Numerical investigation of system reaction-diffusion with diffusion with double nonlinearity

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The article considers the Cauchy problem for the following system of nonlinear parabolic equations.

$$\begin{cases} \frac{\partial u}{\partial t} = \frac{\partial}{\partial x}\left(\left|\frac{\partial u^{k_1}}{\partial x}\right|^{p-2} \frac{du^{m_1}}{dx}\right) \pm cv_1(t)\frac{\partial y}{\partial x} - b_1 u^{q_1}\left|\frac{\partial v^{m_1}}{\partial x}\right|^{p_1} \\ \frac{\partial v}{\partial t} = \frac{\partial}{\partial x}\left(\left|\frac{\partial v^{k_2}}{\partial x}\right|^{p-2} \frac{dv^{m_2}}{dx}\right) \pm cv_1(t)\frac{\partial v}{\partial x} - b_1 v^{q_2}\left|\frac{\partial u^{m_2}}{\partial x}\right|^{p_2} \end{cases} \quad (1)$$

Where $m_i \geq 1$, $p_i > 1+1/m$, $q_i > 0$, $k_i \geq 1$, (i=1,2), p>0 are given as numerical parameters, u_0 (x) and v_0 (x) are continuous positive-definite carrier functions.

(1), (2) boundary value problem plays an important role in mathematical modeling of the diffusion process in non-linear media, fluid flow in porous media, biological population dynamics, polytropic filtration, synergetics, and solving problems in a number of other areas. For instance, $u(x,t)$ and $v(x,t)$ represent the two biological population density or the temperature of the two porous media in the process of heat dissipation [1-4].

As is known, the system of equations (1) is degenerate in the domain $u,v \equiv 0$ and does not have a classical solution. In this case, the generalized solution of problem (1)-(2) is studied as follows:

The given system of equations (1) are degenerate in k(p-2)+m-1>0, so its solution in the Q ={(t, x): 0 < t <T, } domain (1) is understood as a satisfactory generalized solution in distribution of equations. The system (1) represents a number of processes, and is considered as a mathematical model of reaction-diffusion, heat diffusion, filtration, and etc. The given equations (1) describe the slow diffusion process at k(p-2)+m-1>0, and the fast diffusion process at k(p-2)+m-1<0.

It is known that the solution of problem (1), (2) is global or unbounded under specific conditions of numerical parameters. Regarding (1) and (2), several scientists have conducted research with this very question. Including:

Z. Rakhmonov [5] observed the issue of heat conduction in an inhomogeneous environment:

$$\rho_1(x)\frac{\partial u}{\partial t} = div\left(\left|\nabla u^l\right|^{p-2}\nabla u^l\right) + \rho_2(x)u^q \; x\in R^N \; t>0 \quad (3)$$

$$u(x,0) = u_0(x) \; x\in R^N \quad (4)$$

Here:

$$(\cdot) = grad_x(\cdot) \equiv \left(\frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2}, \ldots, \frac{\partial}{\partial x_N}\right)(\cdot), \rho_1(x) = |x|^m, \rho_2(x) = |x|^n$$

It is considered as a model of the combustion process in the medium where in n=m=0 power of heat release is $u^q \geq 0$ and temperature depends on $u = u(x,0) \geq 0$. Automodel solutions of problems (3), (4) were constructed by the author and asymptoreattics of automodel solutions were found.

Sh. Sadullayeva [6,7] studied the issue of heat propagation in the following non-homogeneous medium in the $Q_T = \{(t,x):0\} < t < T, x\in R^N\}$ domain:

$$Au \equiv -\frac{\partial(\rho(x)u)}{\partial t} + \nabla(u^{m-1}\left|\nabla u^k\right|^{p-2}\nabla u^l) \quad (5)$$

$$u|_{t=0} = u_0(x) \geq 0, x\in R^N \quad (6)$$

Equations (11), (12) are considered as models of liquid and gas filtration, heat diffusion processes with a double nonlinear coefficient. Diffusion in the medium, where $\rho(x)$ is the density of the medium. In this paper, numerical solutions of problems (5), (6) were built based on the auto model approach and localization conditions were found.

The scientists Mi, Y.S., Mu, C.L., and Chen, B.T. [1] studied the following nonlinear filtering problem for the case of slow diffusion.

(8)

$$\begin{cases} \dfrac{\partial u}{\partial t} = \dfrac{\partial}{\partial x}\left(\left|\dfrac{\partial u}{\partial x}\right|^{p_1-2}\dfrac{\partial u^{m_1}}{\partial x}\right), x>0, 0<T<0 \\ \dfrac{\partial u}{\partial t} = \dfrac{\partial}{\partial x}\left(\left|\dfrac{\partial u}{\partial x}\right|^{p_2-2}\dfrac{\partial u^{m_2}}{\partial x}\right), x>0, 0<T<0, \end{cases}$$

$$\begin{cases} -\left|\dfrac{\partial u}{\partial x}\right|^{p_1-2}\dfrac{\partial u^{m_1}}{\partial x}\Bigg|_{x=0} = v^{q_1}(0,t), 0<T<0 \\ -\left|\dfrac{\partial u}{\partial x}\right|^{p_2-2}\dfrac{\partial u^{m_2}}{\partial x}\Bigg|_{x=0} = u^{q_2}(0,t), 0<T<0, \end{cases}$$

(9)

$$\mathrm{u}(x,0) = u_0(x) \geq 0 v\,(x,0) = v_0(x) \geq 0 x > 0 (10)$$

They proved that when $q_1q_2 \leq \dfrac{(2p_1-1+m_1)(2p_1-1+m_2)}{p_1p_2}$ any solution to problem (8)-(10) is global. Also, for $q_1q_2 > \dfrac{(2p_1-1+m_1)(2p_1-1+m_2)}{p_1p_2}$:

- if $\max\{l_1-k_1, l_2-k_2\}<0$ any solution to problem (8)-(10) is unlimited;
- if $\min\{l_1-k_1, l_2-k_2\}>0$ and the initial conditions are small enough (8)-(10), it is proved that any solution of the problem is global.

Additionally, Wanjuan Du and Zhong Li [1] (1) studied the conditions of time globality and non-globality of solutions to the following problem:

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial x}\left(\left|\frac{\partial u}{\partial x}\right|^{p-2}\frac{\partial u}{\partial x}\right) + u^{\beta},\quad (x,t)\in R_+\times(0,+\infty), \tag{11}$$

$$\left|\frac{\partial u}{\partial x}\right|^{p-2}\frac{\partial u}{\partial x}(0,t) = u^q(0,t),\quad t>0 \tag{12}$$

The solution of the problems (7), (8) is global and unbounded under specific conditions of numerical parameters. Below are the asymptotics of global and unbounded solutions based on the research of these scientists.

$\beta \leq 1$, $q > 2(p-1)$ case. The automodel solution of (11), (12) problems in the following form is:

$$u_1(x,t) = t^{\alpha}\phi(\xi),\ \xi = xt^{-\gamma}$$

Here: $\alpha = \dfrac{1}{1-\beta}$, $\gamma = \dfrac{p-1-\beta}{p(1-\beta)}$, $\phi(\xi)$ is the solution of the following problem:

$$\frac{d}{d\xi}\left(\left|\frac{d\phi^k}{d\xi}\right|^{p-2}\frac{d\phi}{d\xi}\right)+\gamma\xi\frac{d\phi}{d\xi}-\alpha\phi+\phi^\beta=0 \tag{13}$$

$$\left|\frac{d\phi^k}{d\xi}\right|^{p-2}\frac{d\phi}{d\xi}\Bigg|_{\xi=0}=0 \tag{14}$$

It is proved that the solution of problem (13), (14) with a compact guide has the following asymptotics in

$\xi \to \left(ap/(p-2)\right)^{\frac{p-1}{p}}\gamma^{\frac{1}{p}}$

$$\phi(\xi)=\left(a-\frac{p-2}{p}\gamma^{\frac{1}{p-1}}\xi^{\frac{p}{p-1}}\right)^{\frac{p-1}{p-2}},(1+o(1)),a>0$$

Introduce the following notations to find the automodel solution for the system of equations (1), (2).

$$w_t=f(\xi)\ \xi=\eta\cdot t^{-\lambda}\ z=g(\xi)\ \xi=\eta\cdot t^{-\lambda}\ \lambda=\frac{1}{p} \tag{15}$$

As a result, the system of equations (1) comes to the automodel equation of the following form:

$$\begin{cases}\frac{d}{d\xi}\left(\left|\frac{df^{k_1}}{d\xi}\right|^{p-2}\frac{df^{m_1}}{d\xi}\right)+\frac{1}{p}\xi\frac{df}{d\xi}-b_1f^{q_1}\left|\frac{dg^{m_1}}{d\xi}\right|^{p_1}=0\\ \frac{d}{d\xi}\left(\left|\frac{dg^{k_2}}{d\xi}\right|^{p-2}\frac{dg^{m_2}}{d\xi}\right)+\frac{1}{p}\xi\frac{dg}{d\xi}-b_2g^{q_2}\left|\frac{df^{m_2}}{d\xi}\right|^{p_2}=0\end{cases} \tag{16}$$

Let's consider the following function for the asymptotics of the solution of equation (16):

$$\bar{f}(\xi)=A_1\left(a_1-\xi^{\frac{p}{p-1}}\right)_+^{\alpha_1} \tag{17.1}$$

$$\bar{g}(\xi)=A_2\left(a_2-\xi^{\frac{p}{p-1}}\right)_+^{\alpha_2} \tag{17.2}$$

Here: $A_1,\ A_2>0,\ a_1=const>0,\ a_2=const>0.\ \alpha_1=\dfrac{p-1}{(k_1-1)(p-2)+m_1+p-3},$

$\alpha_2=\dfrac{p-1}{(k_2-1)(p-2)+m_2+p-3}.$

We search the solution of equation (16) in the following form:

$$f=\theta(\tau_1)\bar{f}(\xi),\ \tau_1=-\ln(a_1-\xi^{\frac{p}{p-1}})$$

$$g=Q(\tau_2)\bar{g}(\xi),\ \tau_2=-\ln(a_2-\xi^{\frac{p}{p-1}}) \tag{18}$$

As a result, by putting (18) into the system of equations (16), we get the following expressions:

$$f^{`}=\overline{f}^{`}_{\xi}\theta+\overline{f}\theta^{`}_{z}z^{`}_{\xi}=-A_{1}\frac{p}{k(p-2)+m-1}\xi^{\frac{1}{p-1}}\left(a_{1}-\xi^{\frac{p}{p-1}}\right)^{\gamma-1}\theta+\frac{\xi^{\frac{p}{p-1}}}{a_{1}-\xi^{\frac{p}{p-1}}}\cdot\frac{p}{p-1}A_{1}\left(a_{1}-\xi^{\frac{p}{p-1}}\right)^{\gamma-1}\theta^{`}=$$

$$\xi^{\frac{p}{p-1}}A_{1}\left(a_{1}-\xi^{\frac{p}{p-1}}\right)^{\gamma-1}\left[\frac{p}{p-1}\theta^{`}_{z}-\frac{1}{k(p-2)+m-1}\theta\right]^{p-2}$$

$$g^{`}=\overline{g}^{`}_{\xi}Q+\overline{g}Q^{`}_{z}z^{`}_{\xi}=-A_{2}\frac{p}{k(p-2)+m-1}\xi^{\frac{1}{p-1}}\left(a_{2}-\xi^{\frac{p}{p-1}}\right)^{\gamma-1}Q+\frac{\xi^{\frac{p}{p-1}}}{a_{2}-\xi^{\frac{p}{p-1}}}\cdot\frac{p}{p-1}A_{2}\left(a_{2}-\xi^{\frac{p}{p-1}}\right)^{\gamma-1}Q^{`}=$$

$$\xi^{\frac{p}{p-1}}A_{2}\left(a_{2}-\xi^{\frac{p}{p-1}}\right)^{\gamma-1}\left[\frac{p}{p-1}Q^{`}_{z}-\frac{1}{k(p-2)+m-1}Q\right]^{p-2}$$

Analyzing these two equations using the method presented in [2], we arrive at the following system of equations:

$$\frac{d}{dz}(L(\theta))-\frac{1}{k(p-2)+m-1}L(\theta)+\frac{1}{k(p-2)+m-1}\theta\frac{p}{p-1}\frac{A_{1}}{(A_{1}p)^{p-1}}=0$$

$$\frac{d}{dz}(L(Q))-\frac{1}{k(p-2)+m-1}L(Q)+\frac{1}{k(p-2)+m-1}Q\frac{p}{p-1}\frac{A_{2}}{(A_{2}p)^{p-1}}=0$$

From which

$$\theta=(c_{1})\frac{1}{k_{1}(p-2)+m_{1}+p-3}$$

$$Q=(c_{2})\frac{1}{k_{2}(p-2)+m_{2}+p-3} \tag{19}$$

is figured. As a result, we have the asymptotics of automodel solutions (17.1), (17.2), (19) for problems (1), (2).

Numerical solution. For this, build a net according to the spatial coordinate

$$S_{h}=\{x_{i}=i\cdot h,h>0,i=1,2....,n,n\cdot h=b\}$$

and according to time

$$V_{h}=\{x_{i}=i\cdot h,h>0,i=1,2....,n,n\cdot h=b\}$$

Write the equation (1) in the following form:

$$\begin{cases}\frac{\partial u}{\partial t}=\frac{\partial}{\partial x}\left(B_{1}(u)\frac{\partial u}{\partial x}\right)+c\upsilon_{1}(t)\frac{\partial u}{\partial x}-b_{1}u^{q_{1}}\left|\frac{\partial v^{m_{1}}}{\partial x}\right|^{p_{1}}\\ \frac{\partial v}{\partial t}=\frac{\partial}{\partial x}\left(B_{2}(v)\frac{\partial v}{\partial x}\right)+c\upsilon_{2}(t)\frac{\partial u}{\partial x}-b_{2}v^{q_{1}}\left|\frac{\partial u^{m_{2}}}{\partial x}\right|^{p_{2}}\end{cases} \tag{20}$$

$$B_{1}(u)=m_{1}\left|k_{1}u^{k_{1}-1}\frac{\partial u}{\partial x}\right|^{p-2}u^{m_{1}-1}$$

$$B_{2}(v)=m_{2}\left|k_{2}v^{k_{2}-1}\frac{\partial v}{\partial x}\right|^{p-2}v^{m_{2}-1}$$

And construct a numerical solution scheme with the following initial and boundary conditions.

$$\begin{cases} u(t,0)=\phi_1(t)>0, \;\; u(t,b)=\phi_2(t)=0, \;\; t\in[0,T] \\ v(t,b)=\phi_3(t)=0, \;\; v(t,b)=\phi_4(t)=0, \;\; t\in[0,T] \\ u(x,0)=u_0(x), \qquad v(x,0)=u_0(x), \qquad x\in R_+ \end{cases}$$

Replace problem (2) with an implicit differential scheme and get a differential problem using the balance method (20):

$$\begin{cases} \dfrac{y_i^{j+1}-y_i^j}{\tau}=\dfrac{1}{h^2}\left(a_{i+1}(y)\left(y_{i+1}^{j+1}-y_i^{j+1}\right)-a_i(y)\left(y_i^{j+1}-y_{i-1}^{j+1}\right)\right)+c_1\upsilon_1(t)\dfrac{y_i^{j+1}-y_{i-1}^{j+1}}{h}-G_1\left(y_i^j,g_i^j\right) \\ \dfrac{g_i^{j+1}-g_i^j}{\tau}=\dfrac{1}{h^2}\left(a_{i+1}(g)\left(g_{i+1}^{j+1}-g_i^{j+1}\right)-a_i(g)\left(g_i^{j+1}-g_{i-1}^{j+1}\right)\right)+c_2\upsilon_2(t)\dfrac{g_i^{j+1}-g_{i-1}^{j+1}}{h}-G_2\left(y_i^j,g_i^j\right) \end{cases} \tag{21}$$

$$G_1\left(y_i^j,g_i^j\right)=b_1\left(y_i^j\right)^{q_1}\left|m_1\left(g_i^j\right)^{m_1-1}\frac{g_i^j-g_{i-1}^j}{h}\right|^{p_1}$$

$$G_2\left(y_i^j,g_i^j\right)=b_1\left(g_i^j\right)^{q_2}\left|m_2\left(y_i^j\right)^{m_2-1}\frac{y_i^j-y_{i-1}^j}{h}\right|^{p_2}$$

Here one of the following formulas is used to calculate a(y) and a(g):

$$\begin{cases} a_i(y)=B_1\left(\dfrac{y_i^j-y_{i-1}^j}{2}\right) \\ a_i(g)=B_2\left(\dfrac{g_i^j-g_{i-1}^j}{2}\right) \end{cases} \tag{22}$$

$$\begin{cases} a_i(y)=\dfrac{B_1(y_i^j)+B_1(y_{i-1}^j)}{2} \\ a_i(g)=\dfrac{B_2(g_i^j)+B_2(g_{i-1}^j)}{2} \end{cases} \tag{23}$$

Use the simple iteration method to solve the system of equations (21) and get the following result:

$$\begin{cases} \dfrac{\overset{s+1}{y_i^{j+1}}-\overset{s}{y_i^j}}{\tau}=\dfrac{1}{h^2}\left(a_{i+1}(\overset{s}{y})\left(\overset{s+1}{y_{i+1}^{j+1}}-\overset{s+1}{y_i^{j+1}}\right)-a_i(\overset{s}{y})\left(\overset{s+1}{y_i^{j+1}}-\overset{s+1}{y_{i-1}^{j+1}}\right)\right)+c_1\upsilon_1(t)\dfrac{\overset{s+1}{y_i^{j+1}}-\overset{s+1}{y_{i-1}^{j+1}}}{h}-G_1\left(\overset{s}{y_i^j},\overset{s}{g_i^j}\right) \\ \dfrac{\overset{s+1}{g_i^{j+1}}-\overset{s}{g_i^j}}{\tau}=\dfrac{1}{h^2}\left(a_{i+1}(\overset{s}{g})\left(\overset{s+1}{g_{i+1}^{j+1}}-\overset{s+1}{g_i^{j+1}}\right)-a_i(\overset{s}{g})\left(\overset{s+1}{g_i^{j+1}}-\overset{s+1}{g_{i-1}^{j+1}}\right)\right)+c_2\upsilon_2(t)\dfrac{\overset{s+1}{g_i^{j+1}}-\overset{s+1}{g_{i-1}^{j+1}}}{h}-G_2\left(\overset{s}{y_i^j},\overset{s}{g_i^j}\right) \end{cases} \tag{24}$$

Here: $s=0,1,2,\ldots$.

$\overset{s+1}{y}_i$ and $\overset{s+1}{g}_i$ with respect to (24) differential circuits are linear. the value of y and g is obtained from the previous time step as the initial iteration for $\overset{s+1}{y}_i$, $\overset{s+1}{g}_i$.

$$\overset{0}{y}{}^{j+1}=y^j,\; \overset{0}{g}{}^{j+1}=g^j.$$

The iteration accuracy is given and the process continues until the following conditions are satisfied in the iterative scheme calculation:

$$\max_{0\le i\le n}\left|\overset{s+1}{y}_i-\overset{s}{y}_i\right|<\varepsilon,\; \max_{0\le i\le n}\left|\overset{s+1}{g}_i-\overset{s}{g}_i\right|<\varepsilon$$

Introduce the following designations into the system of differential equations (24):

$$A_{1i} = \frac{\tau}{h^2} a_{i+1}\left(\overset{s}{y}\right) \overset{s+1}{y}{}_{i+1}^{j+1}$$

$$B_{1i} = \left(\frac{\tau}{h^2} a_i\left(\overset{s}{y}\right) - \frac{\tau c}{h} \upsilon\left(t_j\right)\right) \overset{s+1}{y}{}_{i+1}^{j+1}$$

$$A_{2i} = \frac{\tau}{h^2} a_{i+1}\left(\overset{s}{g}\right) \overset{s+1}{g}{}_{i+1}^{j+1}$$

$$B_{2i} = \left(\frac{\tau}{h^2} a_i\left(\overset{s}{g}\right) - \frac{\tau c}{h} \upsilon\left(t_j\right)\right) \overset{s+1}{g}{}_{i+1}^{j+1}$$

$$C_{1i} = A_{1i} + B_{1i} + 1$$

$$C_{2i} = A_{2i} + B_{2i} + 1$$

$$F_{1i} = \overset{s}{y}{}_i^j - \tau G_1\left(\overset{s}{y}{}_i^j, \overset{s}{g}{}_i^j\right)$$

$$F_{2i} = \overset{s}{g}{}_i^j - \tau G_2\left(\overset{s}{y}{}_i^j, \overset{s}{g}{}_i^j\right)$$

As a result, system (24) becomes as follow:

$$\begin{cases} A_{1i}\, \overset{s+1}{y}{}_{i+1}^{j+1} - C_{1i}\, \overset{s+1}{y}{}_{i}^{j+1} + B_{1i}\, \overset{s+1}{y}{}_{i-1}^{j+1} = -F_{1i}, \quad i = 1,2,\ldots,n-1 \\ A_{2i}\, \overset{s+1}{g}{}_{i+1}^{j+1} - C_{2i}\, \overset{s+1}{g}{}_{i}^{j+1} + B_{2i}\, \overset{s+1}{g}{}_{i-1}^{j+1} = -F_{2i}, \quad i = 1,2,\ldots,n-1 \end{cases} \tag{25}$$

Solve the system of algebraic equations (25) using the tridiagonal matrix algorithm. According to this method:

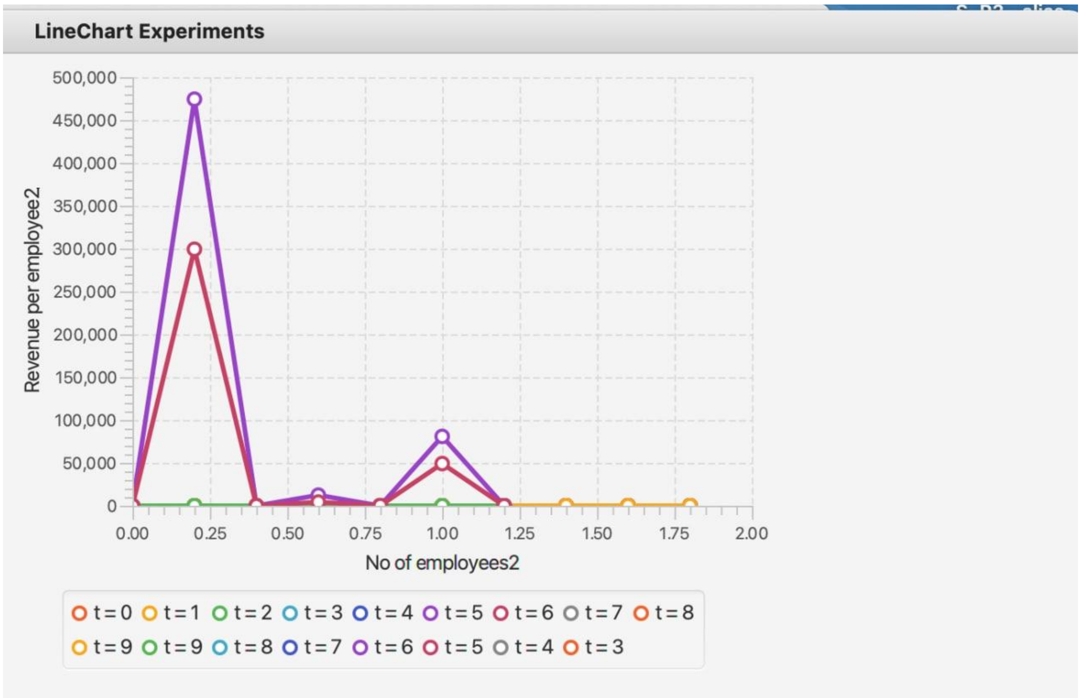
$$\begin{cases} \overline{y}_i = \alpha_{1i}\left(\beta_{1i} + \overline{y}_{i+1}\right), \\ \overline{g}_i = \alpha_{2i}\left(\beta_{2i} + \overline{g}_{i+1}\right), \end{cases} \tag{26}$$

Here the $\alpha_{1i},\ \alpha_{2i},\ \beta_{1i},\ \beta_{2i}$ coefficients are calculated as follows:

$$\begin{cases} \alpha_{1i+1} = \dfrac{B_{1i}}{C_{1i} - \alpha_{1i} A_{1i}}, \\ \alpha_{2i+1} = \dfrac{B_{2i}}{C_{2i} - \alpha_{2i} A_{2i}}, \end{cases}$$

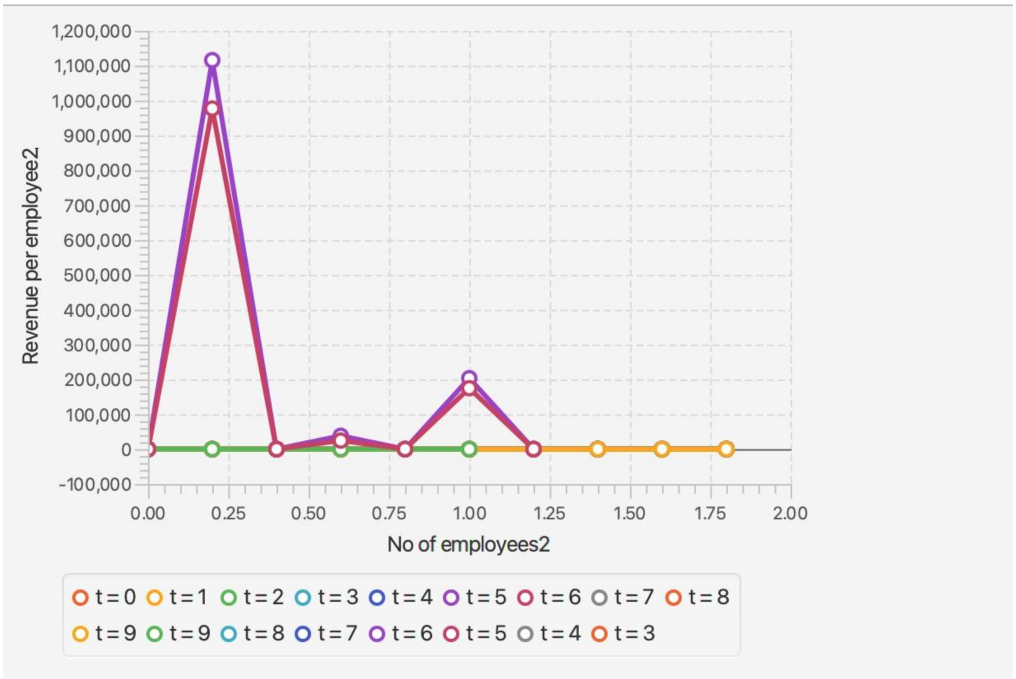
$$\begin{cases} \beta_{1i+1} = \dfrac{A_{1i}\beta_{1i} + F_{1i}}{C_{1i} - \alpha_{1i} A_{1i}}, \\ \beta_{2i+1} = \dfrac{A_{2i}\beta_{2i} + F_{2i}}{C_{2i} - \alpha_{2i} A_{2i}}, \end{cases}$$

where $i = 1,2,\ldots n$. The initial α_{10}, α_{20}, β_{10}, β_{20} values are found from the boundary conditions given above.



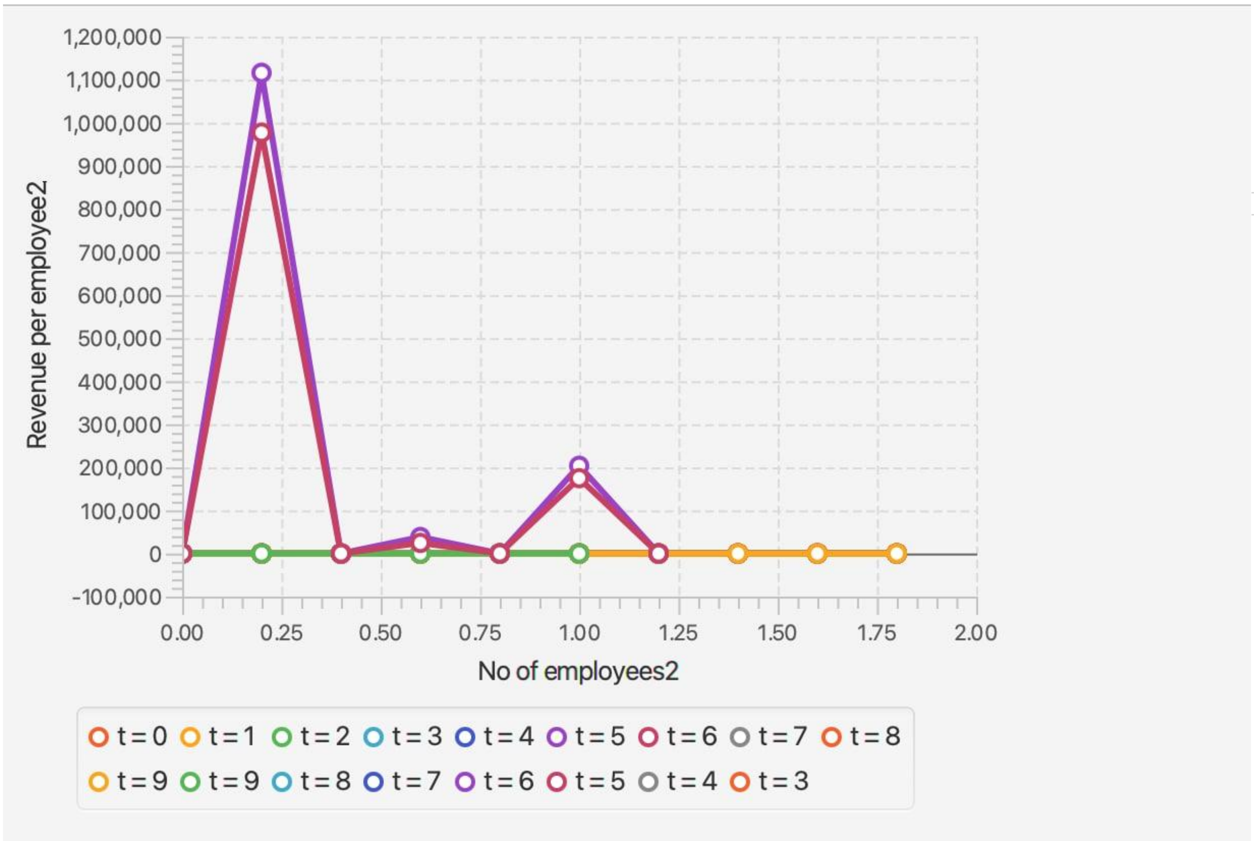
A = 2.5, a=2.3, m=1.5, p=2.6, k=1.5, c=0.5, T=2.0, q1=1.5,b1=2.0;

Obviously, the u(x) fluid trend achieves higher result compared to the v(x) fluid trend. The fluid reaches the highest result at t=0.25. In addition, when t=0.4 and t=0.8, the fluid dispersion becomes zero. Subsequently, when t=1, the value of u(x) reaches to 75, while the value of v(x) reaches up to 50.



A = 2, a=2, T=2, m=2,p1=3, p=2.6, k=1.5, c=0.5, b1=2.0, q1=1.5;

The chart illustrates that the u(x) fluid trend achieves higher result compared to the v(x) fluid trend. The fluid reaches the highest result at t=0.25. However, when t=0.4 and t=0.8, the fluid dispersion becomes zero. Following, when t=1, the values of u(x) and v(x) reach up to 180.



A = 2.8, a=2.4, m=2.5, p=2.6, k=1.5, c=0.5, T=3.3, q1=1.5,b1=2.0;

Here, u(x) fluid trend achieves a higher result than v(x) fluid trend. The fluid reaches the highest result at t=0.25. And when t=0.4 and t=0.8, the fluid spread becomes zero. Later, when t=1, the values of u(x) and v(x) reach almost 100.

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