. L. Kalla, Y. Tang, and J. Huang, "The Grünwald – Letnikov method for fractional differential equations," *Comput. Math. with Appl.*, vol. 62, no. 3, pp. 902–917, 2011, doi: 10.1016/j.camwa.2011.03.054.

[20] R. A. Zitar, M. A. Al-Betar, M. A. Awadallah, I. A. Doush, and K. Assaleh, "An Intensive and Comprehensive Overview of JAYA Algorithm, its Versions and Applications," *Arch. Comput. Methods Eng.*, vol. 29, no. 2, pp. 763–792, 2022, doi: 10.1007/s11831-021-09585-8.

[21] M. Kumar and T. K. Rawat, "Optimal design of FIR fractional order differentiator using cuckoo search algorithm," *Expert Syst. Appl.*, vol. 42, no. 7, pp. 3433–3449, 2015, doi: 10.1016/j.eswa.2014.12.020.