Mathematical model of the dynamics of a freight car on the descent part of the marshaling yard

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Abstract.
Objective: to describe the speed of the car at each section of the descent part of the hump using the principles of classical mechanics; to get a generalized model of the speed of movement of the car at the estimated point of the hump, which allows speeding up the process of building graphs and changing the kinematic characteristics of the car's movement; to present the change in the instantaneous speed of movement of the car along the entire length of the descent part of the hump in the form of a graph of the step function. Research methods: the paper applies mainly theoretical research methods: analysis and modeling. The paper widely uses the methods of classical mechanics. Main results: Based on the application of a single impulse, a mathematical model has been developed for the movement of a car along the entire length of the descent part of the hump. The description of the dynamics of the car rolling from the hump, including sections of brake positions, in a generalized form was made for the first time. Conclusions and their significance for the industry: The change in the instantaneous speed of movement along the entire length of the descent part of the hump is presented in the form of a step function graph for the first time. The proposed model of a generalized mathematical notation of an instantaneous change in the speed of a car rolling down the descent part of the hump has a practical significance. This model allows calculating instantaneous values of the speeds of movement of the car from the top of the hump to the design point in a continuous mode, which makes it possible to accelerate the process of building graphs of changes in accelerations, speeds, and time of car's movement. The resulting model allows quickly analyzing the mode of shunting cars from the humps, the combination of power of brake positions and improve the accuracy of determining the permissible velocity of impact of cars in the sorting yards. This paper is the most important step for solving a promising task of designing an automated system for calculating the dynamic characteristics of a car in a hump yard.
Keywords:railway, station, marshalling hump, wagon, Heaviside discontinuous functions, a generalized mathematical notation of the instantaneous speed of a carriage along a slide profile
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1 Introduction

A series of publications are devoted to the problem of calculating and designing the profile of humps, for example, [1 - 13]. The existing method of calculating the humps [4] is based on the use of the concept of "power of brake positions - h_{br} or brake tools h_{brt}". The speed of movement of the car on all sections of the hump, including brake positions, is determined in [1 - 6, 9] by the formula $v=\sqrt{2g'h}$ (where g' - the acceleration of a freely falling body taking into account the inertia of rotating parts, $h = h_h$ - the height of the fall, and for sections of brake positions: $h = h_{br}$- the power of braking positions) (see p. 186 in [1]), applicable only for *ideal constraint* [14]. Thus, the fallacy of determining the energy height of the hump h_h is the use of the concept of an *ideal* constraint that is incompatible by the physical meaning to solving the problems of the humps, on which the constraints are in fact *non-ideal*.

In [13], it was noted that in [2, 3], the same method was used to determine the energy height of the hump h_h. They made the assumption that at any point on the inclined plane, the energy of a rolling body of mass $M$is equal to the sum of the potential E_p and the kinetic energy E_c. It is assumed that this energy is spent on the work of the forces of resistance to the motion A_r, i.e.$E_c + E_p = A_r$(see p. 8 in [2] and formula (6) in [3]). From here determine the energy height of the hump h_h. However, this approach is contrary to the law of energy conservation [14]. As can be seen from studies [1 - 5, 8], nowadays, the design of the energy height of the hump h_h is performed using the concept of "power of brake positions h_{br} or brake tools h_{brt}". The power of brake positions h_{br} is chosen according to the method [4], in which the free-fall velocity of bodies is used $v = \sqrt{2g'h}$, although $h = h_{br}$is the height of the braking zone of the brake section of the hump, which is an unknown value and should be determined.

In [10], in order to take real account of the operational conditions of the humps, it is recommended to use the parameters of specific resistances to the movement w, which reflect the generalized characteristics of the modern car fleet and the hump yards. Taking into account this factor, formula (2) is given in [10], which, according to the authors of the paper [10], has a developed universal form:

$$v_f^2 = v_{in}^2 + 2g'(i\text{-}w)10^{-3} \cdot l - 2g'h_b. \tag{1}$$

Where

$v_f = v_e$ – final or estimated speed of the car in the corresponding section of the hump, depending on the normalized value [v_{av}] (see Table 4.7 in [4]), m/s;

$v_{in} = v_{or}$ – initial speed or speed of rolling the car from the top of the hump, depending on the power of the hump (see Table 4.6 in [4]), m/s;

g' – acceleration of a freely falling body, taking into account the inertia of rotating parts, m/s^2;

i – slope of the studied section of the track, ‰;

w – specific resistance to the movement of the car in the studied sections of the track, kgf/t;

l – length of the studied section of the track, m;

h_b – height of a zone of braking of a brake section of a hump, m.

However, as the results of studies showed in [9, 12], formula (1) contains a number of inaccuracies and gross errors in its components. Although in [10], it is noted without substantiated evidence supported by calculations that formulas (1) and (2) in [10] can be used on any sections of the humps with a slope i, taking into account the presence of certain values of specific resistance to movement w and power of braking positions hbr (i.e. heights of sections of brake positions) (see the first paragraph of the last column on page 36 in [10]). In the opinion of the authors of the paper [10], the calculations of the humps, which simulate the conditions of movement of the designed runners with different running properties, are performed from this expression (see the first paragraph of the last column on page 36 in [10]). Also, in [10], it is indicated that "... any new proposed designed models for the movement of cars" should be compared with formulas (1) and (2) in [10] (see the second paragraph of the last column on page 36 in [10]).

Let us reveal some of the disadvantages of the formula (1). So, for example, it, as noted in [11], contains two completely disparate mathematical expressions describing the movement of a car in various parts of the hump, where the minuend values are valid for non-ideal constraints, and the subtrahend - for an ideal plane (constraint). Such an approach, as noted in [11], contradicts the elementary principles for solving engineering problems of theoretical mechanics: first, in order to simplify, it is necessary to solve the problem either for an ideal constraint (of course, not taking into account the inertia of rotating masses in the sections of brake positions, since when hump retarders are triggered, there is no rolling, but pure sliding of the wheelset), which is of no scientific and practical value; secondly, or for a non-ideal constraint, which is of scientific and practical interest.

Otherwise, the fallacy of the mathematical notation of formula (1) (see formula (2) in [10]) is that it cannot be given a universal form, "mechanically" combining the minuend and the subtrahend.

Thus, it was revealed that the question of exact mathematical modeling of the movement of a car on the descent part of the humps is still relevant.

Objective:

describe the speed of movement of the car at each section of the descent part of the hump, using the principles of classical mechanics;

get a generalized model of the speed of movement of the car at the estimated point of the hump, which allows speeding up the process of building graphs of the change of the kinematic characteristics of the car's movement;

present in the form of a graph of the step function the change in the instantaneous speed of movement of the car along the entire length of the descent part of the hump.

2 Research methods

Research methods are based on the basic law of the dynamics of a point with a non-ideal constraint (d'Almbert principle) [15] and include the following stages:

— at the first stage, the acceleration of the car's movement in all sections of the profile of the hump as the most important kinematic parameter of the car's movement, the magnitude of which directly determines the other movement parameters (time, speed and path of the studied sections), determined on the basis of the d'Alembert principle [6, 7, 11, 13 - 15], assuming that all active (gravitational force of a car with a load) and reactive (friction, resistance to the wind, resistance to movement from curves and switches, etc.) forces as forces in fractions of the gravitational force of the car with a load are computable values;

— at the second stage, the speed of movement of the car in each section of the hump profile [4] was found using well-known physics formulas [7, 11, 13], based on the fact that the entering speeds of the car for each section and the acceleration of movement in these sections are known;

— at the third stage, at the edges of the sections of the hump, the formulas of the instantaneous speed of movement of the car are interconnected by the method of matching ("stitching"), which is known in mechanics [13, 15 - 17];

— at the fourth stage, the change in the instantaneous speed of movement on the descent part of the hump is presented in the form of a step function graph [16, 18, 19];

— at the fifth stage, a generalized mathematical notation of the change in the instantaneous speed of a car rolling down the discharge section of the hump is presented in a compact form [16, 18, 19].

3 Research results

The formulas of the instantaneous speeds of the car on each section of the hump, according to the simplified method adopted in [6, 7, 11, 13, 17], are written in the form convenient for their calculation. At the same time, these formulas of the instantaneous speed of movement of the car at the edges of the sections can be interconnected by the matching method known in mechanics.

So, for example, let us present the formulas of the speed of the car for each i section of the hump (where *i* = 1,...,9 are the numbers of the sections of the hump) in the form:

— for the first high-speed section of the hump

$$v_{\mathrm{f1}}^2 = v_{\mathrm{in}}^2 + a_1 t_1 \le [v_{\mathrm{av2f}}]; \tag{2}$$

— for the second high-speed section of the hump to the switch

$$v_{\mathrm{f2}}^2 = v_{\mathrm{f1}}^2 + a_2 t_2 \le [v_{\mathrm{av2}S}]; \tag{3}$$

— for the second high-speed section of the hump after the switch

$$v_{\mathrm{f2S}}^2 = v_{\mathrm{f2}}^2 + a_{2S} t_{2S} \le [v_{\mathrm{av1br}}]; \tag{4}$$

— for the section of the first brake position

$$v_{1\mathrm{br}}^2 = v_{2\mathrm{f}}^2 - a_{1\mathrm{br}} t_{1\mathrm{br}} \leq [v_{\mathrm{av4}}]; \quad (5)$$

— for the intermediate section of the hump to the switch

$$v_{\mathrm{f4}}^2 = v_{1\mathrm{br}}^2 + a_4 t_4 \leq [v_{\mathrm{av4}S}]; \quad (6)$$

— for the intermediate section of the hump after the switch

$$v_{\mathrm{f4S}}^2 = v_{\mathrm{f4}}^2 + a_{4S} t_{4S} \leq [v_{\mathrm{av2br}}]; \quad (7)$$

— for the section of the second brake position

$$v_{2\mathrm{br}}^2 = v_{\mathrm{f4S}}^2 - a_{2\mathrm{br}} t_{2\mathrm{br}} \leq [v_{\mathrm{av6}}]; \quad (8)$$

— for the switching area of the hump to the first switch

$$v_{\mathrm{f6}}^2 = v_{2\mathrm{br}}^2 + a_6 t_6 \leq [v_{\mathrm{av6}S1}]; \quad (9)$$

— for the switching area of the hump after the first switch

$$v_{\mathrm{f6S1}}^2 = v_{\mathrm{f6}}^2 + a_{\mathrm{av6}S1} t_{6S1} \leq [v_{\mathrm{av6}S2}]; \quad (10)$$

— for the switching area of the hump to the second switch

$$v_{\mathrm{f6S2}}^2 = v_{\mathrm{f6S1}}^2 + a_{6S2} t_{6S2} \leq [v_{\mathrm{av6}S3}]; \quad (11)$$

— for the switching area of the hump to the third switch

$$v_{\mathrm{f6S3}}^2 = v_{\mathrm{f6S2}}^2 + a_{6S3} t_{6S3} \leq [v_{\mathrm{av3br}}]; \quad (12)$$

— for the first section of the marshalling yard track

$$v_{\mathrm{f7}}^2 = v_{\mathrm{f6S3}}^2 + a_7 t_7 \leq [v_{\mathrm{av3br}}]; \quad (13)$$

— for the section of the third brake position

$$v_{\mathrm{f3br}}^2 = v_{\mathrm{f7}}^2 - a_{3\mathrm{br}} t_{3\mathrm{br}} \leq [v_{\mathrm{av9}}]; \quad (14)$$

— for the second section of the marshalling yard track

$$v_{\mathrm{f9}}^2 = v_{\mathrm{f3br}}^2 + a_9 t_9 \leq [v_{\mathrm{avEP}}]. \quad (15)$$

where

$v_{\mathrm{in}} = v_{\mathrm{or}}$ – as in the formula (1), the initial speed or speed of rolling the car from the top of the hump, depending on the power of the hump (see Table 4.6 in [4]), m/s;

v_{ei} – the estimated (or instantaneous) speed of the car in the corresponding *i* section of the hump, m/s;

a_i – the acceleration of the car's movement in the corresponding *i* section of the hump, calculated value [7, 11, 13, 17] (see, for example, formula (10) in [6] for the high-speed sections of the hump, for sections of brake positions - (6) in [11]), m/s;

t_i– the time of movement of the car in the corresponding *i* section of the hump, calculated in the high-speed sections of the hump according to the formula of elementary physics (see formula (11) in [7], and in sections of brake positions - according to formula (11) in [11]), s;

$[v_i]$ – permissible entering speed of the car to the studied section of the hump (see Table 4.7 in [4]), m/s.

Note that in (2) - (4), (6), (7), (9) - (13), (15) the acceleration of the car a_i is calculated by the formula (10) in [7], and in (5), (6), and (14) - according to the formula (6) in [11], while according to the methodology of works [1–5, 8, 10], there is even no mention of this.

The existing method for calculating humps (see formula (2) in [10]) and method proposed by the authors [7, 11, 13, 17] have distinctive features. Each of them, naturally, has some assumptions, as it is usually in mathematics and mechanics.

Instead of formulas (2) - (15), let us present a generalized view of the mathematical model of the simplified method of the authors' hump calculations [7, 11, 13], considering that the first, second, and third sections of brake positions (1BP, 2BP, and 3BP), for example, can be divided into three conditional sections (the zone of the car's wheelbase entrance (WB), the braking zone of the car (BZ) up to the stop, and the remaining section (AB) after braking, corresponding to the brake position of the car retarder), and representing the movement of the car along the descent of the hump in the form of a Heaviside unit impulse (and/or jump) $f(t)$ or $\sigma_0(t)$ [16, 18, 19].

It should be borne in mind that the task of determining the time t_{br} and the path l_{br} of braking in the braking zone of the car (BZ) until it stops ($v_{br} = 0$) was solved in [11] for the first time.

Considering that the car can be braked, *firstly*, directly when, for example, the front wheelset enters brake positions and/or, *secondly*, when the wheelset of the front carriage enters brake positions, the braking zone can also consist of two sections - the car's braking zone (BZ) (v_{br} = const) up to the stop ($v_{br} = 0$) and the remaining section (AB) after braking (v_{AB} = const).

In addition, in the first and second brake positions (1BP and 2BP), where, as a rule, two car retarders are installed, the braking zone can consist of five sections (car wheelbase entry area (WB) (v_{WB} = const), zone of the first braking of the car (1BZ) (v_{1br} = const) up to the stop ($v_{br} = 0$), and the remaining section (1AB) after braking (v_{1AB} = const), the zone of the second braking of the car (2BZ) (v_{2br} = const.) up to the stop ($v_{2br} = 0$), and the remaining section (2AB) after braking (v_{2AB} = const). Also, the braking zone may consist of four sections - zones of the first braking (1BZ) (v_{1br} = const) up to the stop ($v_{1br} = 0$) and the remaining section (1AB) after braking (v_{1AB} = const), the area of the second braking of the car (2BZ) (v_{2br} = const.) up to the stop ($v_{2br} = 0$), and the remaining area (2AB) after braking (v_{2AB} = const).

In sections of the third braking position (3BP), in which one car retarder is usually installed, the braking zone can consist of two sections: the car braking zone (BZ) (v_{3BP} = const) up to the stop ($v_{3BP} = 0$) and the remaining section (AB) after braking (v_{AB} = const).

The change in the instantaneous speed of movement along the entire length of the descent part of the hump can be represented as a graph of the step function. The graph of the unit impulse $f(t)$ in the form of a step function can be represented as an example in Fig. 1.

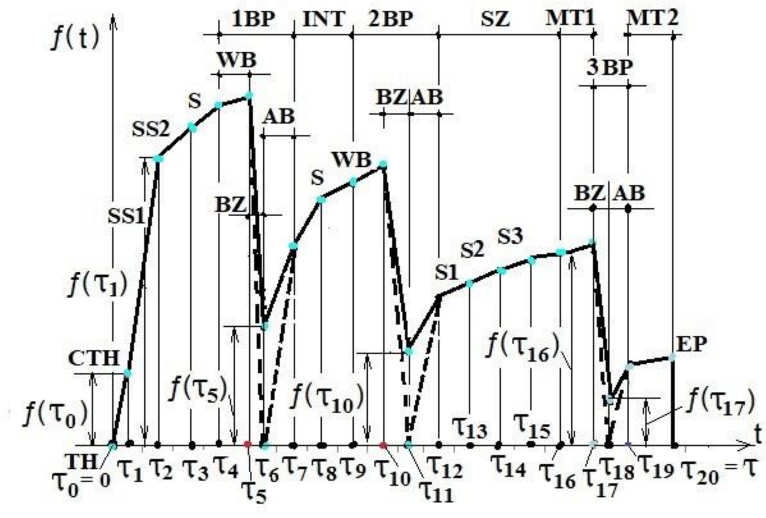

Fig. 1. Graph of the step function representing the change in acceleration and speed of movement along the entire length of the descent part of the hump

In Fig. 1 as well as in Table. 1, Fig. 1 – 3 in [13], the following is denoted:

TH - top of the hump;

CTH - top of the hump, represented by the unit impulse $f(\tau_0)$;

SS1 and SS2 - the first and second speed sections of the hump, represented by unit impulses $f(\tau_1)$ and $f(\tau_2)$;

1BP, 2BP, and 3BP - the first, second, and third brake positions of the hump, represented by unit impulses $f(\tau_4) - f(\tau_6)$, $f(\tau_9) - f(\tau_{11})$, $f(\tau_{16})$, and $f(\tau_{17})$;

INT - intermediate section of the hump, represented by unit impulses $f(\tau_8)$ and $f(\tau_9)$;

SZ - switching zone of the hump, represented by unit impulses $f(\tau_{13}) - f(\tau_{15})$;

MT1 and MT2 - the first and second sections of the marshalling yard track, represented by unit impulses $f(\tau_{16})$ and $f(\tau_{19})$;

S - separation switches represented by unit impulses $f(\tau_3)$ and $f(\tau_7)$;

S1, S2, and S3 – the first, second, and third switches represented by unit impulses $f(\tau_{12})$, $f(\tau_{13})$, and $f(\tau_{14})$;

WB - sections of accounting for the length of the car's wheelbase, represented by unit impulses $f(\tau_4)$ and $f(\tau_9)$;

BZ – car's braking zones represented by unit impulses $f(\tau_5)$, $f(\tau_{10})$, and $f(\tau_{17})$;

AB - remaining parts of the braking positions, represented by unit impulses $f(\tau_6)$, $f(\tau_{11})$ and $f(\tau_{18})$, and corresponding to the braking position of the car retarder;

EP - estimated point represented by the unit impulse $f(\tau_{19})$, with the exception of t - time of the car's movement; τ_j - fixed numbers (j = 1, ..., 19 - numbers of sections of the track profile of the hump).

Besides, the dash-dotted lines in the zones of direct braking (BZ) of the car in the brake positions (1BP, 2BP, and 3BP) correspond to the uniformly decelerated motion of the car (with acceleration $|a_{1br}| = -a_{1br} = -a_5 < 0$, $|a_{10br}| = -a_{2br} = -a_{10} < 0$, and $|a_{17br}| = -a_{3br} = -a_{17} < 0$) and with full use of the power of brake positions, when it is necessary to achieve a complete stop of the car, i.e. in cases when $v_{1br} = v_{2br} = v_{3br} = 0$, and solid lines - to a partial braking of the car when the car moves with acceleration $|a_{1br}| = a_5 < 0$, $|a_{10br}| = a_{10} < 0$, and $|a_{17br}| = a_{17} < 0$ and with the speed $v_{1br} = v_{1BZ}$ = const > 0, $v_{2br} = v_{2BZ}$ = const > 0, and $v_{3br} = v_{5BZ}$ = const > 0.

In this case, we use the fact that the unit impulse (and/or jump) $f(t)$ [18] can be considered as the difference between two unit and/or discontinuous Heaviside functions [13, 17]:

$\boldsymbol{\sigma_0(t)}$ and $\boldsymbol{\sigma_0(t-\tau)}$, i.e. $\boldsymbol{f(t)} = \boldsymbol{\sigma_0(t)} - \boldsymbol{\sigma_0(t-\tau)}$,

and $\tau = \tau_1$, $\tau = \tau_2 > \tau_1$; $\tau = \tau_3 > \tau_2$,....., $\tau = \tau_{19} > \tau_{18}$.

Also, we will keep in mind that when $t < 0$: $f(t) = 0$ - the origin of coordinates;

in the time interval $0 \le t \le \tau_1$: $f(t) = f(\tau_0)$ – top of the hump (TH);

in the time interval $\tau_1 \le t \le \tau_2$: $f(t) = f(\tau_1)$ – first speed section (SS1);

in the time interval $\tau_2 \le t \le \tau_3$: $f(t) = f(\tau_2)$ – second speed section (SS2);

in the time interval $\tau_2 \le t \le \tau_3$: $f(t) = f(\tau_3)$ – separation switch (S), …;

in the time interval $\tau_5 \le t \le \tau_6$: $f(t) = f(\tau_5)$ – braking zone of the first brake position (1BP), ….;

in the time interval $\tau_8 \le t \le \tau_9$: $f(t) = f(\tau_8)$ – intermediate section (INT), ….;

in the time interval $\tau_{10} \le t \le \tau_{11}$: $f(t) = f(\tau_{10})$ – braking zone of the second brake position (2BP), ….;

in the time interval $\tau_{17} \le t \le \tau_{18}$: $f(t) = f(\tau_{17})$ – braking zone of the third brake position (3BP);

in the time interval $\tau_{18} \le t \le \tau_{19}$: $f(t) = f(\tau_{18})$ – remaining section (AB) of the third brake position (3BP);

in the time interval $\tau_{19} \le t \le \tau_{20}$: $f(t) = f(\tau_{19})$ – second section of the marshalling yard track

and when $t > \tau_{20}$: $f(t) = 0$ – estimated point (EP).

In addition, we note that unit impulses импульсы $f(\tau_0)$, $f(\tau_1)$, …., $f(\tau_4)$, $f(\tau_6)$, …., $f(\tau_9)$, $f(\tau_{11})$, $f(\tau_{16})$, $f(\tau_{18})$, and $f(\tau_{19})$ characterize the uniformly accelerated motion of the car with acceleration $a_k > 0$ (where k = 0, 1, ..., 4, 6, ..., 9, 11, ..., 16, 18, and 19), at which it is accelerated in time intervals $\tau_0 = 0 \le t \le \tau_1$, $\tau_1 \le t \le \tau_2$, … , $\tau_3 \le t \le \tau_4$, $\tau_6 \le t \le \tau_7$, … , $\tau_9 \le t \le \tau_{10}$, $\tau_{11} \le t \le \tau_{12}$, … , $\tau_{16} \le t \le \tau_{17}$, $\tau_{18} \le t \le \tau_{19}$, and $\tau_{19} \le t \le \tau_{20}$.

Unit impulses $f(\tau_5)$, $f(\tau_{10})$, and $f(\tau_{17})$ characterize the uniformly decelerated motion of the car with acceleration $|a_j|$ <0 (where j = 5, 10, and 17), at which it is braked in the time interval $\tau_5 \le t \le \tau_6$, $\tau_{10} \le t \le \tau_{11}$, and $\tau_{17} \le t \le \tau_{18}$.

Below we show the mathematical notations of the instantaneous speed of the car in the most characteristic sections of the descent part of the humps.

Let's assume that, for example, the speed of car rolling $v_{\text{in}}(t) = f(t)$ before the time moment t = 0 was equal to zero ($v_{\text{in}}(t) = f(t) = 0$), and then, in the time interval $\tau_0 = 0 \le t \le \tau_1$, it took the value $v_{\text{in}}(t) = f(t)$ = const, and, starting from the moment $t = \tau_1$, the car starts its motion uniformly accelerated with acceleration a_1> 0, picking up speed v_{SS1} = const> 0 (see the initial part of Fig. 1).

Mathematically, this can be written in the following form [13, 16, 17]:

$$v_{\text{in}}(t) = f(t) = \begin{cases} 0 & \text{when } t = \tau_0 = 0, \\ f(t) = \quad f(\tau_0) = v_{\text{in}} = \text{const} & \text{when } \tau_0 = 0 \le t \le \tau_1, \\ f(\tau_1) = v_{\text{SS1}} & \text{when } \quad t > \tau_1. \end{cases} \quad (16)$$

Let us give a mathematical notation of the change in the instantaneous speed of the car v_{SS1} in the first speed section (SS1) of the hump (see Fig. 1). The car enters this section with the initial speed v_{in} = const, and then, in the time interval (τ_1, τ_2), it takes the value $f(t) = f(\tau_1) = v_{\text{SS1}}$ = const > 0, moving *uniformly accelerated* with acceleration a_2> 0, and, starting from the moment $t = \tau_2$, the car continues its movement equally accelerated with acceleration a_3> 0, picking up speed v_{S} = const> 0. In this case, the mathematical notation has the following form [13, 16, 17]:

$$v_{\text{fSS1}}(t) = f(t) = \begin{cases} f(\tau_1) = v_{\text{in}} & \text{when } t < \tau_1, \\ f(t) = \quad f(\tau_1) = v_{\text{SS1}} = \text{const} & \text{when } \tau_1 \le t \le \tau_2, \\ f(\tau_2) = v_{\text{S}} & \text{when } t > \tau_2. \end{cases} \quad (17)$$

Let us describe the change in the instantaneous speed of the car using the example of the first brake position (1BP), bearing in mind that this section consists of three zones: the wheelbase zone of the car (WB), the braking zone (BZ), and the remaining section after braking (AB) (see 1BP section in Fig. 1).

The speed of the car entrance to WB v_{en4WB} is equal to the speed of exit from the section of the separation switch (S) v_{exS} = const> 0, and in the time interval $\tau_4 \le t \le \tau_5$, the car moves uniformly accelerated with acceleration a_4> 0 and speed $f(t) = f(\tau_4) = v_{\text{4WB}}$ = const> 0.

Further, a car retarder is switched on for braking a car that moves with an initial speed $f(t) = f(\tau_5) = v_{\text{in1br}} = v_{\text{in5WB}}$ = const> 0, after which it moves *uniformly decelerated* with acceleration $a_{\text{1br}} = a_5$ = const < 0 (where $|a_5| = -a_5$) and with the speed $f(t) = f(\tau_5) = v_{\text{1br}} = v_{\text{5BZ}}$ = const> 0 if the full power of the car retarder was not used, otherwise, the equality $v_5 = v_{\text{5BZ}} = 0$ should be observed.

In Fig. 1, the case when the condition $v_{\text{5br}} = v_{\text{5BZ}}$ = const > 0 is met is shown by a linearly decreasing solid line, and the case when the equality $v_{\text{5br}} = v_{\text{5BZ}} = 0$ is observed is represented by a linearly increasing dash-dotted line.

Further, the car moves along the remaining length of the section after braking (AB) (see section 1BP in Fig. 1).

The speed of the car entrance to the AB v_{en6AB} is equal to the speed of its exit from the BZ, i.e.$v_{\text{en6AB}} = v_{\text{exBZ}}$ = const> 0, and in the time interval $\tau_6 \le t \le \tau_7$, the car moves uniformly accelerated with acceleration a_6 = const> 0 and speed $f(t) = f(\tau_6) = v_{\text{6AB}}$ = const> 0.

In these cases, the mathematical notation of the instantaneous speeds of the car is as follows:

— when the car moves in the section equal to the wheelbase of the car (WB)

$$v_{\text{fWB}}(t) = f(t) = \begin{cases} f(\tau_4) = v_{\text{S}} & \text{when} \quad t < \tau_4, \\ f(t) = \quad f(\tau_4) = \text{const} & \text{when}\ \tau_4 = 0 \le t \le \tau_5, \\ f(\tau_5) = v_{\text{1br}} & \text{when} \quad t > \tau_5; \end{cases} \tag{18}$$

— when the car is braked (BZ) in the case of incomplete use of the power of the brake positions (is represented by a solid line in Fig. 1)

$$v_{\text{f1br}}(t) = f(t) = \begin{cases} f(\tau_5) = v_{\text{WB}} & \text{when}\ t < \tau_5, \\ f(t) = \quad f(\tau_5) = v_{\text{1br}} = \text{const} & \text{when}\ \tau_5 \le t \le \tau_6, \\ f(\tau_6) = v_6 & \text{when}\ t > \tau_6; \end{cases} \tag{19}$$

— when braking the car (BZ) in case of full use of the power of the brake positions (it is represented by the dash-dotted line in Fig. 1)

$$v_{\text{f1br}}(t) = f(t) = \begin{cases} f(\tau_5) = v_{\text{WB}} & \text{when}\ t < \tau_5, \\ f(t) = \quad f(\tau_5) = 0 & \text{when}\ \tau_5 \le t \le \tau_6, \\ f(\tau_6) = v_6 & \text{when}\ t > \tau_6; \end{cases} \tag{20}$$

— when the car moves along the remaining length of the brake positions (AB)

$$v_{\text{fAB}}(t) = f(t) = \begin{cases} f(\tau_6) = v_6 & \text{when}\ t < \tau_6, \\ f(t) = \quad f(\tau_6) = \text{const} & \text{when}\ \tau_6 \le t \le \tau_7, \\ f(\tau_7) = v_{\text{S}} & \text{when}\ t > \tau_7. \end{cases} \tag{21}$$

Similarly, it is possible to write down the instantaneous speed of movement of the car in other sections of the hump.

Let us also describe the change in the instantaneous speed of the car along the second section of the marshalling yard track (MT2) (see the last section of Fig. 1). The speed of the car entrance to this section of the hump is equal to the exit speed from the AB section of the third brake position (3BP) $v_{3AB} = v_{en19}$ = const > 0, in case if the full power of the car retarder was not used, otherwise, the entrance speed of the car would be $v_{en19} = 0$.

In Fig. 1, cases in which the conditions $v_{en19} = v_{3AB}$ = const > 0 and $v_{en19} = v_{3AB} = 0$ are met are shown by linearly increasing solid lines and dash-dotted lines, respectively.

In the time interval $\tau_{19} \le t \le \tau_{20}$, the car moves uniformly accelerated with acceleration $a_{20} > 0$ and with speed скоростью $f(t) = f(\tau_{19}) = v_{MT2}$ = const> 0, and, starting from the moment $t = \tau_{20}$, it stops its movement after estimated point (EP).

A generalized mathematical notation of the simplified method of the authors' hump calculations [8, 12, 14], corresponding, in a particular case, to the graph of impulse functions in Fig. 1, which characterizes the change in the instantaneous speed of rolling (and in the deceleration sections - sliding speed) of the car along the descent part of the hump $v(t)$, unlike the unsuccessfully and incorrectly presented expanded universal form of formula (2) in [10], we present in the form of a graph of a step function [16, 17]:

$$
\begin{aligned}
& v_{\mathrm{f}}^{2} = v_{\mathrm{in}}^{2}(\sigma_0(t)-\sigma_0(t-\tau_0)) + a_1 t_{1\mathrm{SS}1}(\sigma_0(t)-\sigma_0(t-\tau_0-\tau_1)) + \\
& +a_{2\mathrm{SS}2}t_{2\mathrm{SS}2}(\sigma_0(t)-\sigma_0(t-\tau_0-\tau_1-\tau_2)) + \\
& +a_{3\mathrm{S}}t_{3\mathrm{S}}(\sigma_0(t)-\sigma_0(t-\tau_0-\tau_1-\tau_2-\tau_3)) + \\
& +a_4 t_4(\sigma_0(t)-\sigma_0(t-\tau_0-\tau_1-\tau_2-\tau_3-\tau_4)) - \\
& -a_{1\mathrm{br}}t_{1\mathrm{br}}(\sigma_0(t)-\sigma_0(t-\tau_0-\tau_1-\tau_2-\tau_3-\tau_4-\tau_5)) + \\
& +a_6 t_6(\sigma_0(t)-\sigma_0(t-\tau_0-\tau_1-\tau_2-\tau_3-\tau_4-\tau_5-\tau_6)) + \\
& +a_{7\mathrm{S}}t_{7\mathrm{S}}(\sigma_0(t)-\sigma_0(t-\tau_0-\tau_1-\tau_2-\tau_3-\tau_4-\tau_5-\tau_6-\tau_7)) + \\
& +a_8 t_8(\sigma_0(t)-\sigma_0(t-\tau_0-\tau_1-\tau_2-\tau_3-\tau_4-\tau_5-\tau_6- \\
& -\tau_7-\tau_8)) + a_9 t_9(\sigma_0(t)-\sigma_0(t-\tau_0-\tau_1-\tau_2-\tau_3-\tau_4- \\
& -\tau_5-\tau_6-\tau_7-\tau_8-\tau_9)) - a_{2\mathrm{br}}t_{2\mathrm{br}}(\sigma_0(t)-\sigma_0(t-\tau_1-\tau_2-\tau_3- \\
& -\tau_4-\tau_5-\tau_6-\tau_7-\tau_8-\tau_9-\tau_{10})) + a_{11}t_{11}(\sigma_0(t)-\sigma_0(t- \\
& -\tau_1-\tau_2--\tau_3-\tau_4-\tau_5-\tau_6-\tau_7-\tau_8-\tau_9-\tau_{10}-\tau_{11})) + \\
& +a_{12\mathrm{S}1}t_{12\mathrm{S}1}(\sigma_0(t)-\sigma_0(t-\tau_1-\tau_2-\tau_3-\tau_4-\tau_5-\tau_6- \\
& -\tau_7-\tau_8-\tau_9-\tau_{10}-\tau_{11}-\tau_{12})) + a_{13\mathrm{S}2}t_{13\mathrm{S}2}(\sigma_0(t)-\sigma_0(t-\tau_1- \\
& -\tau_2-\tau_3-\tau_4-\tau_5-\tau_6-\tau_7-\tau_8-\tau_9-\tau_{10}-\tau_{11}-\tau_{12}-\tau_{13})) + \\
& +a_{14\mathrm{S}3}t_{14\mathrm{S}3}(\sigma_0(t)-\sigma_0(t-\tau_1-\tau_2-\tau_3-\tau_4-\tau_5-\tau_6-\tau_7-\tau_8- \\
& -\tau_9-\tau_{10}-\tau_{11}-\tau_{12}-\tau_{13}-\tau_{14})) + a_{15}t_{15}(\sigma_0(t)-\sigma_0(t-\tau_1- \\
& -\tau_2-\tau_3-\tau_4-\tau_5-\tau_6-\tau_7-\tau_8-\tau_9-\tau_{10}-\tau_{11}-\tau_{12}-\tau_{13}- \\
& -\tau_{14}-\tau_{15})) + a_{16}t_{16}(\sigma_0(t)-\sigma_0(t-\tau_1-\tau_2-\tau_3-\tau_4- \\
& -\tau_5-\tau_6-\tau_7-\tau_8-\tau_9-\tau_{10}-\tau_{11}-\tau_{12}-\tau_{13}-\tau_{14}-\tau_{15}-\tau_{16})) + \\
& -a_{3\mathrm{br}}t_{3\mathrm{br}}(\sigma_0(t)-\sigma_0(t-\tau_1-\tau_2-\tau_3-\tau_4-\tau_5-\tau_6-\tau_7-\tau_8- \\
& -\tau_9-\tau_{10}-\tau_{11}-\tau_{12}-\tau_{13}-\tau_{14}-\tau_{15}-\tau_{16}-\tau_{17}) + \\
& +a_{18}t_{18}(\sigma_0(t)-\sigma_0(t-\tau_1-\tau_2-\tau_3-\tau_4-\tau_5-\tau_6-\tau_7-\tau_8- \\
& -\tau_9-\tau_{10}-\tau_{11}-\tau_{12}-\tau_{13}-\tau_{14}-\tau_{15}-\tau_{16}-\tau_{17}-\tau_{18})) + \\
& +a_{19}t_{19}(\sigma_0(t)-\sigma_0(t-\tau_1-\tau_2-\tau_3-\tau_4-\tau_5-\tau_6-\tau_7-\tau_8- \\
& -\tau_9-\tau_{10}-\tau_{11}-\tau_{12}-\tau_{13}-\tau_{14}-\tau_{15}-\tau_{16}-\tau_{17}-\tau_{18}-\tau_{19})).
\end{aligned}
\tag{22}
$$

and/or let us present a generalized mathematical notation of the change in the instantaneous speed of a car rolling down the descent part of the hump in a more compact form [15-17]:

$$
\begin{aligned}
& v_{\mathrm{f}}^{2} = v_{\mathrm{in}}^{2} f(\tau_0) + a_1 t_{1\mathrm{SS}1} f(\tau_1) + a_{2\mathrm{SS}2}t_{2\mathrm{SS}2} f(\tau_2) + a_{3\mathrm{S}}t_{3\mathrm{S}} f(\tau_3) + \\
& +a_4 t_4 f(\tau_4) - a_{1\mathrm{br}}t_{1\mathrm{br}} f(\tau_5) + a_6 t_6 f(\tau_6) + a_{7\mathrm{S}}t_{7\mathrm{S}} f(\tau_7) + \\
& +a_8 t_8 f(\tau_8) + a_9 t_9 f(\tau_9) - a_{2\mathrm{br}}t_{2\mathrm{br}} f(\tau_{10}) + a_{11}t_{11} f(\tau_{11}) + \\
& +a_{12\mathrm{S}1}t_{12\mathrm{S}1} f(\tau_{12}) + a_{13\mathrm{S}2}t_{13\mathrm{S}2} f(\tau_{13}) + a_{14\mathrm{S}3}t_{14\mathrm{S}3} f(\tau_{14}) + \\
& +a_{15}t_{15} f(\tau_{15}) + a_{16}t_{16} f(\tau_{16}) - a_{3\mathrm{br}}t_{3\mathrm{br}} f(\tau_{17}) + a_{19}t_{19} f(\tau_{18}).
\end{aligned}
\tag{23}
$$

It should be noted that the mathematical notation of the instantaneous speeds of the car (22) and/or (23) corresponds to the case when the car is moving relative to the top of the hump (TH) uniformly accelerated with the set speed of rolling (v_{in} = const> 0, for example, v_{in} = 0.8, …, 1.7 m/s, depending on the hump capacity (see Table 4.6 in [4]).

In this case, the time interval $\Delta\tau_0 = \tau_1 - \tau_0$ is very small and can be considered almost equal to zero (see Fig. 1). The case is also considered when the car moves uniformly decelerated (v_{brk} = const ≤ 0) in the i sections of brake positions (1BP, 2BP, and 3BP) with the turned on car retarder in the braking zones.

Here, the time intervals $\Delta\tau_8 = \tau_9 - \tau_8$, $\Delta\tau_{10} = \tau_{11} - \tau_{10}$, and $\Delta\tau_{17} = \tau_{18} - \tau_{17}$ are very small (for example, from 1 to 3 s) and can be considered almost equal to zero, since for a negligibly small period of time, the loaded car picks up speed v_{bri} = const and also quickly stops to $v_{bri} = 0$, continuing to pick up speed v_{iAB} = const in the remaining sections (AB) of the brake positions (see Fig. 1).

On the other speed sections of the hump (SS1, SS2, S, WB, AB, INT, BZ, MT1, and MT2), the calculated runner moves uniformly accelerated with average speeds v_{avi} = const not exceeding the established average speeds of the car [v_{av}] depending on the hump capacity (see Table 4.7 in [4]) for a sufficiently noticeable period of time, for example, $\Delta\tau_2 = \tau_3 - \tau_2 >> 1$ s.

Also note that in the mathematical notation of the instantaneous speeds of the car (23), as well as in formulas (2) - (4), (6), (7), (9) - (13), (15), the acceleration of the car a_k, according to the d'Alembert principle, is calculated by formula (10) in [7].

The time of movement of the car t_i is found according to the formula of elementary physics (see formula (11) in [7]) from the dependence $t_i = f(v_{0i}, |a_i|, l_i)$ (where l_i is the length of the section under study) in the i-th section of the descent part of the hump, except parts of the brake position. Usually, in the zones of braking, the full power of the brake positions is used, ensuring full stop of the car. Therefore, the time of movement t_{bri} and the path of braking l_{bri} of the braked car are found from the condition that the braking speed is zero, i.e.$v_{bri} = 0$ (see formulas (10) and (11) in [11]).

As can be seen, the mathematical notation (22) and/or (23) of the instantaneous speeds of a car of the simplified method of the authors' hump calculations [7, 11, 13], presented in a generalized form, has a significant difference from the expanded universal form of formula (2) in [10], which has significant inaccuracies.

4 Discussion

Thus, on the basis of the conducted studies, we especially note the following results:

1. The formulas for the instantaneous speeds of the car for each section of the hump yard are presented in a convenient form for practical use.
2. The change in the speed of movement of the car on the entire profile of the descent part of the hump is presented in the form of a step function graph.
3. Using Heaviside unit functions in a compact, simplified form, a generalized mathematical notation of the change in the instantaneous speeds of the car rolling down the descent part of the hump is presented.

The presented paper summarizes the results of previously published papers (see, for example, [7, 11, 13, 18]).

The proposed model of a generalized mathematical notation of an instantaneous change in the speed of a car rolling down the descent part of the hump enriches the theory of rolling a car along the descent part of the hump. This model is of practical importance, since it allows calculating instantaneous values of speeds of the car's movement from the top of the hump to the estimated point in a continuous mode, allowing us to speed up the process of plotting the graph of changes in accelerations, speeds, and time of car's movement (see, for example, [17, 19, 20]). The presented model allows quickly analyzing the mode of rolling cars from the humps, the combination of capacity of brake positions, and improving the accuracy of determining the permissible rates of collision of cars in the hump yards.

This paper is the most important step for solving a promising task of designing an automated system for calculating the dynamic characteristics of a car in the hump yard.

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