Quantum Decision-making in Newcomb's Problem: Effect of Reward Size

Nobuye Ishibashi-Ohmura, Taiki Takahashi

ABSTRACT

This study experimentally examined people's decision making in the Newcomb's problem. We observed that Savage's sure-thing principle and Kolmogorovian law of probability was violated. Also, the degree of the violation increased as the reward size increased. By adopting quantum decision theory, we further quantified interference effect as a quantum phase factor. The quantum phase also depended on the reward size; i.e., it increased as the reward size in the unknown box in the Newcomb's problem increased. Future directions in the application of the present theory to studies in quantum decision theory and neuroeconomics are discussed.

Key Words: uncertainty, Newcomb's paradox, quantum decision theory, risk

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1. Introduction

Since the political philosopher Robert Nozick (1969) theoretically analyzed two types of rational principles (i.e., von Neumann and Morgenstern's expected utility theory and Savage's sure-thing principle) in the Newcomb's paradox, several studies of the Newcomb's paradox have been performed in philosophy (Bar-Hillel and Margalit, 1972; Lewis, 1979), psychology (Shafir and Tversky, 1992; Stanovich and West, 1999), and decision theory (Ken Binmore, 2008). For instance, David Lewis (Lewis, 1979) argued that Newcomb's problem is theoretically similar to the Prisoner's Dilemma. However, psychological and behavioral decision theoretical foundations of human decision making in the Newcomb's problem is still unknown. Recently, in order to model paradoxical human decisions in the situation such as the Prisoner's Dilemma (Rapoport and Chammah, 1965; Croson, 1999; Li *et al.,* 2010), Ellsberg's urn problem (Ellsberg, 1961; Aerts *et al.,* 2011) and two-stage gambling task (Shafir and Tversky, 1992), quantum decision theoretical models have been developed (Pothos and Busemeyer, 2009; Busemeyer *et al.,* 2011). These quantum decisions theoretical studies revealed that human decision making is better accounted for by quantum probabilistic models in comparison to classical probabilistic models including Bayesian rational decision theory. In this study, we therefore adopted quantum decision theory to model human decision making in the Newcomb's problem by varying economic incentives. This paper is organized in the following manner. In Section 1, we briefly introduce the Savage's principle (STP) in relation to the law of total probability (LTP) in Kolmogorovian probability theory and how to model the violation of STP with quantum probability theory. In Section 2, the procedures of our behavioral experiments are explained. In

Corresponding author: Taiki Takahashi
Address: Taiki Takahashi and Nobuye Ishibashi-Ohmura, Department of Behavioral Science, Center for Brain Science, Center for Experimental Research in Social Sciences, Hokkaido University, Sapporo, Japan.
Phone: +81-11-706-3057 **Fax**: +81-11-706-3066
e-mail ✉ taikitakahashi@gmail.com, nobuye@gmail.com
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Section 3, we show our experimental data of human decision making in the Newcomb's problem. In Section 4, conclusions of the present study and future directions in decision neuroscience and behavioral neuroeconomics are discussed.

1.1 Quantum probability and Newcomb's paradox

Savage's sure-thing principle (STP) in Bayesian decision theory (one of the most well-known types of rational decision theory prevailing in economics) states that if [a decision-maker prefers option A to option B when she knows the state of the world is X, and she also prefers option A to option B when she knows the state of the world is not X], then she should prefer option A to option B even if she does not know whether the state of the world is X or not X (Savage, 1954). The political philosopher Robert Nozick argued that this fundamental principle in decision theory is incompatible with von Neumann and Morgenstern's expected utility theory, another rational decision theoretical principle developed in game theory (Nozick, 1969). It is to be noted that STP is, mathematically, a special case of the following more general law of total probability in axiomatic probability theory established by Kolmogorov (see Khrennikov, 2010):

$$\mathrm{P(A) = P(A|X)P(X) + P(A|not\,X)P(not\,X).} \quad (1)$$

You can readily confirm that STP is equivalent to LTP when $\mathrm{P(A|X) = P(A|not\,X) = 1}$, by utilizing the relation P(X)+P(not X)=1. Several experimental studies on decision problems such as the Prisoner's Dilemma and two stage gambling reported the violation of this law of total probability (Shafir and Tversky, 1992), indicating that most people do not perform rational decision making under uncertainty. To model these "irrational" decisions under uncertainty (called "disjunction effect" in decision science, Shafir and Tversky, 1992; Tversky and Shafir, 1992; Bar-Hillel and Neter, 1993), quantum probability theoretical models have been introduced in quantum decision and cognitive sciences (Pothos and Busemeyer, 2009; Busemeyer *et al.,* 2011; Asano *et al.,* 2012; Busemeyer and Bruza, 2012; Cheon and Takahashi, 2010; Cheon and Takahashi, 2010; Cheon and Takahashi, 2012):

$$\mathrm{P(A) = P(A\,|\,X)P(X) + P(A\,|\,not\,X)P(not\,X) +} \\ \text{(Qunatum Interference term).} \quad (2)$$

where the quantum intereference term is equal to $2\cos\theta\sqrt{\mathrm{P(A|X)P(X)P(A|not\,X)P(not\,X)}}$ with a quantum phase factor θ.

We now introduce the Newcomb's problem. Robert Nozick (1969) proposed a paradox in rational decision theory. He constructed a decision problem in which the sure-thing principle conflicts with the expected utility theory. Nozick called the example Newcomb's Problem after the physicist, William Newcomb, who first formulated the problem. In the Newcomb's Problem a decision maker may choose either to take an opaque box or to take both the opaque box and a transparent box. The transparent box contains ten dollars that the decision maker can plainly see. The opaque box (the unknown box) contains either nothing or one million dollars, depending on a prediction, about the decision maker's choice, which has already made. If the prediction was that the decision maker will take both boxes, then the opaque box is empty; while if the prediction was that the decision maker will take the opaque box alone, then the opaque box contains a million dollars. The prediction has been quite successful. According to expected utility theory, the decision maker should choose only one box, since the probability of the success of the prediction (and hence the probability that the opaque box contains a million dollars) is very high. On the contrary, according to Savage's sure-thing principle, the decision maker should take both boxes, for the total amount of money is always larger for two boxes than for one box, irrespective of the amount of money in the opaque box. Although the Newcomb's paradox has extensively been discussed theoretically in the domain of philosophy (D. Lewis, 1979; Hurley, 1991; Campbell and Sowden, 1985), and theoretical physics (Piotrowski and Sładkowski, 2003; Mihara, 2010) only few experimental studies have been conducted in psychology and behavioral science (Shafir and Tversky, 1992; Stanovich and West, 1999; Goldberg, 2005). In this study, we therefore conducted behavioral experiments of the Newcomb's problem with different reward sizes in the opaque box, and analyzed the decision makers' choices by utilizing a quantum decision theoretic model.

2. Methods

2.1 Participants

A total of forty (Female 36: Male 4) students from a nursing school in Japan participated in the study. The mean age was 27 years old (standard deviation was 8.03). No student was exposed to decision theory before participation.

2.2 Procedure

The participants were asked to answer their tendency to choose two boxes on 10 point scales (four scales depending on the magnitudes of reward in the *unknown* boxes) in the Newcomb's problems. We linearly transformed the participants' answers to the question with the 10 point scales to real numbers ranging from 0 to 1, which we defined as the participants' probabilities of choosing the options of both boxes. In our experiment, the instruction about the Newcomb's problem was below (in Japanese):

> US army research institute recently developed a computer program predicting human behavior. The past success probability of the predictions made by the computer program was about 90 percent. The computer program now tries to predict your choice, according to your answers to more than one hundred psychological questions. Suppose that you have already answered the psychological questions and the computer program has already made a prediction of your choice in the following problem.
>
> Problem:
>
> Please choose between option A and option B.
>
> A) Black box only
>
> B) Both boxes (black and white boxes)
>
> The white box contains 1,000 yen. The black box contains either one million yen or 0 yen. The state of the black box has already been determined by the following rule:
>
> **Rule:** If the computer program predicted that you would choose option A (Black box only), the black box contains one million yen. If the computer program predicted that you would choose option B (Both boxes), the black box is empty. Note that the state of the black box (empty or full) has already been determined and will not be changed from now.

The participants' answers to this problem were defined as conditional probabilities of choosing two boxes when the state of the black box is either full or empty (unknown); i.e., Pr(two boxes | unknown), after a linear transformation into a real number between 0 to 1.

To examine the effect of size of the prize (the magnitude of the uncertain money) in the Black Box, we changed the reward size (one million yen, 100,000 yen, 10,000 yen, and 1,000 yen) across four experimental conditions. All participants conducted all of the four Newcomb's problem tasks differing in the reward size in the Black Box.

Participants were further asked to answer their choice tendencies when the state (content) of the Black box were known to full of each of the four different magnitudes of reward (defined as Pr(two boxes | known full)) and empty (defined as Pr(two boxes | known empty)), again with 10 point scales. It is noteworthy that this procedure is one of the novel advantages of our present study. Specifically, in the previous experimental studies of Newcomb's problem (Shafir and Tversky, 1992; Stanovich and West, 1999; Goldberg, 2005), probabilities of choosing the "two boxes" were not actually measured. Shafir and Tversky (1992) assumed, without empirical evidence, participants choices in cases the state of the black box were known to full and empty were the "two boxes" in examining the violation of Savage's sure-thing principle (STP). In this study, in contrast, the probabilities of choosing the "two boxes" in cases the black box is known to full and empty, in order to examine the violation of law of total probability in the Newcomb's problem in a precise manner.

2.3 Data analysis

We linearly transformed the participants' answers to the questions with 10 point scales into real numbers ranging from 0 to 1, which we defined as the participants choice probabilities, Pr(two boxes| unknown), Pr(two boxes | known full), and Pr(two boxes | known empty). Furthermore, we also calculated individual quantum phase factors according to quantum probabilistic law of total probability (equation 2). We conducted ANOVAs for examining the effect of reward size on subject's choice probabilities across the four different magnitudes (one million yen, 100,000 yen, 10,000yen, and 1,000 yen) and the effect of the two orders (a descending order and an ascending order) of presenting the four different magnitudes on subject's choice probabilities.

3. Results

3.1 Violation of law of total probability (LTP)

Figure 1 shows participants' choice probabilities of two boxes. Participants didn't completely choose "two boxes" even when the state of the black box were known to both full and empty (one-sample t-tests, all *ps* < 0.05). This indicates that participants' choice in Newcomb's problem didn't meet the antecedent of STP. We thus examined whether LTP (equation 1), rather than STP per se, was violated. According to Pothos and Busemeyer (Pothos and Busemeyer, 2009), we defined the interference effect (disjunction effect under uncertainty) as below:

(interference effect) =
Pr(two boxes | unknown) –
(Pr(two boxes | known full) +
Pr(two boxes | known empty)) / 2. (3)

Calculated sizes of interference effect (Figure 2) were greater than zero (one-sample t-tests, all *ps* < 0.05) for all the magnitudes, implying that law of total probability was violated under all magnitude conditions.

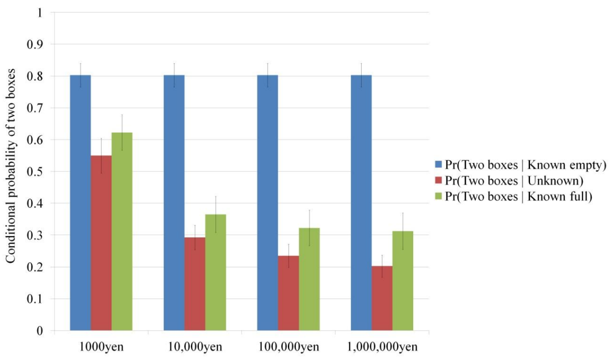

Figure 1. Choice probabilities for option B (two boxes) across conditions. Note that the same data of Pr(Two boxes | Known empty) were presented across four different reward sizes. Pr(Two boxes | Unknown) increases as the reward size of the prize in Black Box decreases. Data are expressed as Mean+/-Standard Error of Mean.

3.2 Effect of magnitude on interference effect

It was observed that Pr(Two boxes | unknown) and Pr(Two boxes | Known full) decreased as reward size in the Black Box increased. A two-way ANOVA analysis with reward size as a within participants' factor and knowledge of the state of the Black box(known empty / unknown / known full) as a within participants' factor revealed that this magnitude effect is significant (F(3, 117)=37.25 , *p<0.0001*). Next, we can see that size (i.e., unsigned absolute value) of the interference effect (disjunction effect) was increased as reward size of the prize in Black Box increased (one-way ANOVA, *F*(3, 117)=5.94 , *p<0.001*). Finally, we calculated quantum phases according to quantum probabilistic law of total probability (equation 2). We again observed that quantum phase increased as rseward size of the prize in Black Box increased (one-way ANOVA, *F*(3, 117)=14.07 , *p<0.001* ; see Figure 3).

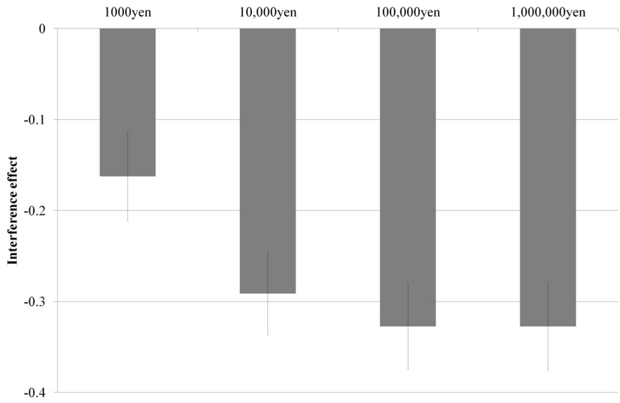

Figure 2. Interference effect in Newcomb's problem. The size of the interference effect (disjunction effect) increased as reward size of the prize in Black Box increased.

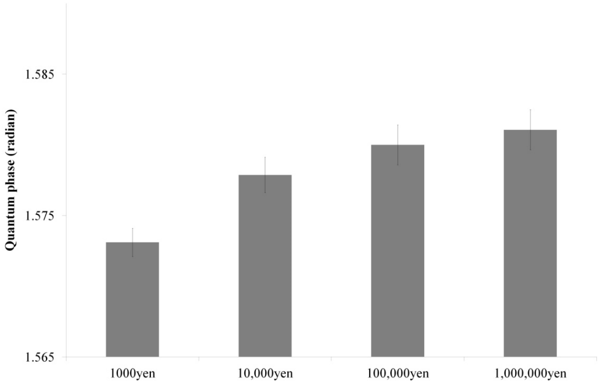

Figure 3. Quantum phase in Newcomb's problem increased as reward size of the prize in Black Box increased.

3.3 Effect of order of presenting magnitude

In order to explanatorily examine the effect of the two orders (a descending order and an ascending order) of presenting the four different magnitudes on subject's choice probabilities, we conducted a three-way ANOVA with reward size as a within participants' factor, knowledge of the state of the Black box (known empty / unknown / known full) as a within participants' factor and order (descending order/ascending order) of the presentation of the four different reward sizes (one million yen, 100,000 yen, 10,000yen, and 1,000 yen) as a between participants' factor. The ANOVA revealed that there was no significant effect of order ($F(1, 38) = 1.82$, $p = 0.185 > 0.05$) but the interaction among three factors was significant ($F(6, 228) = 4.50$, $p < 0.001$), indicating that there was no effect of the order alone but the combination of order, knowledge and magnitude affected subject's choice probabilities.

4. Conclusions and implications for quantum decision theory

As far as we know, this is the first study to demonstrate that law of total probability is violated for a wide range of reward sizes in the Black box in the Newcomb's problem and the violation of law of total probability is increased as reward size of the prize in the Black Box increased, and two-box choices are decreased as reward size in the Black Box increased, by adopting the quantum decision theoretical model.

Previous studies on the violation of Savage's sure-thing principle observed that the violation decreases as the magnitude of the reward (Li *et al.,* 2010) or as the variability of reward dependent on decision maker's choices. In contrast, our results show that the violation increased as the reward size in the uncertain box increased. This suggests that the decision makers are sensitive to a difference between certain and uncertain options.

A recent theoretical study proposed a quantum version of the Newcomb's box (Cavalcanti, 2010). According to the theoretical study, under quantum mechanical conditions, one box choice may be more rational than two box choice, due to non-causal quantum correlations between the prediction and decision-maker's choices. Future quantum decision theoretical studies therefore should examine the beliefs of decision makers who chose the one box option are more quantum than classical in terms of correlations and causalities (Gibbard and Harper, 1978; David Lewis, 1981) between events occurring in the universe.

Recent decision science studies proposed the field of computational psychiatry (Montague, 2012) to quantitatively model maladaptive decision making in psychiatric disorders. Our present study implies that Newcomb's problem could be utilized to capture psychiatric patients' impaired decision with a quantum phase parameter in computational psychiatry (Takahashi, 2012) and neuroeconomics (Takahashi, 2009).

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