An Origami Graph with One Path on the Outer Vertex Has Locating-Chromatic Number Four

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Abstract:
The locating-chromatic number denote by $\chi_L(G)$, is the smallest s such that G has a locating s-colouring. In this research, we study the locating-chromatic number of an Origami graph with one path on the outer vertex. An origami graph with one path on outer vertex namely $L^1_{O_n}$ is a graphs with $V(L^1_{O_n}) = \{u_i, v_i, w_i, x_i, y_i : i \in \{1, \dots, n\}\}$ and $E(L^1_{O_n}) = \{u_iw_i,\ u_iv_i,\ v_iw_i, v_ix_i, w_iy_i : i \in \{1, \dots, n\}\} \cup \{u_iu_{i+1},\ w_iu_{i+1}\ : i \in \{1, \dots, n-1\}\} \cup \{u_nu_1,\ w_nu_1\}$. Also, we determined the locating-chromatic number of an Origami graph with one path on the outer vertex.

Keywords: locating-chromatic number, origami, graphs, one path
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1. INTRODUCTION

The concept of metric dimension was first introduced by Slater [1] in 1975. Then Melter and Harary [2] in 1976 gave the concept of the metric dimension of a graf sort known at this time. Application metric dimension can be seen in some areas, such as navigation robots that are modeled by graf [3], the chemical compound classification [4], and optimizing the threat detection sensors [5]. The partition dimension of a graph is given by Chartrand et al [6] which is an extension of the concept of metric dimension. The locating-chromatic number of a graph was first discovered by Chartrand et al. [7] which is a combination of two graph concepts, colouring vertices and partition dimension of a graph. The locating-chromatic number of a graph denote by $\chi_L(G)$, is the smallest s such that G has a locating s-colouring. Furthermore, in 2003 Chartrand et al. [8] they constructed a tree of order $n \geq 5$ which has a locating-chromatic number s where $s \in (3, 4, \dots, n-2, n)$.

Furthermore, it has been determined the locating-chromatic number for several types of graphs, including Asmiati and Baskoro [9] have characterized all graphs containing cycles. Behtoei dan Anbarloei [10] is the joining of two arbitrary graphs. Purwasih et al. [11] subdivide the graph on one side. Syofyan et al. [12] homogeneous lobster graph. Furthermore Welyyanti et al. [13] complete $an\ n$-ary tree graph. Syofyan et al. [14] certain tree graph. Then Syofyan et al. [15] tree embedded in a 2-dimensional grid.

The locating-chromatic number of certain barbell graphs on complete graphs and generalized Petersen graphs have been determining by Asmiati et al. [16]. For rainbow connection number of Origami graph O_n, has been discovered by Nabila and Salman. [17]. In 2021 Irawan et al.[18] has succeeded determine the locating-chromatic number of O_n and subdivision on the outer edge. Next, Irawan et al. [19, 20] determined certain barbell operations of Origami graphs and subdivision of certain barbell operations of Origami graphs obtained by locating-chromatic number 5. Motivated by that, in this research, we will determine the locating-chromatic number of the Origami graph with one path on the outer vertex, namely $L^1_{O_n}$ for $n = 3,4,5$. The definition of the following Origami graph is taken from [17].

Definition 1.1. [17] An origami graph O_n, $n \geq 3$ is a graph with $V(O_n) = \{u_i, v_i, w_i\colon i \in [1,n]\}$ and $E(O_n) = \{u_iw_i, u_i v_i, v_iw_i\colon i \in [1,n]\} \cup \{u_iu_{i+1}, w_iu_{i+1}: i \in [1, n-1]\} \cup \{u_nu_1, w_nu_1\}$.

To work out the lower bound of the locating-chromatic number of a graph, we use the basic theorems taken from [7, 18]. The set of neighbors of vertex b in G, denoted by $N(b)$.

Theorem 1.1 [7] Let c be a locating-coloring in a connected graph G. If a and b are different vertices of G such that $d(a,w) = d(b,w)$ for all $w \in V(G) - \{a,b\}$, then $c(a) \neq c(b)$. In particular, if a and b are non-adjacent vertices of G such that $N(a) \neq N(b)$, then $c(a) \neq c(b)$.

Theorem 1.2 [18] $\chi_L(O_n) = \begin{cases} 4, & 3 \leq n \leq 6 \\ 5, & \text{otherwise.} \end{cases}$

2. RESEARCH METHODS

The method used to determine the locating-chromatic number of Origami graph with one path at the outer vertex, namely $L^1_{O_n}$, for $n = 3,4,5$ as follows :

1) Define an Origami graph with one path on the outer vertex for $n \in \mathbb{N}$, with $n \geq 3$.
2) Determine the colour classes of $L^1_{O_n}$, for $n = 3,4,5$ through the approximation locating-chromatic number of a graph.
3) Determine the lower bound of $\chi_L(L^1_{O_n})$ for $n = 3,4,5$.
4) Determine the upper bound of $\chi_L(L^1_{O_n})$ for $n = 3,4,5$.
5) Formulate the results obtained in the form of theorems and prove them. This theorem contains the locating-chromatic number of the Origami graph with one path on the outer vertex $\chi_L(L^1_{O_n})$ for $n = 3,4,5$.

3. MAIN RESULTS

In this part, we will determine the locating-chromatic number of the Origami graph with one path on the outer vertex $L^1_{O_n}$ for $n = 3,4,5$. First, we define an origami graph with one path on the outer vertex. Let $n \in \mathbb{N}$, with $n \geq 3$. An origami graph with one path on outer vertex $L^1_{O_n}$ is a graphs with $V(L^1_{O_n}) = \{u_i, v_i, w_i, x_i, y_i\colon i \in \{1, \dots, n\}\}$ and $E(L^1_{O_n}) = \{u_iw_i, u_iv_i, v_iw_i, v_ix_i, w_iy_i\colon i \in \{1, \dots, n\}\} \cup \{u_iu_{i+1}, w_iu_{i+1} : i \in \{1, \dots, n-1\}\} \cup \{u_nu_1, w_nu_1\}$.

Theorem 2.1 $\chi_L(L^1_{O_3}) = 4$.

Proof. The first, we assign the lower bound of $\chi_L(L^1_{O_3})$. Let $L^1_{O_3}$, with $n \geq 3$ Origami graph with one path on the outer vertex, with $V(L^1_{O_3}) = \{u_i, v_i, w_i, x_i, y_i\colon i \in \{i = 1, 2, 3\}\}$ and $E(L^1_{O_3}) = \{u_iw_i, u_iv_i, v_iw_i, v_ix_i, w_iy_i\colon i = 1,2,3\} \cup \{u_iu_{i+1}, w_iu_{i+1} : i = 1,2\} \cup \{u_3u_1, w_3u_1\}$. An Origami graph with one path on the outer vertex $L^1_{O_3}$ contains Origami graph O_3, then by Theorem 1.2. $\chi_L(L^1_{O_3}) \geq 4$.

Furthermore, we assign the upper bound of $L^1_{O_3}$. To show that $\chi_L(L^1_{O_3}) \leq 4$, consider the 4-colouring c on $L^1_{O_3}$ as follow,

$C_1 = \{u_1, x_1, y_1\}$;
$C_2 = \{u_3, v_2, x_3, y_3\}$;
$C_3 = \{u_2, v_1, v_3, x_2, y_2\}$;
$C_4 = \{w_1, w_2, w_3\}$.

The colouring c will build partition Π on $V(L^1_{O_3})$. We will show that the colour codes of all vertices in $L^1_{O_3}$ are distinct. We have $c_\Pi(u_1) = (0,1,1,1)$; $c_\Pi(u_2) = (1,1,0,1)$;$c_\Pi(u_3) = (1,0,1,1)$; $c_\Pi(v_1) = (0,2,0,1)$; $c_\Pi(v_2) = (2,0,1,1)$; $c_\Pi(v_3) = (2,1,0,1)$; $c_\Pi(w_1) = (1,2,1,0)$;$c_\Pi(w_2) = (2,1,1,0)$; $c_\Pi(w_3) = (1,1,1,0)$; $c_\Pi(x_1) = (0,2,1,2)$; $c_\Pi(x_2) = (3,1,0,2)$; $c_\Pi(x_3) = (3,0,1,2)$; $c_\Pi(y_1) = (0,3,2,1)$;$c_\Pi(y_2) = (3,2,1,0)$; $c_\Pi(y_3) = (2,0,2,1)$. Because the colour codes of all vertices $L^1_{O_3}$ are distinct, thereby c is a locating-colouring. So $\chi_L(L^1_{O_3}) \leq 4$.

□

Fig.1 is illustrated a locating-colouring of an Origami graph with one path on the outer vertex $L^1_{O_3}$ with $\chi_L(L^1_{O_3}) = 4$.

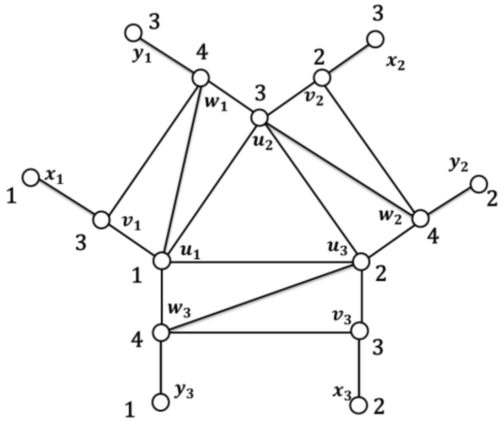

Fig.1 A graph $L^1_{O_3}$ with $\chi_L(L^1_{O_3}) = 4$

Teorema 2.2 $\chi_L(L^1_{O_4}) = 4$.

Proof. The first, we assign the lower bound of $\chi_L(L^1_{O_4})$. Let $L^1_{O_4}$, with $n \geq 3$ Origami graph with one path on the outer vertex, with $V(L^1_{O_4}) = \{u_i, v_i, w_i, x_i, y_i : i = 1,2,3,4\}$ and $E(L^1_{O_4}) = \{u_iw_i, u_iv_i, v_iw_i, v_ix_i, w_iy_i : i = 1,2,3,4\} \cup \{u_iu_{i+1}, w_iu_{i+1} : i = 1,2,3\} \cup \{u_4u_1, w_4u_1\}$. An Origami graph with one path on the outer vertex $L^1_{O_4}$ contains Origami graph O_4, then by Theorem 1.2. $\chi_L(L^1_{O_4}) \geq 4$.

Furthermore, we assign the upper bound of $L^1_{O_4}$. To show that $\chi_L(L^1_{O_4}) \leq 4$, consider the 4-colouring c on $L^1_{O_4}$ as follow,

$C_1 = \{v_1, w_4\}$;
$C_2 = \{u_1, u_3, v_2, x_1, y_1, x_3, y_3\}$;
$C_3 = \{u_2, u_4, v_3, x_2, y_2, x_4, y_4\}$;
$C_4 = \{v_4, w_1, w_2, w_3\}$.

The colouring c will build partition Π on $V(L^1_{O_4})$. We will show that the colour codes of all vertices in $L^1_{O_4}$ are distinct. We have $c_\Pi(u_1) = (1,0,1,1)$; $c_\Pi(u_2) = (2,1,0,1)$;$c_\Pi(u_3) = (2,0,1,1)$; $c_\Pi(u_4) = (1,1,0,1)$; $c_\Pi(v_1) = (0,2,0,1)$; $c_\Pi(v_2) = (3,0,1,1)$; $c_\Pi(v_3) = (3,1,0,1)$;$c_\Pi(v_4) = (1,2,0,1)$; $c_\Pi(w_1) = (1,1,1,0)$; $c_\Pi(w_2) = (3,1,1,0)$; $c_\Pi(w_3) = (2,1,1,0)$; $c_\Pi(w_4) = (0,1,1,1)$; $c_\Pi(x_1) = (1,0,3,2)$; $c_\Pi(x_2) = (4,1,0,2)$; $c_\Pi(x_3) = (4,0,1,2)$; $c_\Pi(x_4) = (2,3,0,1)$; $c_\Pi(y_1) = (2,0,2,1)$;$c_\Pi(y_2) = (4,2,0,1)$; $c_\Pi(y_3) = (3,0,2,1)$; $c_\Pi(y_4) = (2,0,2,1)$. Because the colour codes of all vertices, $L^1_{O_4}$ are distinct, thereby c is a locating-colouring. So $\chi_L(L^1_{O_4}) \leq 4$.

□

Fig. 2 is illustrated a locating-colouring of an Origami graph with one path on the outer vertex $L^1_{O_4}$ with $\chi_L(L^1_{O_4}) = 4$.

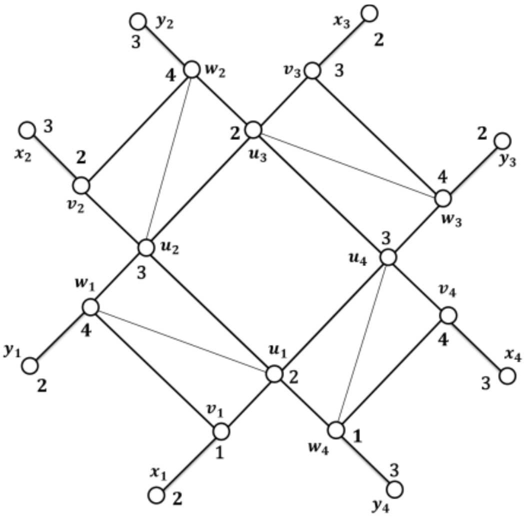

Fig.2 A graph $L^1_{O_4}$ with $\chi_L(L^1_{O_4}) = 4$

Teorema 2.3 $L^1_{O_5}$, $\chi_L(L^1_{O_5}) = 4$.

Proof. The first, we assign the lower bound of $\chi_L(L^1_{O_5})$. Let $L^1_{O_5}$, with $n \geq 3$ Origami graph with one path on the outer vertex, with $V(L^1_{O_5}) = \{u_i, v_i, w_i, x_i, y_i : i = 1, 2, 3,4,5\}$ and $E(L^1_{O_5}) = \{u_i w_i, u_i v_i, v_i w_i, v_i x_i, w_i y_i : i = 1,2,3,4,5\} \cup \{u_i u_{i+1}, w_i u_{i+1} : i = 1,2,3,4\} \cup \{u_5 u_1, w_5 u_1\}$. An Origami graph with one path on the outer vertex $L^1_{O_5}$ contains Origami graph O_5, then by Theorem 1.2. $\chi_L(L^1_{O_5}) \geq 4$.

Furthermore, we assign the upper bound of $L^1_{O_5}$. To show that $\chi_L(L^1_{O_5}) \leq 4$, consider the 4-colouring c on $L^1_{O_5}$ as follow,

$C_1 = \{u_1, u_4, v_5; x_1, x_4, y_5\}$;
$C_2 = \{u_2, v_1, v_3, x_2, y_1\}$;
$C_3 = \{u_3, u_5, v_2, v_4, x_3, y_2, x_5, y_4\}$;
$C_4 = \{w_1, w_2, w_3, w_4, w_5\}$.

The colouring c will build partition Π on $V(L^1_{O_5})$. We will show that the colour codes of all vertices in $L^1_{O_5}$ are distinct. We have $c_\Pi(u_1) = (0,1,1,1)$; $c_\Pi(u_2) = (1,0,1,1)$; $c_\Pi(u_3) = (1,1,0,1)$; $c_\Pi(u_4) = (0,2,1,1)$; $c_\Pi(u_5) = (1,2,0,1)$; $c_\Pi(v_1) = (1,0,2,1)$; $c_\Pi(v_2) = (2,1,0,1)$;$c_\Pi(v_3) = (2,0,1,1)$; $c_\Pi(v_4) = (1,3,0,1)$; $c_\Pi(v_5) = (0,3,1,1)$; $c_\Pi(w_1) = (1,1,2,0)$; $c_\Pi(w_2) = (2,1,1,0)$;$c_\Pi(w_3) = (1,1,1,0)$; $c_\Pi(w_4) = (1,3,1,0)$; $c_\Pi(w_5) = (1,2,1,0)$; $c_\Pi(x_1) = (0,1,3,2)$; $c_\Pi(x_2) = (3,0,1,2)$; $c_\Pi(x_3) = (3,1,0,2)$; $c_\Pi(x_4) = (0,4,1,2)$; $c_\Pi(x_5) = (1,4,0,2)$; $c_\Pi(y_1) = (2,0,3,1)$;$c_\Pi(y_2) = (3,2,0,1)$; $c_\Pi(y_3) = (0,2,2,1)$; $c_\Pi(y_4) = (2,4,0,1)$; $c_\Pi(y_5) = (0,3,2,1)$. Because the colour codes of all vertices $L^1_{O_5}$ are distinct, thereby c is a locating-colouring. So $\chi_L(L^1_{O_5}) \leq 4$.

□

Fig. 3 is illustrated a locating-colouring of an Origami graph with one path on the outer vertex $L^1_{O_5}$ with $\chi_L(L^1_{O_5}) = 4$.

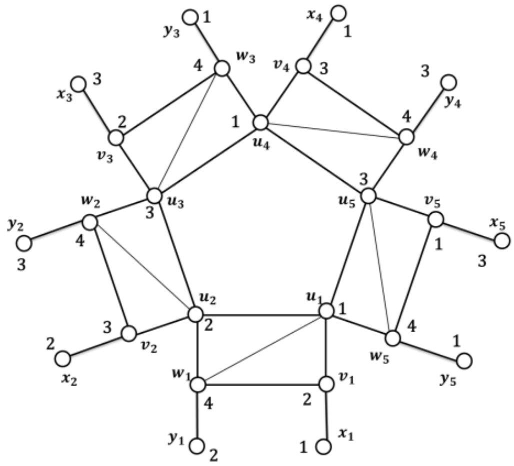

Fig.3 A graph $L^1_{O_5}$ with $\chi_L(L^1_{O_5}) = 4$

4. CONCLUSIONS

In this paper, the author examines the locating-chromatic number of Origami graphs with one path on the outer vertex $L^1_{O_n}$ for $n = 3,4,5$ where the following results are obtained:

- $\chi_L(L^1_{O_3}) = 4$
- $\chi_L(L^1_{O_4}) = 4$
- $\chi_L(L^1_{O_5}) = 4$

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