



# Effect of Fractional Profile-index on Nonlinearity and Dispersion in Single-Mode Optical Fibers

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## Abstract

In this paper, some properties of optical fiber have been studied numerically by changing the values of graded-index as non-integer numbers using the finite element method (FEM) in the COMSOL environment. The results showed that the small graded orders gives a strange properties, such as a great value at nonlinearity and a new forms of dispersion relation are obtained. The paper enables us to choose the graded order and wavelength to achieve the desired values of dispersion and nonlinearity.

**Key Words:** Graded Order, Single Mode Fiber, Nonlinearity, Dispersion.

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## Introduction

Light propagation in graded-index (GR-IN) media was investigated during the mid-1970s, motivated mostly by their applications in optical communication systems [1,2]. GR-IN fibers constitute an important transmission medium for optical signals. Except for a few special refractive index profile shapes that admit of explicit field solutions, the guided modes capable of propagating along the fiber must be determined by approximate methods [3]. In this type of fiber the refractive index continuously decreases from the central axis to the cladding. It is for example possible to design fibers with a parabolic refractive index profile, where the rays are continuously deflected instead of propagating along straight lines between two reflections. In this case, the intermodal dispersion induced by the fiber is decreased because the difference between the optical paths of small and large incident angles is minimized. In addition, the numerical aperture ( $NA$ ) varies as a function of the radial distance ( $\rho$ ). It is maximum at the center and decreases when the rays enter closer to the cladding [4]. The number of propagated modes

depends on the core size and  $NA$ . However, when the last parameters decrease, the number of modes decrease too. Although, fiber imperfections, like index inhomogeneity, core ellipticity and eccentricity, and bends, introduce coupling between modes, even if a light pulse is launched into a single mode, it tends to couple to other modes. That means, any change or fluctuations of such fiber parameters will change and affect the characteristics of the beam propagating along the fiber, although GR-IN silica fibers are rarely used for long-haul links, the use of this type of fiber has decreased, it has many applications in the medical field and as biosensor [5,6].

There are no closed-form analytical solutions for the mode fields of GR-IN fibers with arbitrary refractive index distributions [2,7]. For this reason, in this work, we decided to explore numerically the effect of arbitrary values of graded orders on some an important properties of single mode-fibers.

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**Theory**

Consider the propagation of a continuous-wave beam of frequency  $\omega$  inside a GR-IN fiber. the

$$n^2(x, y) = n_1^2 [1 - 2\Delta(\rho/a)^g] \tag{1}$$

where  $\rho = \sqrt{x^2 + y^2}$ , the parameter  $\Delta = (n_1 - n_2)/n_1$ ,  $n_1$  is the refractive index in the center of the core,  $n_2$  the refractive index of the cladding ( $n_1\sqrt{1 - 2\Delta} = n_2$ ),  $a$  is the core's radius, and  $g$  is the graded-order or so-called the profile index. All those parameters dependent on the dopant and its concentration. Therefore, the above equation May present a certain nonlinearity that the index-profile will obey with a single  $g$ .

$$u_{n,m}(\rho, \varphi, z) = N_{nm} e^{i(n\varphi + \beta_{nm}z)} \begin{cases} \frac{J_n(A_{nm}\rho/a)}{J_n(A_{nm})} & , \text{ core} \\ \frac{K_n(B_{nm}\rho/a)}{J_n(B_{nm})} & , \text{ clad} \end{cases} \tag{2}$$

where  $N_{nm}$  is a normalization constant,  $J_n$  and  $K_n$  are the first-kind Bessel function and the modified Bessel function of second-kind, respectively, the integer number  $n$  represents the order of Bessel function and the integer  $m$  will be mentioned later.

$$V = k_o a \sqrt{n_1^2 - n_2^2} \tag{3}$$

In  $(\rho, \varphi, z)$ , the  $V$  is related with the normalized propagation constant  $A$  and  $B$  for the core and cladding, respectively, as the form  $V^2 = A^2 + B^2$ .

$$\frac{AJ_n(A)}{J_{n+1}(A)} = \frac{BK_n(B)}{K_{n+1}(B)} \tag{4}$$

From the intersection order between the graphs Eq.(4) and  $V^2 = A^2 + B^2$ , one may determine  $m$ .

In GR-IN, the modal dispersion is computed by a complicate procedure which depend on many approximations, where the refractive index is a function of radius, the multimode optical fibers is

$$\frac{\eta'_{core}(R)}{\eta_{core}(R)} = - \frac{\ell K_\ell(w) + w K_{\ell-1}(w)}{K_\ell(w)} \tag{5}$$

where  $R = \rho/a$  and  $\ell$  counts the number of maximum intensities that may appear in the radial. The fiber dispersion plays a critical role in the propagation of short optical pulses because different spectral components associated with the pulse travel at different. Dispersion induced pulse broadening. In the nonlinear regime, the combination of dispersion and nonlinearity can

$$D_{ch} = D_M + D_W \tag{6}$$

where  $D_M, D_W$  are the material dispersion and the

power- low or so called profile-grading can be defined as[1,4]

For weakly guiding approximation the polarization is linear and both the electric and magnetic fields lie in a plane transverse to the fiber's axis. Their modal distribution  $F_{nm}(x, y)$  and propagation constant  $\beta_{nm}$  are known. Because of the cylindrical symmetry of fibers, it is useful to express the  $F_{nm}(x, y)$  in cylindrical coordinates  $(\rho, \varphi, z)$ , so the field distribution has the form [7,8]

The number of modes supported by a specific fiber at a given wavelength depends on its design parameters, such as ( $a$  and  $\Delta$ ). An important parameter for each mode is the normalized frequency ( $V$ ), which is defined as [4]

And, for all modes one can find values of these parameters using the relation of dispersion [8]

described by  $\eta_i(\rho, \varphi, z) = D_i(\rho, \varphi)e^{i(wt - \beta_i z)}$ . Consequently, the general solution when  $g$  takes an integer values that essentially depending on Laguerre polynomial will be [10,11]

result in a qualitatively different behavior. The dispersion in single-mode fiber comes from the fact that the refractive index of the material used to make an optical fiber is a function of the wavelength ( $\lambda$ ). This type is called the chromatic dispersion that defines as [12]

waveguide dispersion that depend basically on  $g$  as



well as to  $V$ . Mathematically accounted by expanded the propagation constant by Taylor series around a

$$D_M = -\frac{\lambda}{c} \frac{d^2 n_{eff}}{d\lambda^2} \tag{7}$$

where  $c$  is the velocity of light in vacuum,  $n_{eff}$  is the effective refractive-index that has an analogous meaning for light propagation in a waveguide with

$$n_{eff} = \begin{cases} n_1 & \text{if } V \rightarrow \infty \\ n_2 & \text{if } V \rightarrow V_o \end{cases} \tag{8}$$

Using Eqs.(1) and (3), (8) may be rewritten as

$$n_{eff} = \sqrt{n_1^2 - (n_1^2 - n_2^2)(2a/V)^2} \tag{9}$$

where  $V_o$  is the cut-off frequency. That means; when  $g \rightarrow \infty$  the effect of core will be dominated, whereas  $g \rightarrow 0$  dominates the effect of cladding. In other words, the characteristics of material

$$n(w) = n_o(w) + N_2 |E|^2 \tag{10}$$

where  $n_o$  is the linear refractive-index,  $N_2$  is the non-linear refractive-index (nonlinear Kerr parameter),  $|E|^2 = P/A_{eff}$  is the optical light intensity inside the fiber,  $A_{eff}$  is the effective cross-section area and  $P$  is the light power inside fiber.

$$A_{eff} = \frac{\left[ \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} |F(x, y)|^2 dx dy \right]^2}{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} |F(x, y)|^4 dx dy} \tag{11}$$

where  $F(x, y)$  is the modal distribution of the fundamental mode. Obviously,  $A_{eff}$  depends on fiber parameters such as ( $a$  and  $\Delta$ ). If  $F(x, y)$  is approximated by a Gaussian distribution, the effective area will be  $A_{eff} = \pi w^2$ , where  $w$  is the

$$\gamma = \frac{2\pi N_2}{\lambda A_{eff}} \tag{12}$$

Our study is based on the effects of fractional graded order. Hence, we are going to use FEM in COMSOL environment to solve the wave equation through optical fibers. Knowing that the FEM are used a single value for the normalized frequency to determine the associated effective refractive index. So, the process will repeat dozens of times to draw the dispersion relationship for each graded order. In turn, the associated parameters of dispersion, effective area and nonlinearity will be determined.

center frequency  $w_o$ , then the martial dispersion will be [4,12]

restricted transverse extension as  $n_2 \leq n_{eff} \leq n_1$ .

In fact, fraction of the mode will propagate in the cladding, so  $n_{eff}$  can express as [13]

dispersion will control by the graded order.

Most of the nonlinear effects in optical fibers originate from nonlinear refraction, at high pulse intestines the n dependent on intensity as

The distribution of light along the fiber depends on the cross-section area. The parameter  $A_{eff}$  is known as the effective mode area that is very important for nonlinear manifestation, which is defined as [14]

mode width parameter. The width parameter  $w$  depends on the  $V$  parameter of fiber. The another important nonlinear parameter, which inversely proportional with the effective area, is called the nonlinearity factor that is defined as

### Results and Discussion

The graded order in the optical fiber controls the spot size and number of modes furthermore dispersion and nonlinearity. That properties are studied mathematically by taking the different graded orders includes the step-index fiber (infinity graded order) and some of the integer values of the graded order. Complex mathematics are used to calculate the integer graded orders whereas



fractional one is more complexity and not studied yet. So, to achieve that we using finite element method in COMSOL environment makes it easier. The radius of core, clad and the maximum refractive index are  $a = 8\mu\text{m}$  and  $b = 30\mu\text{m}$  respectively,  $n_1 = 1.4378$ , and  $\Delta = 0.02$  are chosen to be constant at calculations.

Fig.(1) shows the refractive index profile through the core using different graded orders. Whereas, graded order 1 represents a straight line for the refractive index to change from the top until we reach the value of the refractive index of the clad at the boundary between the core and the clad. Graded order greater than 1 it is tend to convex gradually until we reach an step change at the infinity graded order. For graded order less than 1, the shape tends to concave, and concavity increases with the graded order decreasing. At high values of graded orders the effective refractive index is expected to be closer to the top of the refractive index and for small values it tends to the clad value. Fig.(2) represents a different samples of the fundamental mode at different graded orders and wavelengths at all the graded orders. It is clear from the figure that the spot size of mode increases with increasing wavelength, while changing the graded order will lead to a decrease or increase in the size of the spot size. We will show later that the lowest spot size for different lengths ranges within the range (0.44 -1) of graded orders.

Fig.(3) represents the effective area as a function of graded order using different wavelengths. It is evident from the figure that the effective area decreases to a small value within the range (0.44 - 1) of graded order and then returns to increase and settles at high graded order values. On the other hand, we notice that the lower wavelength achieves less effective area and greater effective area for the greater wavelength.

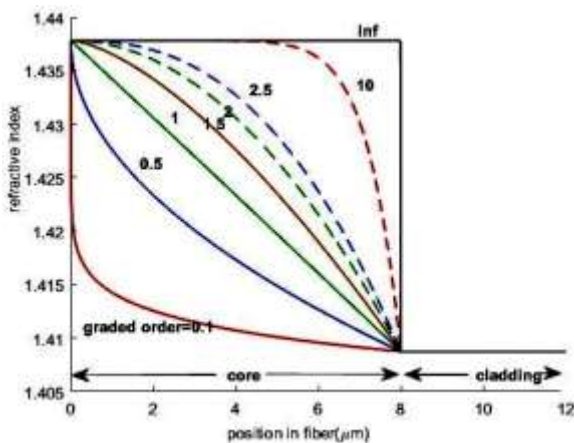


Fig. 1. Refractive index profiles for different graded orders

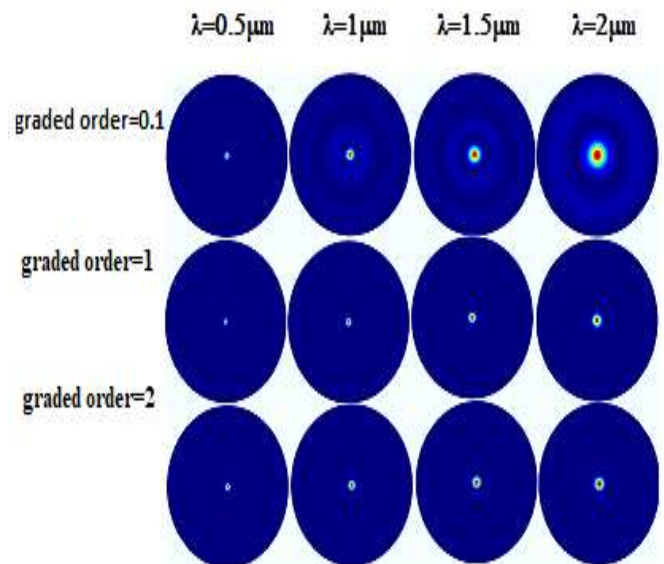


Fig. 2. Different samples of the fundamental mode

Fig.(4) represents the effective refractive index as a function of the graded order for different wavelengths. In general, the effective refractive index of begins  $n_{clad}$  at low graded order values and increases until it stabilizes at  $n_{core}$  for high graded order values. The change is almost exclusively in graded orders less than 10, and then stabilizes at an constant value close to the refractive index  $n_{core}$ . Note that the lower wavelength achieves the high effective refractive index and vice versa.

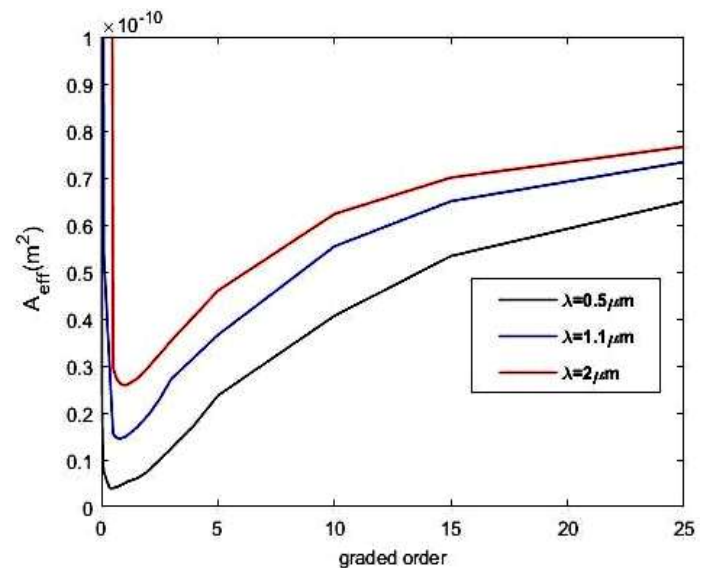


Fig. 3. Effective area as a function of graded order for different wavelengths

Fig.(5) represents the nonlinearity coefficient as a function of graded order for different wavelengths. It is clear from the figure that the nonlinearity takes a great value at the range (0.44 -1) of graded order



and then returns to stability at a fixed value with an increase in the degree of graded order. Note that the lower wavelength achieves a higher nonlinearity coefficient and vice versa, and the nonlinearity values are large at lower graded order values. Fig.(6) represents the graded order that makes the largest nonlinear coefficient as a function of wavelength. It is evident from the figure that the relationship is semi-linearly increasing. That is; the graded order that produces the largest nonlinearity increases almost linearly with wavelength. For the wavelength range used, we notice that the degree of graded orders extent is limited in the range (0.44-1). Fig.(7) represents the highest nonlinearity achieved within the range of different graded orders as a function of wavelength. From it we notice that the greatest nonlinearity decreases exponentially with increasing wavelength. In other words, to achieve high nonlinearity, we can use small wavelengths and graded order values within the range (0.44-1). Fig.(8) represents the effective refractive index as a function of the wavelength for different gradient degrees. We note that the highest values of the effective refractive index are achieved at high graded orders, while the low graded orders will achieve a low effective refractive index, and this is due to the refractive index profile of core in Fig.(1) for different graded orders. In addition, the graded clearly controls the relationship of the effective refractive index with the wavelength, where we notice that the shape of the curve changes a lot with the graded order change. Fig.(9) represents the material dispersion curves for various graded orders. It is evident in the figure that the change of graded order strongly affects the shape of the material dispersion curve. For small graded order, a negative material dispersion curve can be obtained for most wavelengths. At the graded order 2, the relation between the effective refractive index and wavelength is perfect linear, so the material dispersion becomes zero for all wavelengths, and this accompanies the elimination of the multi-mode dispersion. For higher graded orders, the material dispersion curve tends to positive values and its change becomes slight after the graded order 10.

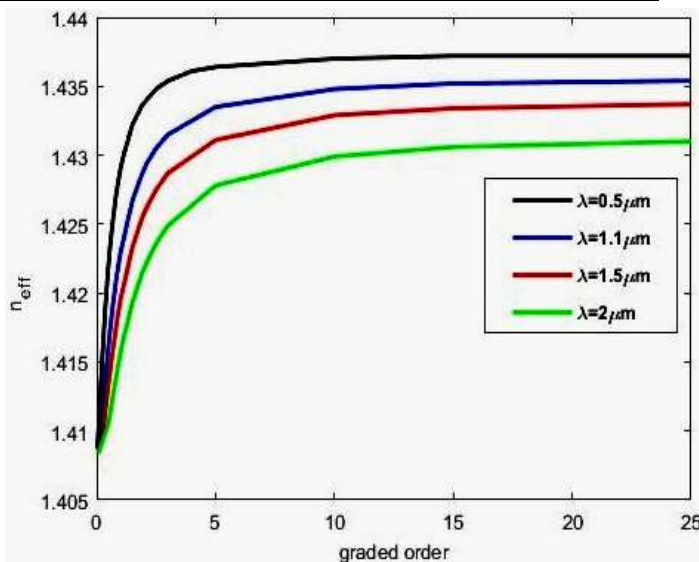


Fig. 4. Effective refractive index as a function of graded order for different wavelengths

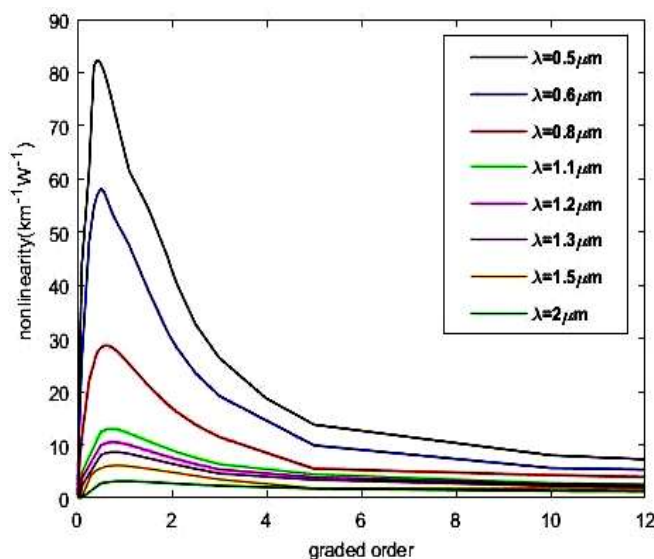


Fig. 5. Nonlinearity as a function of graded order for different wavelengths

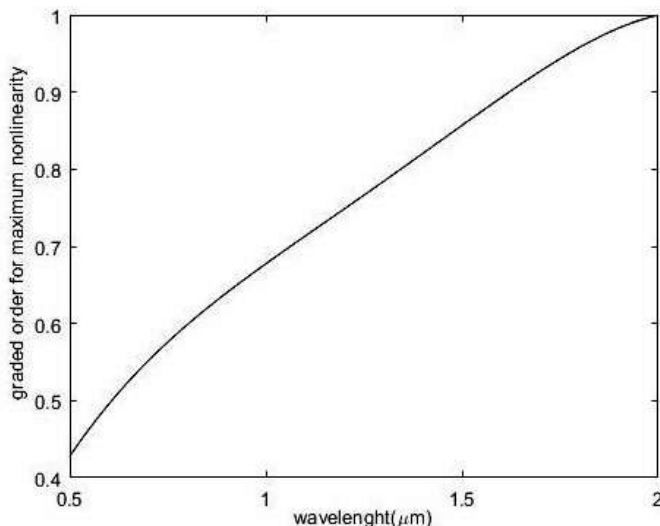


Fig. 6. Graded order for maximum nonlinearity as a function of wavelengths



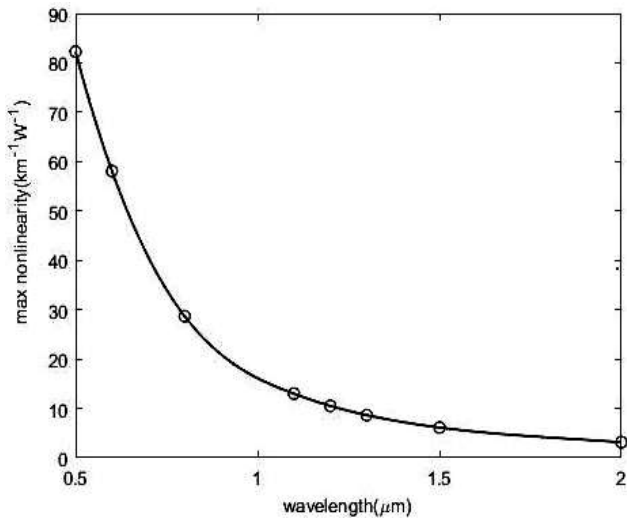


Fig. 7. Maximum nonlinearity as a function of wavelngths

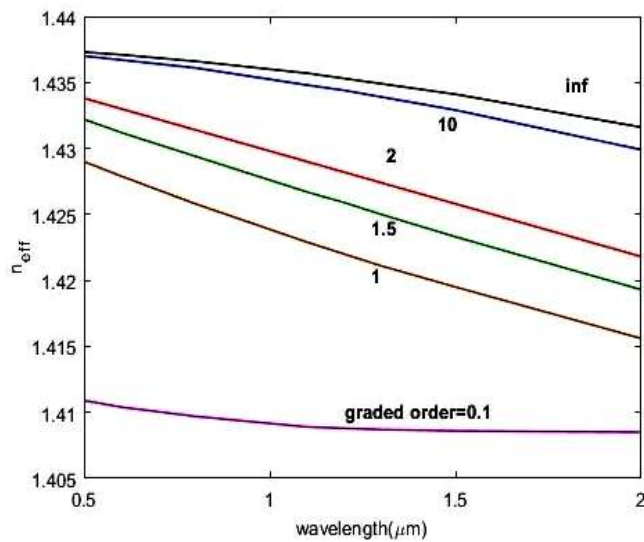


Fig. 8. Effective refractive index as a function of wavelngths for different graded orders

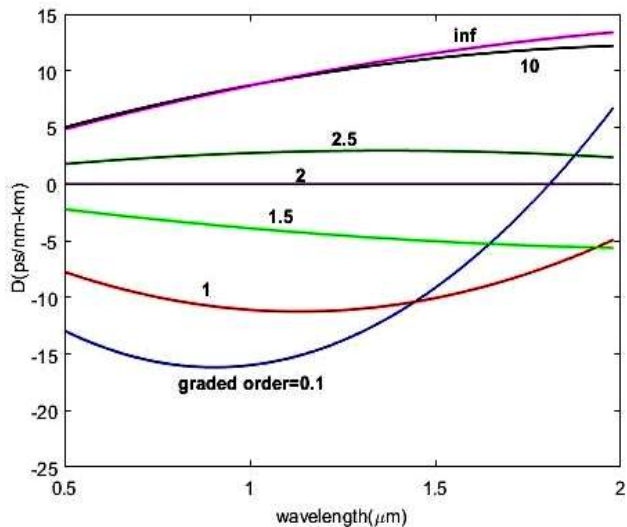


Fig. 9. Material dispersion for different graded orders

## Conclusions

As a conclusion, the properties of modes (particularly the fundamental mode) with the variation of wavelength and graded order can obtain the desired nonlinearity and dispersion values. The appropriate change is concentrated in the small wavelengths and graded order in the range (0.44-1).

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